Poster: Experiments with Volumetric Data Interpolation

Alexej Kolcun
Dep. of Appl. Math&Inf.
Institute of Geonics CAS, Ostrava
Czech Republic
kolcun@ugn.cas.cz

Daniel Pravda
Faculty of Science
University of Ostrava
Czech Republic
R02299@student.osu.cz

ABSTRACT

Marching Cubes and Marching Tetrahedra methods expect data in the nodes of used grid. However, very often we need the isosurface extraction from data defined in the cells of the grid. In this paper we compare the criteria for interpolation values from cells to vertices of the structured grid.

Keywords

Volumetric data, interpolation, weighted mean, structured mesh.

1. INTRODUCTION

Marching Cubes [Lore87] (Marching Tetrahedra [Payn90]) algorithms are powerful tools for extracting isosurfaces from scalar volumetric data. These algorithms expect data located at *vertices* of used mesh. In [Pate05] the sensitivity of resulting interpolation on used mesh is analyzed. However, very often we need the isosurface extraction for data defined in the *cells* of the mesh. There are two possibilities how to use “marching” algorithms in this case:

a) We preserve the analyzed values and we make a mesh with new geometry. In the simplest case a dual mesh is constructed (nodes of dual mesh corresponds to cells of original one; nodes of dual mesh are neighbor if cells of original one have common edge).

b) We preserve the geometry of the mesh and the values from cells to vertices are interpolated.

In the paper the second approach is used. We shall use next abbreviation:

$C = \{c_i, i = 1, \cdots n_C\}$ – conform set of polygonal cells in 2D (polyhedral in 3D),

$|C| = n_C$ – cardinality of the set C, $\|c_i\|$ – volume of the element c_i,

$V = \{v_i, i = 1, \cdots n_V\}$ – set of the mesh vertices,

C_v – set of the cells, which are incident to vertex *v*.

Four different weighted averages are analyzed for simple test tasks. Structured meshes are used.

2. INTERPOLATION STRATEGIES

Let the values of the scalar field F are known for the cells of the mesh. We can calculate the value of this field at the vertex v as the weighted arithmetic mean of the values in incident cells:

$$F^{calc}(v) = \sum_{c \in C_v} a_c F(c) \cdot$$

We shell analyze four methods for weights computation.

Simple Arithmetic Mean $a_c = 1 / |C_v|$.

Due to simplicity of computation this strategy is used very often.

Volume Weighted Mean $a_c = \|c\| \Big/ \sum_{d \in C_v} \|d\|$.

Idea of the volume averaging is that the influence of coincident cells at the common vertex is proportional to their size.

Volume Reciprocal Mean $a_c = \frac{1}{\|c\|} \Big/ \sum_{d \in C_v} \frac{1}{\|d\|}$

This strategy seems to be in contradiction with previous one. However, it can be proved that for

1. linear functions $F\left(x_1,\cdots x_n\right)=r_0+\sum_{i=1}^{n} r_i x_i$,
2. orthogonal meshes, with
$V=\left\{x_{1,1},\cdots x_{1,m_1}\right\}\times\cdots\times\left\{x_{n,1},\cdots x_{n,m_n}\right\}$
3. the value of the cell $F(c)=F(\bar{v})$, where $\bar{v}$ is the center of the cell c,

this strategy gives the exact value: $F^{calc}(v)=F(v)$.

Angle Averaging $a_c=\alpha_{v,c}/q$.

Here $\alpha_{v,c}$ is the angle at the vertex v within the cell c. $q=2\pi$ in 2D and $q=4\pi$ in 3D.

3. TWO-DIMENSIONAL MESH TESTS

We compare the strategies mentioned above for two-dimensional grids. The goal is to gain a basic idea of behavior of the interpolation strategies while changing the shape and size of cells surrounding the given vertex.

Two types of 3x3 grid with 4 rectangular cells (Fig. 1a, 1b) are used. We move the central vertex of the grid and thus we gain grids with different geometries. Different paths for moving central vertex are defined: 1–3 for regular grid (Fig. 1a)), 4–5 for orthogonal one (Fig.1b)). Each starts at the position, when the grid is rectilinear, and passes along the line. In the Fig.1c) one of the grids for the path 2 is shown.

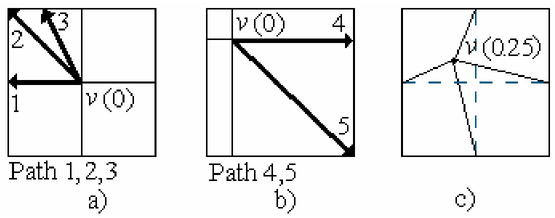

Figure 1.

For each position of the central vertex *v*, the value of each cell is calculated as

$$F(c)=\frac{\iint_c F(x,y)\,dxdy}{\|c\|}.$$

Fig. 2 shows the values of the relative difference

$$\frac{F^{calc}(v(t))-F(v(t))}{F(v(t))}\times 100$$

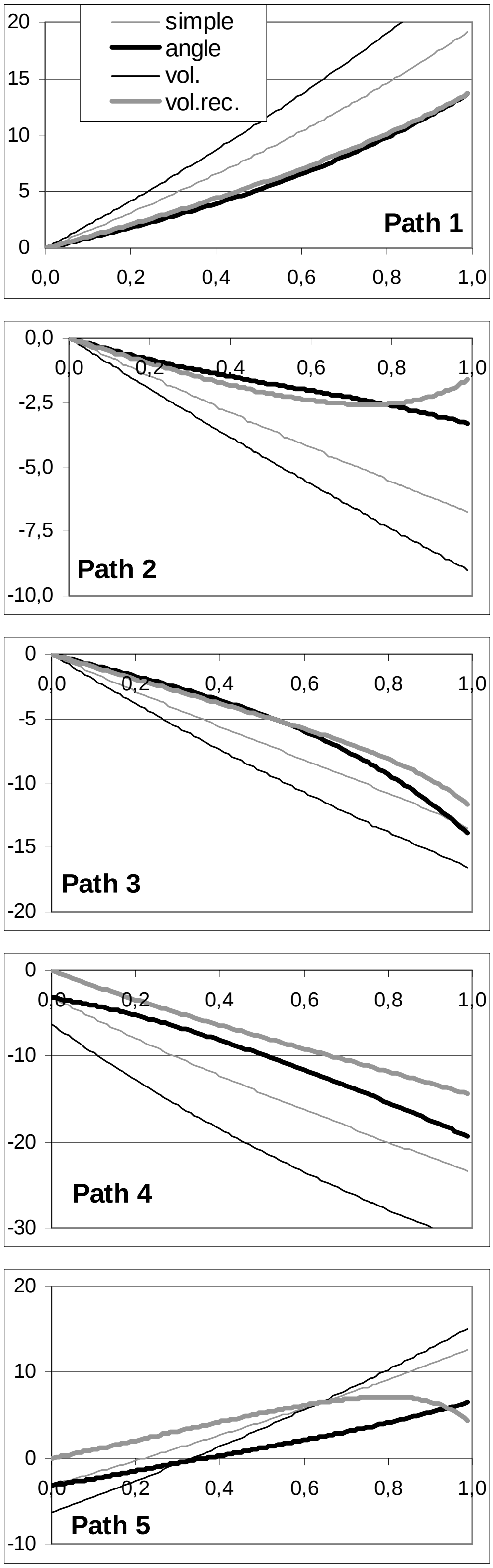

Figure 2.

at the vertex $v(t)$ for the liner function $F(x,y)=2x+3y$. The argument of the horizontal

 ISBN 978-80-86943-99-2

axis $t \in \langle 0,1 \rangle$ denotes the position of the vertex $v(t)$ according to the Fig. 1.

We can see that the strategies of volume reciprocal mean and angle averaging give similar results and are significantly better then simple mean. The volume averaging strategy produces the worst results.

4. TESTS IN 3D

We use two three-dimensional structured grids for a comparison of the interpolation strategies.

First one is based on regular rectangular grid with 11x11x11 vertices. Coordinates of inner vertices are randomly shifted, 20 % of the length of the edge at most.

Second one is based on rectangular grid with 8x8x8 vertices refined near one corner – Fig.3. Coordinates of inner vertices are randomly shifted, 5% of the length of the edge at most.

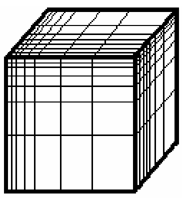

Figure 3.

Tests are made for hexahedral meshes and also for tetrahedral ones. We obtain tetrahedral mesh dividing each hexahedral cell according to the scheme in the Fig. 4.

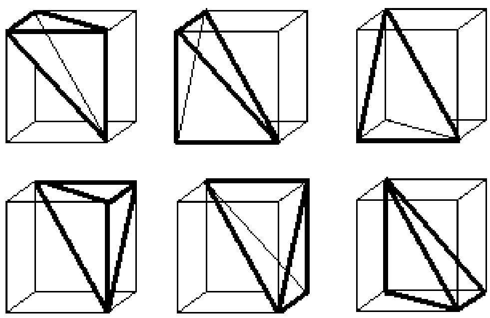

Figure 4.

Linear function $F(x,y,z) = 2x + 3y + 4z$ is used.

Values of the global difference

$$D = \sum_{v \in V} \left(F^{calc}(v) - F(v) \right)^2$$

are in the Table 1 for analyzed strategies.

	Regular grid		Refined grid	
	Hex.	Tetra.	Hex.	Tetra.
Simple mean	215,0	163,3	516,9	358,1
Vol. Weighted	291,7	294,4	2041,8	947,1
Vol. reciprocal	176,1	88,1	5,1	91,6
Angle	33,9	39,8	690,5	157,7

Table 1

5. CONCLUSION

Tests show that the strategy of volume reciprocal averaging, which gives exact solution for rectangular meshes, gives good results in more general cases too (Tab.1 – Refined grid).

For more deformed meshes the strategy of angle averaging gives better results in 3D case (Tab.1 – Regular grid). However, its computational complexity is much larger.

Important result of tests is that the strategy of volume weighted mean appears as improper.

In the future we would like to compare the quality of isosurfaces made by the Marching Cube and by the Marching Tetrahedra algorithms on interpolated data.

ACKNOWLEDGEMENTS

Work is supported by the project of National Research Program of the Czech Republic, project number 1ET400300415.

REFERENCES

[Lore87] Lorensen, W.E., Cline, H.E.:Marching Cubes: A High Resolution 3D Surface Construction Algorithm. Computer Graphics, Vol.21(4), 1987. pp.163-169.

[Pate04] Patera, J.: Iso-Surface Extraction and Approximation Error. Tech.Report DCSE/TR-2004-15. Univ.of West Bohemia, Plzeň 2004.

[Pate05] Patera, J., Skala, V.: Centered Cubic Lattice Method Comparision. In: Proc. of Conf. ALGORIT-MY 2005. STU Bratislava 2005. pp.270-279.

[Payn90] Payne, B.A., Toga, A.V.: Surface Mapping Brain Function on 3D Models. IEEE CG&A 10(5). 1990. pp. 33-41.