

## Inverse Dynamics Control Method Applied to the Standard Servo Control System to Suppress Two-mass System Vibration

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### Abstract:

This paper describes a method for two-mass system torsional vibrations suppression. These vibrations are evoked due to torsion plasticity of mechanism kinematical chain links and large moment of inertia. This strategy belongs to the group of feedforward control techniques. By means of the knowledge about the system parameters and the two-mass system mathematical model it is easy to derivate its transfer function. The main idea of this method is to create the inverse transfer function of problematic second mass of the system and use it in the control structure as a feedforward controller. And in this way suppress the activity of system imaginary poles that cause the residual vibrations. This method was firstly tested on mathematical model simulations and then integrated into the cascade structure of standard servo control unit Siemens Sinamics S120. The results from experiments demonstrate the ability of this method to effectively compensate the torsional vibrations of the mechanism end-link.

### INTRODUCTION

The multi-mass systems are widely represented in processing machines. The typical representatives are machine-tools, robots, turntables etc. The first mass is typically formed by electrical servomotor electromagnetic stator-rotor coupling and the rotor moment of inertia. The elasticity together with high inertia of the mechanism links forms the second and typically the rest of the masses. It is possible to characterize them by their natural frequency and dumping constant. In other words, such mechanism can be expressed by the second or higher order system with complex poles.

Excitation of such systems evokes at the end link except for the required motion also unwanted torsional oscillations. This may seriously affect the servomechanism positioning accuracy. Their frequencies refers to the second and higher mass natural frequencies. Depending on the excitation function and its speed each mass may produce various amplitude of torsional vibration frequency. Widely used conventional control strategy like position cascade structure can hardly affect against the residual oscillations. Some supplement control strategy must be used. Various control strategies have been proposed to this topic. In general they can be divided into feedback [1], [2], [6] and feedforward [3], [4], [5] methods. Feedback methods use for its function actual system values in contrast of feedforward methods, which are adjusting the setpoint values to avoid the oscillations. However, feedforward methods sometimes use the measured

feedback values too, for adjusting its control structure, in case that the two-mass model parameters are variable.

Our department cooperates with Research Institute of Textile Machines Liberec (VUTS) on a joint project whose main goal is devoted to the problems of substitution of stepping mechanisms like turntables, indexing gearboxes etc. by means of electronic cams. The main objective is to increase the production speed of machines with flexible parts in the kinematical chain. Our aim is to explore suppression methods that can be applied in standard electrical servo drive control system (CU). Past years we were focused to the feedback methods with or without direct position feedback [6].

This article deals with the well known Inverse Model Control (IMC) [7], which is the member of the feedforward strategies. The method described in this article was a part of diploma thesis [11]. The brief overview about the multi-mass system modeling refers the section II. The main principal of control method, its application to the standard servo controller and the experiment results are presented in the subsequent sections.

### MATHEMATICAL MODEL OF TWO-MASS SYSTEM

To the design filter it is necessary to create an accurate mathematical model of the two-mass system that can be divided into two or three parts: model of servomotor, its control structure and mechanical chain.

## Mathematical Model of PMSM and Cascade Structure

The first mass of the system forms the electromagnetic compliance of the stator and the rotor. It can be represented by the D-Q model of PMSM [9]. This model is based on the separate controlling two components of current vector (torque-generating and field-generating). The model has the great match with the real system, but for design of inverse controller is inappropriate. Because of product terms of  $i_d$  and  $i_q$  is nonlinear. For the purpose of IMC it is necessary find a linear system transfer function. We can linearize this model as it is described in [10]. It is assumed that the  $i_d$  part of the current is normally forced to be zero. Mathematical model could be then described by following equation

$$U_q = R \cdot i_q + L_q \frac{d}{dt} i_q + \omega_s \cdot \Phi_B \quad (1)$$

$$M_e = \frac{3}{2} p_p \cdot i_q \cdot \Phi_B \quad (2)$$

Where  $U_q$  represents voltage,  $i_q$  stator current,  $L_q$  inductance in q-axis.  $R$  is the stator resistance. Both nonlinear and linear models show a great match with the real servomotor responses. This is why we can use the simple model to derive the system transfer function and use it for design an inverse filter.

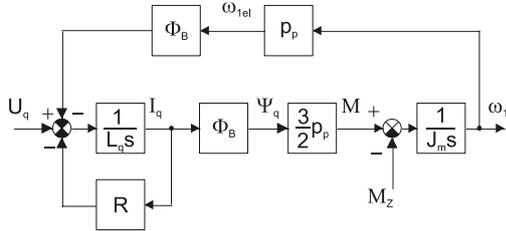


Fig. 1: Model of permanent magnet synchronous motor.

Control structure model consist of three closed control loops. The innermost and the fastest is a current control loop with PI controller. Speed loop is a superior to the current control loop and consist as well from PI controller. The outer loop controls the position by means of P controller. The transfer function of the PI controller is as follows

$$R(s) = K \cdot \left( 1 + \frac{1}{T_s} \right) = K \cdot \frac{T_s + 1}{T_s} \quad (3)$$

## Model of Mechanics with Flexible Members

In order to set the load model correctly, it is necessary to know the parameters of each member of the mechanism kinematical chain.

For the purposes of experiment was designed a massive stand by VUTS, which allows to set up mechanism with known mechanical parameters.

These parameters were counted by means of the finite element method engineering software. The correctness of the parameters could be also verified experimentally by measuring and analyzing the natural frequency of the two-mass system. The parameters for setting up the mathematical model of the synchronous servomotor were measured by means of the servo-control unit functions. The required parameters for setting up the model of the two-mass system are summarized in Tab. 1.

Tab. 1: Two-Mass System Parameters

<i>Mechanism parameters</i>	
$c_{32}$ - Torsion stiffness	1000 [Nm/rad]
$J_3$ - Flywheel moment of inertia	0,105525 [kgm <sup>2</sup> ]
$J_1$ - Gearbox input moment of inertia	0,003935 [kgm <sup>2</sup> ]
$J_2$ - Gearbox output moment of inertia	0,000798 [kgm <sup>2</sup> ]
$p$ - Gearbox ratio	33
<i>Servomotor parameters</i>	
$L_q$ - inductance	9,9767 [mH]
$R$ - Stator and cables resistance	1,081 [Ω]
$k_e$ - Voltage constant	144 [V]
$J_m$ - Moment of inertia	0,0048 [kgm <sup>2</sup> ]
$P_p$ - Pole pair number	4

The experimental stand for testing two-mass system vibration suppression methods consist of a synchronous servomotor with reduced speed by means of a preloaded backlash-free gearbox. The second mass is formed by flexible shaft that connects the gearbox output and the end-link formed by a fly wheel. For the measuring purposes, an external position sensor is connected to the flywheel. The main interest of the project is devoted to the problems of substitution of stepping mechanisms like turntables, indexing gearboxes etc. by means of electronic cams. That's reason why the conception of the mechanism is just like this. The principal chart of the experimental mechanism is shown in Fig.2.

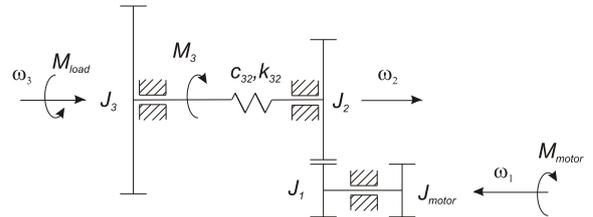


Fig. 2: Principal chart of two-mass system mechanical chain.

In comparison with other links of the kinematical chain the gearbox elasticity is negligible. Therefore, the parameters such as stiffness and damping constants are taken as ideal. Under this assumption there is possible to involve the gearbox moment of inertia into the motor moment of inertia and express the motor with the gearbox as the one-mass system. This kind of simplification cannot be done with the

elastic shaft that connects the gearbox with the flywheel. This is because the flexibility of the coupling shaft and the flywheel at the end of the mechanism is too high and evokes the low frequency residual vibration. That's why must be expressed as a second mass of the system. The model of the two-mass system can be described with the following equation

$$J_{Tot} = J_{motor} + J_1 + \frac{J_2}{p^2} \quad (4)$$

$$M_{motor} = J_{Tot} \frac{d\omega_1}{dt} + \frac{M_3}{p} \quad (5)$$

$$\begin{aligned} M_3 &= \left(\frac{\varphi_1}{p} - \varphi_3\right)(c_{32} + \frac{d}{dt} k_{32}) = \\ &= J_3 \frac{d\omega_3}{dt} + M_{Load} \end{aligned} \quad (6)$$

Where  $c_{32}$  is constant of torsional stiffness and  $k_{32}$  is constant of viscous damping.  $\varphi_1$  and  $\varphi_3$  are torsion angles.  $J_{Tot}$  is the sum of all moments of inertia described above and  $J_3$  the flywheel moment of inertia.  $\omega_1$  represents the angular velocity of the motor and  $\omega_3$  the velocity of the flywheel. Using the Laplace transformation there is possible to transform into the  $s$ -domain and to illustrate as the block diagram shown in Fig. 3.

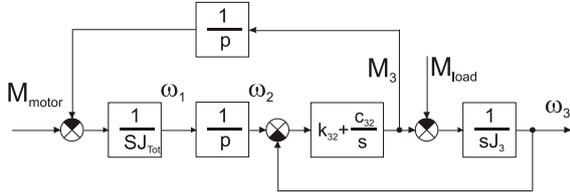


Fig. 3: Model of two-mass system.

If we express the transfer function from  $\omega_1$  to  $\omega_3$ , we can get the formula for calculating the oscillation of the mechanism, so called anti-resonance frequency

$$\frac{\omega_3}{\omega_1} = \frac{1}{p} \cdot \frac{k_{32}s + 1}{\frac{s^2}{\Omega_L^2} + \frac{2\zeta_L}{\Omega_L} s + 1} \quad (7)$$

Where  $\Omega_L$  is the locked motor frequency (natural frequency of the mechanism) and  $\zeta_L$  is the damping ratio of the mechanism

$$\Omega_L = \sqrt{\frac{c_{32}}{J_3}}, \quad \zeta_L = \frac{k_{32}}{2} \cdot \sqrt{\frac{1}{J_3 \cdot c_{32}}} \quad (8)$$

The transfer function from  $\omega_1$  to  $M_1$  gives the relation for the natural frequency of the entire system, so called resonance frequency

$$\frac{\varphi_1}{M_1} = \frac{p^2}{s^2} \cdot \frac{\frac{s^2}{\Omega_L^2} + \frac{2\zeta_L}{\Omega_L} s + 1}{(p^2 J_1 + J_3) \left( \frac{s^2}{\Omega_{LM}^2} + \frac{2\zeta_{LM}}{\Omega_{LM}} s + 1 \right)} \quad (9)$$

Where  $\Omega_{LM}$  is the resonance frequency and  $\zeta_L$  is the damping ratio of the two-mass system

$$\begin{aligned} \Omega_{LM} &= \sqrt{\frac{(p^2 J_1 + J_3) c_{32}}{p^2 J_1 J_3}} \\ \zeta_L &= \frac{k_{32}}{2} \cdot \sqrt{\frac{(p^2 J_1 + J_3)}{p^2 J_1 J_3 c_{32}}} \end{aligned} \quad (10)$$

Fig. 4 shows the frequency characteristic of transfer functions from  $M_1$  to  $\omega_1$ . The arrows highlight two important points, resonant and anti-resonant frequencies, which refer to the (8) and (10).

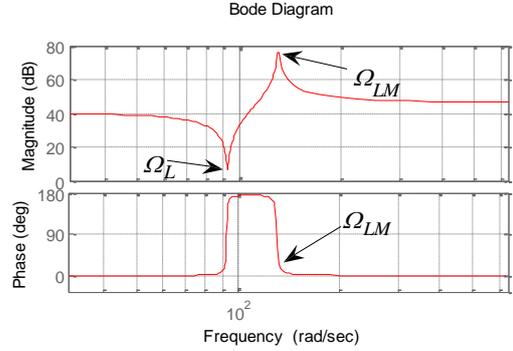


Figure 4: Frequency characteristic of two-mass system.

The damping constant of such systems has usually negligible value. That's why in such cases it is possible to consider the dumping as a zero.

## OVERVIEW OF THE STRATEGY

The control strategy is based on Inverse Model Control (IMC) described in [7]. The main idea of this method is to use as a controller (vibration compensator) inverse transfer function of controlled system. Since the controller has the inverse behavior with regard to the controlled system, thus its dynamics acts against the controlled system dynamics. The actual value of the end link position almost exactly copies the setpoint. The successful application of this control strategy needs to know exact description of the controlled system. If the linear system transfer function is known, then the controller design is simple.

Inverse controller is obtained simply by the controlled system transfer function inversion, thus swapping the numerator and denominator of controlled system transfer function. But this operation often generates a not physically realizable transfer

function of the controller. This happens always when the polynomial order in the denominator is higher than in numerator. Acquired transfer function of controller has a derivative character, thus has lower denominator polynomial order than numerator and is not physically realizable. If we want to ensure feasibility, we must add into controller transfer function additional polynomial  $(\lambda \cdot s + 1)$ .  $\lambda$  must be chosen so as to not cause the system instability and without to not slow down the control process significantly. Multiplicity of added poles must be at least such that the numerator and denominator polynomial, in other words the number of zero and poles, are the same.

However, this control structure in the form of the transfer function is more suitable for use in special signal processors. Standard servo control unit typically use to control the servomotor position or speed cascade control structure. In fact the control structure is more complicated than just the close loops and PI controllers. It is equipped with a controlled values limiters, setpoints filters, etc. Into existing control structure it is quite problematic to implement something. More comfortable way than to realize the special control structure is to use one of the current setpoint filters (see Fig. 5).

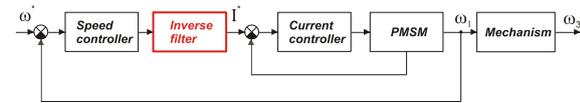


Fig. 5: The control structure with enhanced speed controller.

In addition to its function the speed controller generates the current setpoint. The current loop is innermost and therefore the fastest. That's why is the vibration compensator (filter) embedded just behind the speed controller to get the fastest control action. The filter increases order of speed controller. Its task is to adjust the controller action variable to suppress the residual oscillations. Speed Controller is used only as an accelerating component of control process. The filter can be design two different ways. We can analyze the frequency response and try to suppress the resonant frequency or establish the exact model of a system and try to find an inverse transfer function. We'll describe design of the inverse transfer function base on the method IMC and show the analogy from the frequency respond analysis.

To the design filter, we need to know the transfer function of the system. Because the compensator will be placed after the speed controller (see Fig. 5) must consists of closed servo current control loop and mechanical chain. The desired transfer function (11) is possible to express from the marked line in Fig.6.

$$\frac{\varphi_3(s)}{I_1^*(s)} = \frac{2.2e^5 s^2 + 4.3e^8 s + 1.6e^{11}}{s^5 + 2.8e^3 s^4 + 1.4e^6 s^3 + 3.6e^7 s^2 + 1.4e^{10} s} \quad (11)$$

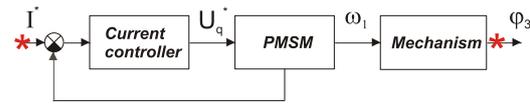


Fig. 6: Block diagram with marked direct link of desired transfer function.

As it is clear from transfer function (11) the denominator has fifth order polynomial. Fifth order filter design and its implementation into the control unit would be complicated. However as will be shown it is sufficient to set up filter from the problematic poles of the system, which evokes the vibration.

When we display poles and zeros from (11) in the complex plane, we can see that the total transfer has only one pair of complex poles. From the chart we can also read a frequency cause by these poles. We can read the same frequency from frequency response in figure 4. This frequency is consistent with frequency of residual oscillations its value is approximately 100 rad/sec. By suppressing just these complex poles can be achieved desired reduction of residual vibrations.

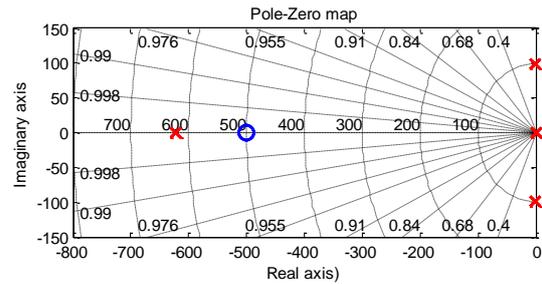


Fig. 7: Distribution of poles and zeros except of remote pole - 2100.

Elimination of system complex poles could be done by put the same complex roots to the compensator numerator. However because the controller has a derivative character it is necessary to add the same number of poles, to get physically realizable controller transfer function. Adding the poles follow the same principles as for the IMC described above. After adding a double real pole  $(\lambda \cdot s + 1)$ , where  $\lambda = 0,005$  is a filter transfer function as follows

$$F_{IF} = \frac{1e^{-4} s^2 + 6.7e^{-3} s + 1}{(5e^{-3} s + 1) \cdot (5e^{-3} s + 1)} \quad (12)$$

## IMPLEMENTAION FILTER INTO THE STANDARD CU

From the transfer is evident that this it is a filter with infinite response (IIR). The filter is second order, because the numerator and denominator are second order polynomials. The Sinamics S120 CU control structure as well as other CUs from other producers is equipped with several degrees of speed or current setpoint filters. In case of S120 it is possible to set

those filters as a low-pass second order filter, low-pass with reduction by constant value, band-stop, or general second order filter. To implement the filter into the drive control structure we can use a general 2nd order filter, but it is necessary to modify the form of the controller by the following formula to get the filter numerator and denominator natural frequency and dumping

$$F_{IF} = K \cdot \frac{s^2 + 2\xi_N \omega_N s + \omega_N^2}{s^2 + 2\xi_D \omega_D s + \omega_D^2} \quad (13)$$

## EXPERIMENTS ON THE STANDARD SERVO CONTROL UNIT

As is mentioned above for the experiment was used the PMSM with servo control unit Sinamics S120 from the portfolio of automation components Siemens Company. Servo controller uses for controlling the position cascade control structure with using the DSC dynamic active component of position controller. DSC accelerates position loop by using a speed feedforward and special algorithm described in [12]. That enables to increase position gain in relation to the sampling times and thus increases the dynamic response of the drive.

An inverse dynamic controller was implemented into the structure by using one of the current setpoint filters. To do this was necessary to express denominator and numerator natural frequency and dumping from the inverse filter transfer function. This can be done simply by comparing equal squares coefficients of equations (12) and (13).

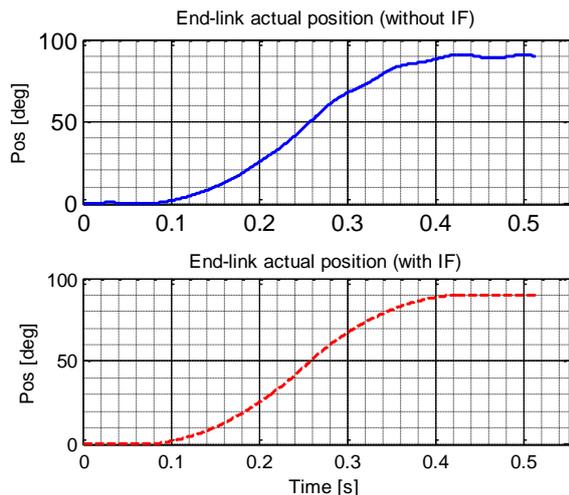


Fig. 8: End-link actual position response with disabled (blue-solid) and enabled (red-dashed) inverse filter vibration compensator.

It also exist another option to implement inverse dynamic controller into the S120 control structure. The programmable memory, open control structure and programming environment DCC Chart enable to

create the transfer function and put it at the specific place in the cascade structure. The main disadvantage of this approach is an execution time of the DCC structures its minimal value is 1ms. Using such filter in the current control loop, which works with low-order cycle times would be inefficient.

In the figure 8 is shown the responses of the experimental two-mass system end-link position to the parabolic cam excitation. The blue solid line shows the response of the system without using inverse filter. The unsmoothed residual vibration is clearly evident. The second chart shows the response of the system with using inverse filter as a second order setpoint filter.

## CONCLUSION

The inverse dynamic control method described in the article is the effective method for two-mass system residual vibration compensation. The article describes their principals, design and its implementation into the control unit. The results of experiments on the real system prove its ability to suppress the residual vibration. The main advantage of this method is that it is easily implementable to the conventional servo controllers.

The residual frequency depends on mechanical parameters. This method has robustness in a small scale against the system parameters changing. In case that we want to apply this controller for suppress residual vibration of system with variable mechanical parameters would be necessary extend the structure with vibration frequency detector and accordingly change the inverse filter parameters. But this extension wasn't experimentally proved.

## ACKNOWLEDGMENT

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