POSTER: Finding Direction of Intersection Curve in Critical Cases of Surface-surface Intersection

Min-jae Oh
Department of Naval Architecture and Ocean Engineering
Seoul National University
Seoul 151-744, KOREA
Mjoh80@snu.ac.kr

Seok Hur
Department of Mathematical Sciences
Seoul National University
Seoul 151-747, KOREA
estone@snu.ac.kr

Tae-wan Kim
Department of Naval Architecture and Ocean Engineering
Seoul National University
and RIMSE
Seoul 151-744, KOREA
taewan@snu.ac.kr

ABSTRACT

Determining the topology of intersection curves is one of the important issues of surface-surface intersection problem used in Computer Aided Geometric Design and Computer Graphics. To compute the intersection curves, we first need to determine the topology of the curves. Thomas A. Grandine[Gr97] presented an algorithm to determine topology using partial derivatives of surface intersection equations. When the two surfaces meet tangentially, the differential values of the parameters of the surfaces are not determined in the intersection equations. These cases are called critical cases. In [Ye99] a method of finding the values of the differentials is presented for the case of the contact of order 1. We present general methods for the case of the contact of higher order ≥ 1 using perturbation method. With these results, we can decide starting or ending of the critical boundary point.

Keywords

surface-surface intersection(SSI), topology decision, critical case, tangential intersection, perturbation method.

1. INTRODUCTION

Surface-surface intersection is an important issue**,** which is used in Computer Aided Design, Computer Aide Geometric Design, Computer Graphics and so on. A topology decision is one part of surface-surface intersection (SSI). To find the intersection curves of two surfaces, first of all, we should figure out the topology of intersection curves of two surfaces. There are many methods are presented in SSI [Ba87, Ba90, Gr97, Gr00, Ho93, Wu99] and, in particular, in finding topology [Gr91, Se88, Se89]. But, in this paper, we focus on the extension of the algorithm presented by T. A. Grandine, which is applied for determining the topology of intersection curves of two surfaces. He presented a method using partial derivatives in surface intersection equations. With Grandine's method, one can determine the topology only in the cases of transversal intersection. So we present an extension of the algorithm for the case of tangential intersection case.

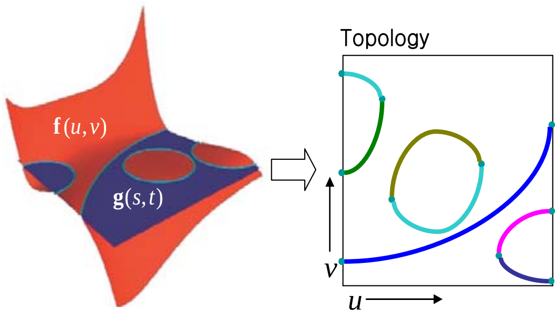

Figure 1. Example of topology

Figure 1 shows an example of topology. There are 7 intersection curves in Figure 1. At each intersection curve, one may determine starting or ending at the boundary and turning points using Grandine's topology decision method. Grandine decides starting or ending by the values of derivatives at each point. But, the intersection is not transversal (that is, two surfaces intersect tangentially) one can not decide the values of derivative with Grandine's method. These cases are called critical cases. In this paper, we consider the critical cases and we present an extension of Grandine's method for such critical cases.

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2. PROBLEM DEFINITION

In this paper, we use T. A. Grandine's topology determination method [Ga97] to find topology of SSI. He suggested starting or ending conditions on the boundary points as followings. When two parametric surfaces $\mathbf{f}(u,v)$ and $\mathbf{g}(s,t)$ are given, the next equation would be used for deciding starting or ending of the boundary point $P(s=0)$;

$$\tan^{-1}\frac{\frac{du}{ds}\cos\theta+\frac{dv}{ds}\sin\theta}{\frac{du}{ds}\sin\theta-\frac{dv}{ds}\cos\theta}. \qquad (1)$$

When the value of the equation (1) is positive [resp. negative], the boundary point will be a starting point [resp. ending point]. If the quantity is positive, the intersection curve heads in the parameter space (u,v) of $\mathbf{f}(u,v)$ to the increasing panel direction as s increases.

The values du/ds and dv/ds can be determined by differentiating SSI equation in the case of transversal intersection. The SSI equation writes

$$\mathbf{f}(u,v)-\mathbf{g}(s,t)=0. \qquad (2)$$

Differentiating the equation (2) by the variable s, we get the equation

$$\frac{du}{ds}\mathbf{f}_u+\frac{dv}{ds}\mathbf{f}_v-\mathbf{g}_s-\frac{dt}{ds}\mathbf{g}_t=\mathbf{0}. \qquad (3)$$

The equation (3) can be represented 3×3 linear system of equations

$$\begin{pmatrix} f_u^x & f_v^x & -g_t^x \\ f_u^y & f_v^y & -g_t^y \\ f_u^z & f_v^z & -g_t^z \end{pmatrix}\begin{pmatrix} \frac{du}{ds} \\ \frac{dv}{ds} \\ \frac{dt}{ds} \end{pmatrix}=\begin{pmatrix} g_s^x \\ g_s^y \\ g_s^z \end{pmatrix}. \qquad (4)$$

When the 3×3 matrix is invertible (that is, if two surfaces intersect transversally), we can find the unique vector value $(du/ds, dv/ds, dt/ds)^T$. Then we can decide whether the boundary point $P(s=0)$ is a starting point or an ending point by the equation (1). But, when 3×3 matrix is not invertible (that is, if two surfaces intersect tangentially), we can not determine the topology with the Granidin's method. These cases are called critical cases. We consider such critical cases and suggest a general method of determining the vector value $(du/ds, dv/ds, dt/ds)^T$.

When two surfaces meet tangentially at boundary $P(s=0)$, we cannot determine the values du/ds and dv/ds with the equation (4). For the simple tangential intersection, that is, for the contact of order one, Ye-Ma[Ye99] presented a method of finding the tangent vector of the intersection curve. They used the fact that the two surfaces have the same normal curvatures on the intersection curve. If we know the tangent vector of the intersection curve we can find the vector $(du/ds, dv/ds, dt/ds)^T$ [Oh06]. But the intersection is contact order at least 2 this method is not applicable.

3. TOPOLOGY RESOLUTION OF CRITICAL CASES

Let S_1 and S_2 be two analytic surfaces in R^3 defined by the following parametric representations.

$$\begin{aligned} S_1 &= \{\mathbf{f}(u,v)=(f^x(u,v), f^y(u,v), f^z(u,v)) : 0\le u,v\le 1\} \\ S_2 &= \{\mathbf{g}(s,t)=(g^x(s,t), g^y(s,t), g^z(s,t)) : 0\le s,t\le 1\}. \end{aligned} \qquad (5)$$

We assume that the two surfaces intersect tangentially on an intersection curve l. We want to find the vector value $(du/ds, dv/ds, dt/ds)^T$ of the intersection curve on the boundary $s=0$. If we assume that the intersection curve l is parameterized locally in a small neighborhood of $P(s=0)$ by the variable s, the variables u, v and t can be represented analytically by the variable s in small neighborhood of $P(s=0)$. Since two surfaces have the same tangent plane on l, there is an invertible matrix $A(s)$ such that

$$(\mathbf{f}_u(s) \quad \mathbf{f}_v(s))A(s)=(\mathbf{g}_s(s) \quad \mathbf{g}_t(s)). \qquad (6)$$

And, on l, the fact $\partial/\partial s(\mathbf{f}(u,v)-\mathbf{g}(s,t))=0$ implies the equation (3).

Let $A(0)=\begin{pmatrix}\alpha & \gamma \\ \beta & \delta\end{pmatrix}$ be the invertible matrix at $s=0$. According to the relation (6), we can write

$$\frac{du}{ds}\mathbf{f}_u+\frac{dv}{ds}\mathbf{f}_v=\frac{dt}{ds}\gamma\mathbf{f}_u+\frac{dt}{ds}\delta\mathbf{f}_v+\alpha\mathbf{f}_u+\beta\mathbf{f}_v. \qquad (7)$$

This equation implies the relation

$$\begin{pmatrix}\frac{du}{ds} \\ \frac{dv}{ds}\end{pmatrix}=\begin{pmatrix}\alpha & \gamma \\ \beta & \delta\end{pmatrix}\begin{pmatrix}\frac{ds}{ds} \\ \frac{dt}{ds}\end{pmatrix}=\begin{pmatrix}\alpha & \gamma \\ \beta & \delta\end{pmatrix}\begin{pmatrix}1 \\ \frac{dt}{ds}\end{pmatrix}. \qquad (8)$$

Now we have the following lemma,

Lemma If two surfaces intersect tangentially at the boundary point $P(s=0)$, finding the vector value $(du/ds, dv/ds, dt/ds)^T$ is equivalent to finding the value dt/ds.

Now we perturb analytically one of the surface equations, for instance, $S_2(\varepsilon)=\{\mathbf{g}(s,t,\varepsilon):0\le s,t\le 1\}$ so that the intersection is no longer tangential for sufficiently small $\varepsilon\neq 0$. We remark that the Sard's theorem [Sp99] guarantees the existence of such perturbations and that the method of such perturbations will be various. Analytic perturbation means the vector valued function $\mathbf{g}(s,t,\varepsilon)$ is an analytic function with respect to ε and $\mathbf{g}(s,t,0)=\mathbf{g}(s,t)$. After such perturbation the equation (4) can be written

$$\begin{pmatrix}\mathbf{f}_u & \mathbf{f}_v & -\mathbf{g}_t(\varepsilon)\end{pmatrix}\begin{pmatrix}\frac{du}{ds}(\varepsilon) & \frac{dv}{ds}(\varepsilon) & \frac{dt}{ds}(\varepsilon)\end{pmatrix}^T=\mathbf{g}_s(\varepsilon). \quad (9)$$

Since the perturbation is analytic with respect to ε the values $\mathbf{g}_t(\varepsilon),\begin{pmatrix}\frac{du}{ds}(\varepsilon) & \frac{dv}{ds}(\varepsilon) & \frac{dt}{ds}(\varepsilon)\end{pmatrix}^T$ and $\mathbf{g}_s(\varepsilon)$ are analytic with respect to ε for sufficiently small $\varepsilon\neq 0$. These facts write

$$\begin{aligned}
&\mathbf{g}_s(\varepsilon)=\mathbf{g}_s(0)+\sum_{k\ge1}\varepsilon^k\mathbf{g}_s^k=\alpha\mathbf{f}_u+\beta\mathbf{f}_v+\sum_{k\ge1}\varepsilon^k\mathbf{g}_s^k,\\
&\mathbf{g}_t(\varepsilon)=\mathbf{g}_t(0)+\sum_{k\ge1}\varepsilon^k\mathbf{g}_t^k=\gamma\mathbf{f}_u+\delta\mathbf{f}_v+\sum_{k\ge1}\varepsilon^k\mathbf{g}_t^k,\\
&\frac{du}{ds}(\varepsilon)=\frac{du}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{du}{ds}\right)_k\\
&\qquad=\alpha+\gamma\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{du}{ds}\right)_k, \qquad (10)\\
&\frac{dv}{ds}(\varepsilon)=\frac{dv}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{dv}{ds}\right)_k\\
&\qquad=\beta+\delta\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{dv}{ds}\right)_k,\\
&\frac{dt}{ds}(\varepsilon)=\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{dt}{ds}\right)_{k1}.
\end{aligned}$$

Then the equations (9) and (10) imply

$$\begin{aligned}
&\mathbf{f}_u\left[\alpha+\gamma\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{du}{ds}\right)_k\right]\\
&+\mathbf{f}_v\left[\frac{dv}{ds}(0)\beta+\delta\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{dv}{ds}\right)_k\right]\\
&-\left[\gamma\mathbf{f}_u+\delta\mathbf{f}_v+\sum_{k\ge1}\varepsilon^k\mathbf{g}_t^k\right]\left[\frac{dt}{ds}(0)+\sum_{k\ge1}\varepsilon^k\left(\frac{dt}{ds}\right)_k\right]\\
&-\left[\alpha\mathbf{f}_u+\beta\mathbf{f}_v+\sum_{k\ge1}\varepsilon^k\mathbf{g}_s^k\right]=0.
\end{aligned}$$

Consequently, we have an expansion with respect to ε

$$\begin{aligned}
&\mathbf{f}_u\left[\alpha+\gamma\frac{dt}{ds}(0)\right]+\mathbf{f}_v\left[\beta+\delta\frac{dt}{ds}(0)\right]\\
&-(\gamma\mathbf{f}_u+\delta\mathbf{f}_v)\frac{dt}{ds}(0)-(\alpha\mathbf{f}_u+\beta\mathbf{f}_v)\\
&+\varepsilon\begin{bmatrix}\mathbf{f}_u\left(\frac{du}{ds}\right)_1+\mathbf{f}_v\left(\frac{dv}{ds}\right)_1\\ -(\gamma\mathbf{f}_u+\delta\mathbf{f}_v)\left(\frac{dt}{ds}\right)_1-\mathbf{g}_t^1\frac{dt}{ds}(0)-\mathbf{g}_s^1\end{bmatrix} \qquad (11)\\
&+\sum_{k\ge2}\varepsilon^kR_k=0.
\end{aligned}$$

This equality implies the vanishing conditions $R_k\equiv 0$, for all $k\ge 1$.

The vanishing of constant term is trivial. And the vanishing of the coefficient of ε gives

$$\begin{aligned}
&\mathbf{f}_u\left(\frac{du}{ds}\right)_1+\mathbf{f}_v\left(\frac{dv}{ds}\right)_1-(\gamma\mathbf{f}_u+\delta\mathbf{f}_v)\left(\frac{dt}{ds}\right)_1-\mathbf{g}_s^1 \qquad (12)\\
&=\mathbf{g}_t^1\frac{dt}{ds}(0).
\end{aligned}$$

If $\varepsilon\neq 0$, two surfaces sustains transversal intersection, the values $(du/ds)_1,(dv/ds)_1$ and $(dt/ds)_1$ can be evaluated by the equation (4). The values $\mathbf{g}_s^k$ and $\mathbf{g}_t^k$ can be calculated directly by the assumption of analyticity of the perturbation. So we can find the value $dt/ds(\varepsilon=0)$ in above equation when the vector $\mathbf{g}_t^1$ does not vanish. If the value $\mathbf{g}_t^1=0$ we can not find exact value of $dt/ds(\varepsilon=0)$ in the equation (12) and there are two situations depending on the value of the left side of the equality (12). As the first case, left side does not vanish, and then, consequently, there is no such value $dt/ds(\varepsilon=0)$ satisfying the equality. This means the point $P(s=0)$ is an isolate tangential intersection point or the direction of intersection is t-axis, that is, $dt/ds=\infty$. Secondly, the left side vanishes, and then the equality (12) satisfies for any value of $dt/ds(\varepsilon=0)$. So we must use the next vanishing condition $R_2\equiv 0$. If we can not decide the value $dt/ds(\varepsilon=0)$ with the condition $R_2\equiv 0$ we continue this procedure till we could find the value $dt/ds(\varepsilon=0)$. We remark the functions R_k are polynomials with respect to the parameters. Since the ring of polynomials is Noetherian, the ideal generated by R_k has finite generators. Vanishing of these finite generators gives the value $dt/ds(\varepsilon=0)$.

We remark that the difficulty of calculation depends on the method of perturbations. In practical calculation, we can use the following simplified form $dt/ds\left(\varepsilon=0\right)=\lim_{\varepsilon\to 0} dt/ds\left(\varepsilon\right)$.

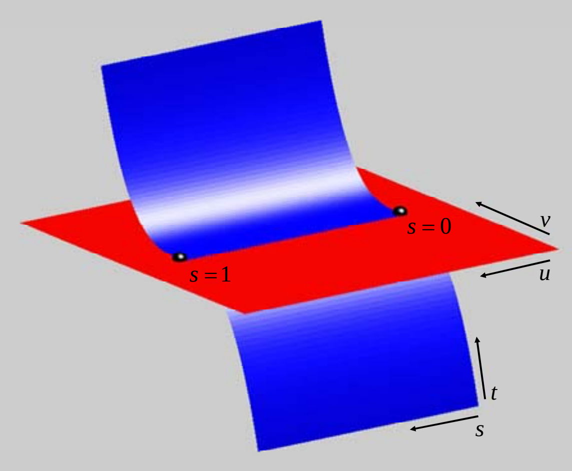

Figure 2. The critical boundary points of contact of order two

As an example (see Figure2), we consider a perturbation

$$S_1=\{\mathbf{f}(u,v)=(u,v,0):0\le u,v\le 1\}$$
$$S_2=\{\mathbf{g}(s,t)=(0.7s+0.15,0.7t+0.15,(t-0.5)^3+\varepsilon):0\le s,t\le 1\}.$$

The vanishing condition gives $dt/ds\cdot\left(\varepsilon\right)^{\frac{2}{3}}=0$. This means $dt/ds=0$ for $\varepsilon\neq 0$ and consequently we have $dt/ds\left(\varepsilon=0\right)=\lim_{\varepsilon\to 0} dt/ds\left(\varepsilon\right)=0$.

4. FUTURE WORKS

We have considered critical case of boundary points. Another critical case is concerning the turning point in the Grandine's method. For example two pipes intersect with cross intersection curves. In such case we cannot decide starting or ending of the cross turning point with the Grandine's method.

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