Line segment similarity criterion for vector images

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ABSTRACT

Vector representation of the images, maps, schematics and other information is widely used, and in computer processing of these data, comparison and similarity evaluation of two sets of line segments is often necessary. Various techniques are already in use, but these mostly rely on the algorithmic functions such as minimum/maximum of two or more variables, which limits their applicability for many optimization algorithms. In this paper we propose a novel area based criterion function for line segment similarity evaluation, which is easily differentiable and the derivatives are continuous in the whole domain of definition. The second important feature is the possibility of preprocessing of the input data. Once finished, it takes constant time to evaluate the criterion for different transformations of one of the input sets of line segments. This has potential to greatly speed up iterative matching algorithms. In such case, the computational complexity is reduced from $O(pt)$ to $O(p+t)$, where p is the number of line segment pairs being examined and t is the number of transformations performed.

Keywords
Vector, Line Segment, Similarity, Distance, Criterion.

1 INTRODUCTION

Evaluating similarity of two scenes is a frequent task to deal with in many technical applications. Due to inevitable error of the measuring devices and dynamic nature of the world around us, we can rarely (if ever) get two exactly same results and test them for equality. Instead, we usually get similar results in similar situations, and this is where some metrics of similarity becomes essential. A human observer usually spots the similarity unconsciously due to our evolutionarily developed sense for pattern matching, but a machine, which did not have millions of years for its development, is reliant on numerically evaluable algorithms.

Although the notion of similarity may look straightforward at first, it is a quite complex task to solve in general. Some situations require invariance to all affine transformations [1]. Other applications require invariance only to a subset of possible transformations as described in [2] and [3], where invariance only to rigid transformations and scaling is demanded. On the other hand, all transformations are important in motion estimation applications [4], [5], path planing [6] and simultaneous localization and mapping (SLAM) problem in robotics [7]. Many applications exist for 3D object detection [8] and 3D scene matching [9], where incomplete line segments often appear and the similarity criterion should take this into account. Another set of algorithms is used for polygons [10] or polyline curves [11]. Mathematically well described is the Frechet distance [12], but its domain of operation are polylines and in case of isolated line segments it simplifies to bare comparison of distances, similar to algorithms above.

Wideness of requirements being put on the similarity criterion leads to a set of algorithms with different properties for different situations, rather than a single overcomplicated method for everything. Specialization also enables deeper optimization, which is necessary in time and resource critical applications. Our research is focused on mobile robotics and SLAM, which requires 2D similarity criterion variant to all transformations with zero output for any two line segments lying on the same line. This behaviour is of a great importance, because the robot, due to an obstructed view, rarely observes an object as a whole. Partial information about the edges of the surrounding objects results into uncertainty about their real dimensions, because we do not know, which part of the real edge has the robot actually sensed. This uncertainty is expressed as a perfect match anywhere along a line defined by an infinite extension of the static line segment. Only two or more skew line segments can solve this ambiguity and define a single transformation between the new and the static data.

Solutions mentioned above represent possible approaches, but are hard to describe mathematically because of inbuilt conditional statements and require repeated recomputation in the iterative fitting algorithms, which limits performance significantly. The method described further in this paper satisfies the requirements and solves the issues described.

2 STATE OF THE ART

The most relevant criteria to our demands are based on the Hausdorff distance. Valuable comparison of the established line segment distance functions [13] evaluates three of these similarity metrics.

Basic Hausdorff distance function for line segments, as described in [10], is defined as:

$$\begin{aligned} d_{l_1,l_2}(l_1,l_2) &= \sup_{\boldsymbol{P}\in l_1} \inf_{\boldsymbol{Q}\in l_2} d(\boldsymbol{P},\boldsymbol{Q}), \\ d_{l_2,l_1}(l_1,l_2) &= \sup_{\boldsymbol{P}\in l_2} \inf_{\boldsymbol{Q}\in l_1} d(\boldsymbol{P},\boldsymbol{Q}), \end{aligned} \tag{1}$$

$$d_{crit}(l_1,l_2) = \max(d_{l_1,l_2}, d_{l_2,l_1}), \tag{2}$$

where $d(\mathbf{P},\mathbf{Q})$ is some distance metric (Euclidean in most cases), d_{l_1,l_2} is longest perpendicular distance from l_2 to l_1 and vice versa in the second case. The criterion distance is the higher value of these two.

Modified Hausdorff line segment distance originates from [14] and the definition is as follows:

$$d_{crit}(l_1,l_2) = \min(\| l_1 \|, \| l_2 \|)\sin(\alpha), \tag{3}$$

where $\| l_x \|$ denotes length of the particular line segment and α is the angle formed by l_1 and l_2.

Modified perpendicular line segment Hausdorff distance [11] relies on the perpendicular distance between an end point of one line segment and the corresponding line segment as a whole. If a line segment is described as $l_x = \{\boldsymbol{P}_{x1}, \boldsymbol{P}_{x2}\}$ then the perpendicular distance can be written as $d_\perp(l_x, \boldsymbol{P}_{yz})$, where $x,y,z \in \{1,2\}$ are appropriate indices. The criterion is then defined by the equations:

$$\begin{aligned} d_{\perp 1} &= \min(\max(d_\perp(l_1,\boldsymbol{P}_{21}), d_\perp(l_1,\boldsymbol{P}_{22})), \\ &\qquad \max(d_\perp(l_2,\boldsymbol{P}_{11}), d_\perp(l_2,\boldsymbol{P}_{12}))), \\ d_{\perp 2} &= \min(\min(d_\perp(l_1,\boldsymbol{P}_{21}), d_\perp(l_1,\boldsymbol{P}_{22})), \\ &\qquad \min(d_\perp(l_2,\boldsymbol{P}_{11}), d_\perp(l_2,\boldsymbol{P}_{12}))), \end{aligned} \tag{4}$$

$$w_i = \frac{d_{\perp i}}{d_{\perp 1} + d_{\perp 2}} \quad \text{for } i = \{1,2\}, \tag{5}$$

$$d_{crit}(l_1,l_2) = \frac{1}{2}\left(w_1 d_{\perp 1} + w_2 d_{\perp 2}\right). \tag{6}$$

Many other criteria can be found in the literature, but although they fulfil slightly different requirements, in general, the concept is quite similar - usage of distances between points, rarely the angle of the examined line segment pair, all of them combined using min()/max() functions. Basic Hausdorff distance (2), and many others not mentioned here, does not even give zero results for line segments lying on the same line. On the other hand, modified version of this criterion (3) gives zero results for every collinear pair of line segments, which is not desirable as well.

The mentioned criteria definitely fulfil the task they were designed for, but the properties do not meet our requirements and their formulation prevents possible optimizations for better performance. The next section describes a novel criterion, which overcomes these limitations.

3 AREA BASED CRITERION FOR LINE SEGMENT SIMILARITY

The main thought behind the design of the presented criterion is following: If the similarity of two points (zero dimensional objects) in higher dimensional spaces is a distance, then the similarity of two line segments (1D objects) should be defined by an area (in more than one dimensional spaces). For the purpose of the criterion, which should return zero as an extremum for a certain input, we are going to work with the square of the area. This approach deals with a possible negative sign of the area without need to employ an absolute value and ensures, that a zero is the minimal possible output of the computation.

Let us have two arbitrary line segments $\boldsymbol{AB}$ and $\boldsymbol{CD}$, as depicted in the following Figure 1.

One of the requirements stated above demands zero output, if both examined line segments belong to the same line. This is satisfied by the squared area of the parallelogram defined by the vectors $\mathbf{x}$ and $\boldsymbol{y}$ (as depicted in Figure 1):

$$S^2_{ABCD}(l_1,l_2) = \| (\boldsymbol{B}-\boldsymbol{A}) \times (\boldsymbol{D}-\boldsymbol{C}) \|^2 = \| \boldsymbol{x} \times \boldsymbol{y} \|^2. \tag{7}$$

To keep the notation lucid, we use a magnitude of a cross product even for a 2D problem. For every cross-product in this paper a condition of zero z coordinate for any vector involved is applied. A brief intuitive examination of the equation (7) reveals the same weakness as has the Modified Hausdorff distance criterion (3): The

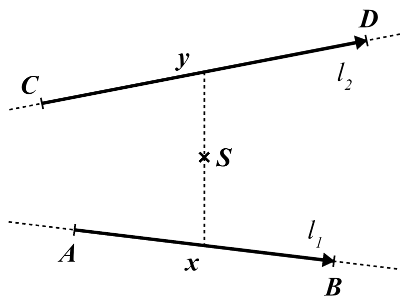

Figure 1: An example of two line segments l_1 defined by points $\boldsymbol{AB}$ and l_2 defined by points $\boldsymbol{CD}$ and the middle point $\boldsymbol{S}$. Further equations rely on a substitution: $\mathbf{x} = \boldsymbol{B} - \boldsymbol{A}$ and $\boldsymbol{y} = \boldsymbol{D} - \boldsymbol{C}$.

result is zero for any collinear pair $\{l_1, l_2\}$. To obtain fully functional criterion, this rule is further narrowed by introduction of the squared area of the triangle $\boldsymbol{ABS}$, which is defined by:

$$S_{ABS}^2(l_1, l_2) = \frac{1}{4} \parallel (\boldsymbol{B} - \boldsymbol{A}) \times (\boldsymbol{S} - \boldsymbol{A}) \parallel^2, \tag{8}$$

where $\boldsymbol{S} = (\boldsymbol{A} + \boldsymbol{B} + \boldsymbol{C} + \boldsymbol{D})/4$. This function is zero for any pair $\{l_1, l_2\}$, where $((\boldsymbol{C} + \boldsymbol{D})/2) \in (\boldsymbol{A} + t\mathbf{x})$ and $t \in \mathbb{R}$.

By summation of the equations (7) and (8) we get the final criterion:

$$S_{crit}(l_1, l_2) = S_{ABCD}^2 + 64 S_{ABS}^2. \tag{9}$$

Coefficient 64 balances the influence of both parts of the criterion, because S_{ABCD} is significantly larger than S_{ABS}. The exact value of the constant is justified in the following subsection, where the criterion is examined deeper.

3.1 Properties of the criterion

So far, the criterion was developed using intuitive understanding of the geometrical properties of the cross product, but this can hardly prove the concept to be working at all conditions.

To provide a mathematical proof of the existence and shape of the minimum of the criterion function, we need to examine every possible mutual position and length of both line segments l_1 and l_2, which means eight variables in total. Situation can be simplified by understanding the geometrical properties of the criterion. Both area functions (7) and (8) are independent of the position of the origin, because all vectors resulting form the subtractions become invariant to translation. Invariance of the area functions with respect to rotation of both line segments is evident from an alternative form of equation (7):

$$S_{ABCD}^2(l_1, l_2) = \parallel \boldsymbol{x} \times \boldsymbol{y} \parallel^2 = \parallel \boldsymbol{x} \parallel^2 \parallel \boldsymbol{y} \parallel^2 \sin^2(\alpha). \tag{10}$$

Since lengths of the vectors and the angle between them are not affected by the rotation of the whole pair and formula (8) can be rewritten in the exact same way, we can claim, that the criterion provides results independent on that kind of transformation.

The value of the criterion function (9) is definitely dependent on the scale, but for the purpose of minimum search, that dependency is irrelevant. Multiplying a parabolic function by a constant does not affect the location of its minimum.

Combination of the previous findings implies a remarkable simplification of the minimum search task. Thanks to the invariance of the criterion to the translation and rotation and omission of the scale factor, we can fix one line segment at a constant position and examine only the remaining four independent variables defining position and length of the second one. Minimum of the criterion function (9) is then given by the solution of the following system of equations:

$$\begin{aligned} \frac{\partial S_{crit}}{\partial C_x} &= 0, \\ \frac{\partial S_{crit}}{\partial C_y} &= 2(C_y - D_y) + 2(C_y + D_y) = 0, \\ \frac{\partial S_{crit}}{\partial D_x} &= 0, \\ \frac{\partial S_{crit}}{\partial D_y} &= -2(C_y - D_y) + 2(C_y + D_y) = 0, \\ &\text{all for } l_1 = \{\boldsymbol{A}, \boldsymbol{B}\} = \{\{0,0\},\{1,0\}\}. \end{aligned} \tag{11}$$

The equations are not simplified, because at this stage we can easily compare magnitudes of the first derivatives of both components of the criterion function (9). The coefficient 64 was chosen to equalize these magnitudes, which are now 2 for both components of the non-zero equations.

System of equations (11) directly provides conditions for critical points of the criterion function:

$$\begin{aligned} C_y = D_y = 0, \\ C_x, D_x \in \mathbb{R}, \end{aligned} \tag{12}$$

which means any line segment lying on the x axis.

To reveal nature of the critical points, we compute the Hessian of the criterion function (9). Generally it is defined as:

$$\mathbf{H}_{i,j} = \frac{\partial f(x_1, \ldots, x_n)}{\partial x_i \partial x_j} \qquad \text{for } 1 \le i, j \le n. \tag{13}$$

As l_1 is set constant at the beginning of this examination, S_{crit} is a function of l_2 only, therefore we can write it as $S_{crit}(C_x, C_y, D_x, D_y)$. The Hessian is then:

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 4 & 0 & 4 \end{bmatrix}. \quad (14)$$

H is positive semi-definite, which implies, that only minima or saddle points can exist in the critical points defined by the conditions (12). Since $S_{crit}(C_x, C_y, D_x, D_y)$ under conditions (12) is always zero, the continuous subspace of the critical points can only be the minimum of the function (9).

Now we can claim, that the criterion (9) truly satisfies requirements formulated in the semifinal paragraph of the Introduction. The only remaining feature to be described is an optimized procedure for similarity evaluation of multiple line segment pairs at once.

3.2 Expansion for a set of line segment pairs

In practice, there are a lot of situations, where two sets of line segments are being tested for similarity. An overall similarity is then given by a sum of similarities of all corresponding pairs from both sets. During scan to map matching in robotics, or image to image registration in computer vision applications, one set is often considered static (remains constant during computation) and the other is dynamic (i.e. manipulated using rigid transformation). The transformation is iteratively adjusted to minimize the overall similarity criterion. The established similarity criteria require transformation of the second set of line segments and recalculation of the output value any time, the transformation changes. Our criterion allows to precompute the result and then transform it in constant time, regardless the number of pairs being examined.

Let the line segment $l_1 = \{\boldsymbol{A}, \boldsymbol{B}\}$ be static and the $l_2 = \{\boldsymbol{C}, \boldsymbol{D}\}$ belong to the transformed set. The transformation is described by a rotation matrix **R** and translation vector $\boldsymbol{t}$:

$$\mathbf{R} = \begin{bmatrix} \cos(\theta) & -\sin(\theta) \\ \sin(\theta) & \cos(\theta) \end{bmatrix}, \qquad \boldsymbol{t} = \begin{bmatrix} t_x \\ t_y \end{bmatrix}, \quad (15)$$

where θ is the angle of rotation and t_x and t_y are translations in the direction of the x and y axes and affects an arbitrary point in accordance with equation $\boldsymbol{P'} = \mathbf{R}\boldsymbol{P} + \boldsymbol{t}$. Equations (7) and (8), with transformation of l_2 included, look as follows:

$$S^2_{ABCD}(l_1, l_2) = \| (\boldsymbol{B} - \boldsymbol{A}) \times (\mathbf{R}(\boldsymbol{D} - \boldsymbol{C})) \|^2, \quad (16)$$

$$S^2_{ABS}(l_1, l_2) = \frac{1}{64} \| (\boldsymbol{B} - \boldsymbol{A}) \times (\boldsymbol{A} + \boldsymbol{B} + \mathbf{R}(\boldsymbol{C} + \boldsymbol{D}) + 2\boldsymbol{t} - 4\boldsymbol{A}) \|^2. \quad (17)$$

The overall similarity for a set of N line segment pairs is then given by:

$$\begin{aligned} S_{tot} = & \sum_{i=1}^{N} S_{crit,i} \\ = & \; s^2 \sum (\boldsymbol{x}_i \cdot \boldsymbol{y}_i)^2 \\ & + c^2 \sum \| \boldsymbol{x}_i \times \boldsymbol{y}_i \|^2 \\ & + 2cs \sum (\boldsymbol{x}_i \cdot \boldsymbol{y}_i) \| \boldsymbol{x}_i \times \boldsymbol{y}_i \| \\ & + \sum \| \boldsymbol{x}_i \times \boldsymbol{v}_i \|^2 \\ & + 2s \sum (\boldsymbol{x}_i \cdot \boldsymbol{z}_i) \| \boldsymbol{x}_i \times \boldsymbol{v}_i \| \\ & + 2c \sum \| \boldsymbol{x}_i \times \boldsymbol{z}_i \| \| \boldsymbol{x}_i \times \boldsymbol{v}_i \| \\ & + 4t_y \sum x_{x,i} \| \boldsymbol{x}_i \times \boldsymbol{v}_i \| \\ & - 4t_x \sum x_{y,i} \| \boldsymbol{x}_i \times \boldsymbol{v}_i \| \\ & + s^2 \sum (\boldsymbol{x}_i \cdot \boldsymbol{z}_i)^2 \\ & + 2cs \sum (\boldsymbol{x}_i \cdot \boldsymbol{z}_i) \| \boldsymbol{x}_i \times \boldsymbol{z}_i \| \\ & + c^2 \sum \| \boldsymbol{x}_i \times \boldsymbol{z}_i \|^2 \\ & + 4st_y \sum x_{x,i} (\boldsymbol{x}_i \cdot \boldsymbol{z}_i) \\ & - 4st_x \sum x_{y,i} (\boldsymbol{x}_i \cdot \boldsymbol{z}_i) \\ & + 4ct_y \sum x_{x,i} \| \boldsymbol{x}_i \times \boldsymbol{z}_i \| \\ & - 4ct_x \sum x_{y,i} \| \boldsymbol{x}_i \times \boldsymbol{z}_i \| \\ & + 4t_y^2 \sum x_{x,i}^2 \\ & - 8t_x t_y \sum x_{x,i} x_{y,i} \\ & + 4t_x^2 \sum x_{y,i}^2, \end{aligned} \quad (18)$$

where $\boldsymbol{x} = \boldsymbol{B} - \boldsymbol{A}$, $\boldsymbol{y} = \boldsymbol{D} - \boldsymbol{C}$, $\boldsymbol{z} = \boldsymbol{D} + \boldsymbol{C}$, $\boldsymbol{v} = \boldsymbol{B} - 3\boldsymbol{A}$ and $c = \cos(\theta)$, $s = \sin(\theta)$, both from the rotation matrix **R**. Separation of the variables related to the rigid transformation and the sums originating from the initial description of every examined line segment is a crucial result. It allows us to precompute the sums only once for the whole set and then evaluate the cumulative similarity for any transformation in constant time. Assuming p is the number of line segment pairs being examined and t is the number of transformations performed, the computational complexity of the whole task is reduced from $O(pt)$ for established criteria to $O(p+t)$ for our criterion. This could lead to significant performance improvement of iterative matching algorithms, which rely on completely static, or partially updated data set. The examples are iterative closest line algorithms by [15] and [7].

4 EXPERIMENTS AND RESULTS

Empirical verification is essential, when any new method is being released for practical applications. In this section, we are going to present synthetic tests proving features theoretically described in Sec. 3.1 and 3.2 and show some performance tests demonstrating advantages of our method, when similarity for a set of line segment pairs under different rigid transformations is to be computed.

All experiments were implemented in the C++ programming language and compiled with Microsoft Visual Studio 2015, using the -O2 optimization setting. No other optimizations were made to keep the tests as general as possible. The machine used for running the experiments had a four-core / eight threads, 64-bit Intel Core-i7-4790K CPU, running on 4.0 GHz. Processor cache memory is large enough to hold all of the data of every test performed. Visualization of the results was done using MATLAB 2015 computing environment.

4.1 Feature verification

For feature verification of our criterion, we have decided to adopt the methodics introduced in [13]. It clearly visualizes properties of the criterion under wide variety of conditions and an interested reader may find results for other line segment distance functions in the cited paper in the given format. We believe, that unification of testing methods will help the readers to compare the methods and choose the right criterion function for their needs.

Initial arrangement of the experiment is depicted in Figure 2. There is a static line segment $l_1 = \{\boldsymbol{A}, \boldsymbol{B}\} = \{\{0,0\}, \{100,0\}\}$ and a dynamic l_2 transformed by various transformations according to the particular test. Figure 2 shows the positive direction of the rotation and the x axis. Rotations are always performed about the origin of the coordinates.

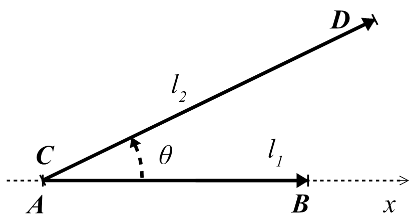

Figure 2: Schematic depiction of the positions of the line segments during the verification process as introduced in [13].

The first two tests directly follow [13]. Figure 3 depicts a situation, where l_2 is first translated in the direction of the x axis and then rotated by a given angle. The graph clearly shows, that for l_2 lying on the x axis, the criterion returns zero and as the translation increases and the angle closes to $\frac{\pi}{2}$ and $\frac{3}{2}\pi$ the output value grows rapidly.

Figure 4 shows another important case, where the length of l_2 is varied and then the line segment is

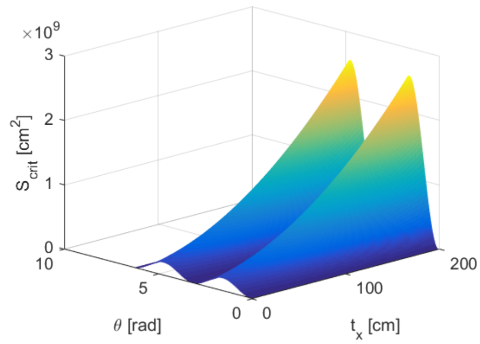

Figure 3: Criterion verification: Translation of l_2 by $t_x \in [0; 200]$ centimetres followed by a rotation by $\theta \in [0; 2\pi]$ radians.

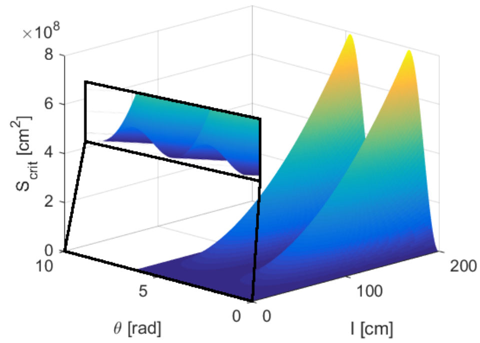

Figure 4: Criterion verification: Length scaling of l_2 in the interval $[1; 200]$ centimetres followed by a rotation by $\theta \in [0; 2\pi]$ radians. Sub-plot shows the detail of the graph for very small lengths of l_2.

rotated by the angle θ. Again, the criterion gives zero output for any l_2 coincident with the x axis and increases as the length grows and θ closes to $\frac{\pi}{2}$ or $\frac{3}{2}\pi$, which is desirable. Even for very short l_2 the nature of the criterion is consistent and the detail magnified in the sub-plot in Figure 4 corresponds to the harmonic shape observed in Figure 3.

The two following tests are meant to demonstrate, that for any l_2 moving along the l_1 (i.e. x axis in these experiments), the value of the criterion remains the same. Figure 5 shows a situation, where l_2 is first rotated and then translated along the x axis. The graph clearly demonstrates, that the translation has no effect and the enumerated similarity remains the same.

Similar behaviour appears, when l_2 is scaled at first, then rotated by the fixed angle $\frac{\pi}{4}$ and translated along l_1. The output of the criterion (see Figure 6) is quadratically dependant on the length of l_2, but the translation does not affect it in any way.

All four experiments prove the theoretical findings from Section 3. If l_2 and l_1 lie on the same line, the criterion returns zero, which corresponds to conditions (12). The tests also illustrate the fact, that the value of the crite-

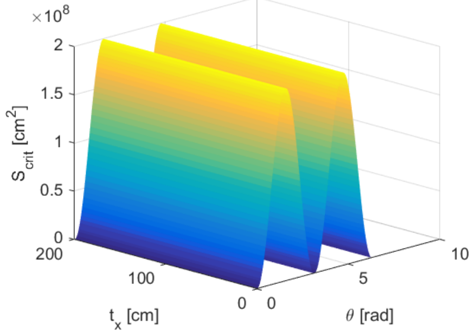

Figure 5: Criterion verification: Rotation of l_2 by $\theta \in [0; 2\pi]$ radians followed by a translation by $t_x \in [0; 200]$ centimetres.

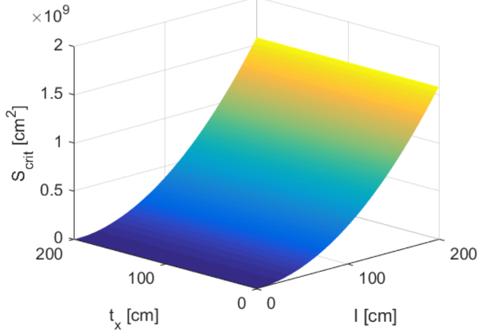

Figure 6: Criterion verification: Length scaling of l_2 in the interval $[1; 200]$ centimetres followed by a rotation by a fixed angle $\pi/4$ radians and translated by $t_x \in [0; 200]$ centimetres at the end.

rion is not affected by translation of l_2 in the direction of l_1, which is mathematically proved in the system of equations (11).

4.2 Performance verification

Though the equation (18) might seem enormous at first, there are many repeating terms, so the sums can be precomputed with reasonable amount of additions and multiplications. The same applies to later evaluation for various transformations. In fact, careful examination of equations (16) and (17) reveals, that the number of basic floating point operations is roughly the same in both cases.

In this set of tests the performance of a naive and the optimized algorithms is compared. The naive implementation of the criterion function for a set of line segment pairs stems from the equations (16) and (17). First, it computes the transformation of the dynamic line segments and then the errors. Contrary, the optimized algorithm first computes sums of the equation (18) and then evaluates the criterion for each particular transformation.

Figures 7 and 8 show the timings of the proposed criterion function. Both exhibit the predicted behaviour sta-

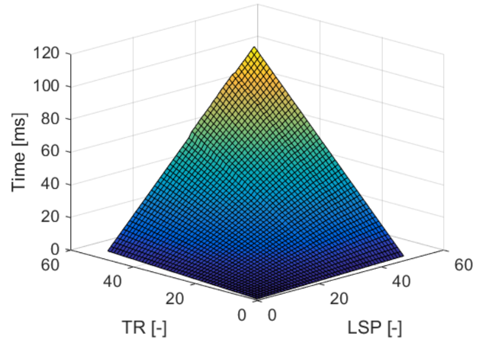

Figure 7: Processing time of a naive implementation of the criterion for various number of line segment pairs being processed (LSP) and transformations performed (TR).

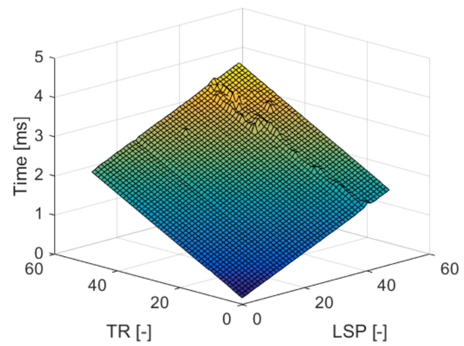

Figure 8: Processing time of the optimized implementation criterion for various number of line segment pairs being processed (LSP) and transformations performed (TR).

ted in Section 3.2. Naive implementation corresponds to $Q(pt)$ computation complexity, while the optimized algorithm is $O(p+t)$. To emphasize the benefits of the optimization Figure 9 shows the speed-up over the basic version. The performance gains are present even for the lowest numbers of transformations and line segment pairs, but the small percentage of improvement is highly implementation dependant and redeemed by a larger memory footprint of the optimized algorithm. The most significant improvements can be observed, when both p and t grow up, which is a frequent case. In such situation, the large performance gains are doubtless.

5 CONCLUSION

This paper presents a novel area-based line segment similarity criterion. Contrary to usual alternatives, the criterion function is fully differentiable and the derivatives are continuous in the whole domain of definition. The criterion is designed to give zero output for any two line segments lying on the same line, which makes it well applicable, wherever a small vector image is to

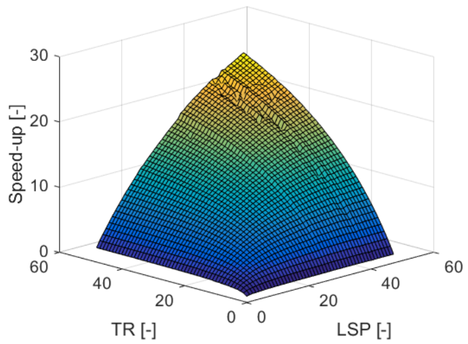

Figure 9: Seed-up of the optimized criterion over a naive implementation for various number of line segment pairs being processed (LSP) and transformations performed (TR).

be fitted into a larger one (e.g. scan to map matching algorithms in robotics).

The criterion also supports precomputation. Many algorithms iteratively transform a line segment set by a rigid transformation and compare it to a static set. Precomputation reduces computational complexity of such algorithms from $O(pt)$ to $O(p+t)$ (p is a number of line segment pairs being examined and t a number of transformation performed).

All of these features were theoretically derived and practically tested in the appropriate parts of the paper. Testing procedure was selected to correspond with other publications to provide the reader with consistent information. We hope, that this decision will help him to compare various approaches and choose the most relevant for his needs.

Future work is going to be focused on application of the presented approach in a mapping system for mobile robots and measurement of the practical performance gains. An interested user of the algorithm may benefit from multithreaded implementation, SIMD instructions, loop unrolling and many other techniques, which can improve the performance significantly, but are highly platform dependant. We hope, that features of the described criterion will open new possibilities not only in robotics, but in shape registration tasks in general.

6 ACKNOWLEDGEMENTS

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