

Three-body segment musculoskeletal model of the upper limb

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Abstract

The main aim is to create a computational three-body segment model of an upper limb of a human body for determination of muscle forces generated to keep a given loaded upper limb position. The model consists of three segments representing arm, forearm, hand and of all major muscles connected to the segments. Muscle origins and insertions determination corresponds to a real anatomy. Muscle behaviour is defined according to the Hill-type muscle model consisting of contractile and viscoelastic element. The upper limb is presented by a system of three rigid bars connected by rotational joints. The whole limb is fixed to the frame in the shoulder joint. A static balance problem is solved by principle of virtual work. The system of equation describing the musculoskeletal system is overdetermined because more muscles than necessary contribute to get the concrete upper limb position. Hence the mathematical problem is solved by an optimization method searching the least energetically-consuming solution. The upper limb computational model is verified by electromyography of the biceps brachii muscle.

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Keywords: upper limb musculoskeletal model, muscle modeling, Hill-type muscle model, EMG measurement

1. Introduction

Human body modeling becomes a powerful tool for safety studies in automobile industry, helps to propose working environments or operations tools, helps to improve endoprosthesis or therapies. Proposed study is motivated by comfort problems.

The main aim of this study is to develop a computational model of an upper limb for determining muscle forces generated to keep a given loaded position.

The upper arm model consists of three segments (arm, forearm, hand) and of all major muscles. The segments are represented by rigid bodies and connected by rotational joints. Muscles are represented by string elements. Their origins and insertions are fixed to the segments according to a real anatomy. Muscle behaviour is defined by the Hill-type muscle model consisting of a contractile element and a viscoelastic element. The segments and the muscles are positioned in a 3D space, nevertheless an limb movement is performed only in a 2D space. The whole upper limb is fixed in the shoulder due to neglecting of pectoral muscles.

A principle of virtual work is used to solve a static balance problem. Defined musculoskeletal system is overdetermined. It means that more muscles cooperate to get the given loaded position than it is necessary. Therefore this problem is solved using an optimization method searching the least energetically-consuming solution. Since it is a complicated control system, three different objective functions are used and obtained results are compared.

The upper limb model is verified by previously measured data using a standard clinical method called electromyography. The measurement is performed with the biceps brachii muscle, the biggest muscle of the upper limb.

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2. Segmental model of an upper limb

Proposed study is focused on a computational model of an upper limb. Morphological proportions of this model correspond to values of a mid-sized man [6].

2.1. Structure of the segmental model

The upper limb model consists of three segments representing arm, forearm and hand. All segments are modeled by rigid bodies connected by rotational joints. These connections represent shoulder, elbow and wrist. The rigid bodies are defined by centers of gravity and by masses. All segments are positioned in a 3D space. Joint movements are restricted in a planar rotation around joint midpoints. The whole upper limb is fixed in the shoulder.

Presented model includes muscles modeled by string elements. The muscles are fixed to the segments according to their real anatomical location [8]. Only main flexors and extensors of the upper limb are considered.

2.2. Coordinate systems

A global coordinate system is defined by a right-handed cartesian coordinate system. A center of the global coordinate system is located in so called H-point [8]. The H-point is a point which is situated in the middle of the hip joints connection (Fig. 1). X-axis is situated in a transversal plane in ventral direction. Y-axis is in a same plane in mediolateral direction. Z-axis is lying in the middle axis of a human body in cranial direction.

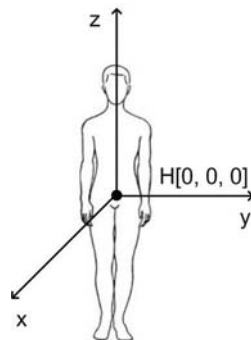


Fig. 1. Global coordinate system of the human body complex, H-point, [6]

Each segment includes own local coordinate system. The center of each local system is situated in the middle of joints connection (i.e. arm – shoulder, forearm – elbow, hand – wrist). The initial position is defined so as the upper limb hangs free along the human body, the palm is rotated forward. In the initial position, the axis of the local systems are oriented in parallel with the global system (Fig. 2). The local systems are fixed to the segments during the segment motions.

2.3. Muscle modeling

Implemented muscles enable to keep a given static position of the upper limb. They are modeled by string elements. Muscle origin and insertion are fixed to the relevant segments. Muscles cross one or more joints. To avoid them some method for muscle wrapping must be implemented, such as described in [9, 12]. To simplify the muscle model special points prescribing the muscle course around the joint are defined (Fig. 3).

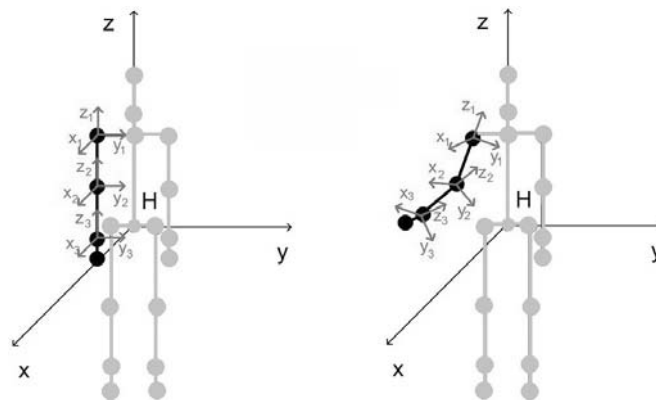


Fig. 2. Local coordinate systems created by segments; initial position (left), rotated general position (right)



Fig. 3. Prescribing of the muscle course around the joint; muscle without prescribed course (left), muscle with prescribed course (right)

Determination of the special point localizations are described using the simplified example including two segments and one muscle string. The special point is defined in the initial position of the model (Fig. 4). The plane, Ω , is oriented in parallel with the plane xy . The joint connection of segments is in Ω plane. Intersection point of the muscle string and the Ω plane is the special point prescribing the muscle course. This point belongs to the local coordinate system of the second segment. Thus, movement of the point depends on the movement of the segment. The point prescribing the muscle course is on the midline of angle formed by segments, see Fig. 4.

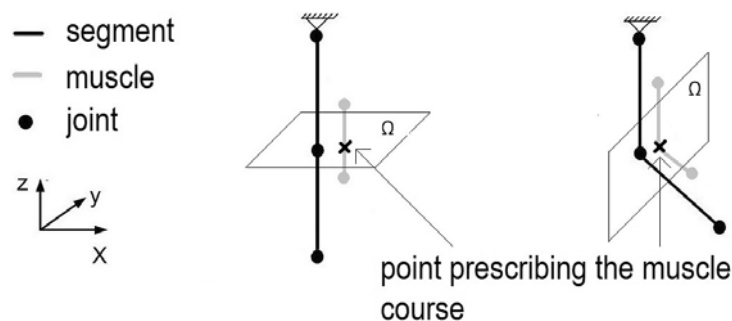


Fig. 4. Localization of the point prescribing the muscle course; initial position of the model (left), general position of the model (right)

Muscle properties correspond to real anatomical and physiological data [8]. Parameters characterizing of each muscle are: the maximal force [4], an optimal muscle length, a physiology cross section area (PCSA), their origin and insertion location kind of muscle, i.e. if the muscle belongs to a group of flexors or extensors.

Muscles including more muscle heads, such as biceps brachii, or having large physiological cross-sectional areas are modeled by several number of independent strings.

2.4. Static balance problem

The mechanical system representing the upper limb model consists of three rigid bodies depicted in Fig. 5. The rigid bodies are defined by given lengths, l_i , and masses, m_i , where $i = 1, 2, 3$. Center of gravity of each rigid body is placed in the middle of appropriate joint connection. The system is fixed to a stationary frame. Rigid bodies are connected by rotational joints. Joint movements are restricted in 2D space. The system is influenced by the gravitational field, by muscle forces and external loads.

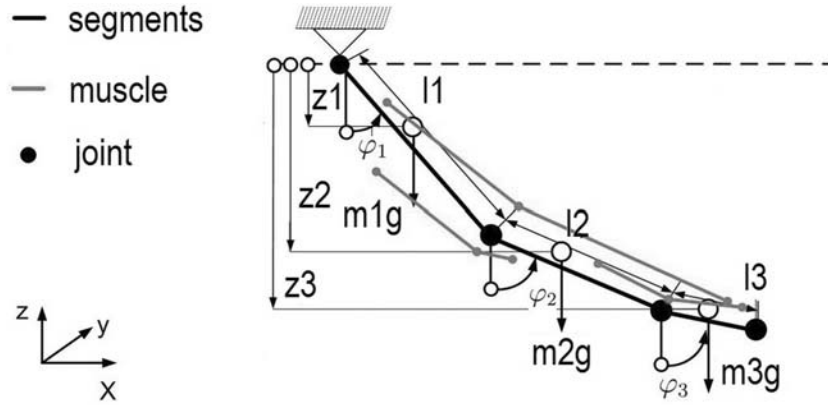


Fig. 5. Mechanical system representing the upper limb; segment parameters: l_i – length, m_i – mass, φ_i – rotation angle

The solution of balance problem comes out of the principle of virtual work [7]:

$$Q_i = -\frac{\partial E_p}{\partial q_i}, \quad (1)$$

where Q_i represents generalized force (joint torque), q_i is generalized coordinate (joint angles) and E_p is potential energy. E_p is given by the following relation:

$$E_p = \sum_{i=1}^M m_i \cdot g \cdot h_i, \quad (2)$$

where m_i is rigid body mass, g is gravitational constant ($g = 9.81 \text{ ms}^{-2}$), h_i is the distance between instantaneous rigid body position and a surface of zero potential energy. M denotes the number of rigid bodies (segments).

The generalized force, Q_i , is given by the sum of unknown muscle forces, F_k , and external loads, \bar{F}_l :

$$Q_i = \sum_{k=1}^K F_k \cdot r_{ik} + \sum_{l=1}^L \bar{F}_l \cdot \bar{r}_{il}, \quad (3)$$

where K is the total number of all muscles and L is the number of external loads. Further r_{ik} and \bar{r}_{il} denotes corresponding moment arm with respect to the i th joint center as depicted in Fig. 6.

Merging Eqs (1) and (3), the energy balance is obtained as:

$$\sum_{k=1}^K F_k \cdot r_{ik} + \sum_{l=1}^L \bar{F}_l \cdot \bar{r}_{il} = -\frac{\partial E_p}{\partial q_i}. \quad (4)$$

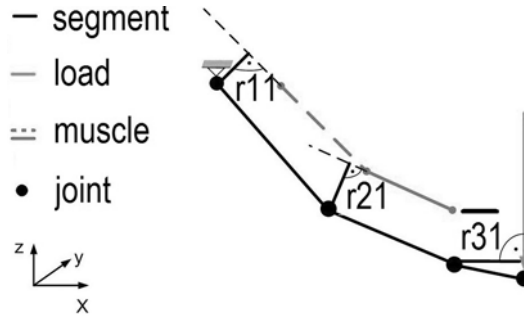


Fig. 6. Arms of forces generated by muscles and external loads

Finally Eq (4) can be written in a matrix notation:

$$\mathbf{A}\mathbf{F} + \bar{\mathbf{A}}\bar{\mathbf{F}} = \mathbf{b}. \quad (5)$$

Used matrices and vectors in (5) can be itemized as:

$$\mathbf{A} = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1K} \\ r_{21} & r_{22} & \dots & r_{2K} \\ & & \ddots & \\ r_{M1} & r_{M2} & \dots & r_{MK} \end{bmatrix}, \quad \bar{\mathbf{A}} = \begin{bmatrix} r_{11}^- & r_{12}^- & \dots & r_{1L}^- \\ r_{21}^- & r_{22}^- & \dots & r_{2L}^- \\ & & \ddots & \\ r_{M1}^- & r_{M2}^- & \dots & r_{ML}^- \end{bmatrix}, \quad (6)$$

$$\mathbf{F} = \begin{bmatrix} F_1 \\ F_2 \\ \vdots \\ F_K \end{bmatrix}, \quad \bar{\mathbf{F}} = \begin{bmatrix} \bar{F}_1 \\ \bar{F}_2 \\ \vdots \\ \bar{F}_L \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -\partial E_p / \partial q_1 \\ -\partial E_p / \partial q_2 \\ \vdots \\ -\partial E_p / \partial q_M \end{bmatrix}, \quad (7)$$

where \mathbf{F} represents the vector of unknown muscle forces, $\bar{\mathbf{F}}$ are known external loads, M is the number of segments, K represents the total number of muscle forces and L is the total number of external loads.

The number of muscle forces is higher than the number of rigid bodies ($M \ll K$). Thus, this system of algebraic equations is overdetermined.

2.5. Optimization

Demanded static position of the upper limb is the result of the complex muscles cooperation. Each muscle contributes with another muscle activation rate. This rate is established by a central nervous system.

The upper limb model is represented by the overdetermined system of Eqs (5). Many combinations of muscle activations exist to keep each given position. The number of unknown muscle forces is much more higher than the number of equations. Thus, the optimization method searching the least energetically-consuming solution is used [9]. The optimization problem is defined as:

- the unknown muscle forces: $\mathbf{F} = \{F_k\}, k = 1, 2, \dots, K,$
- objective function: $F_0(\mathbf{F}),$
- equality constraint function: $f_m(\mathbf{F}) = 0, m = 1, 2, \dots, M,$
- inequality constraint function: $F_i^{lower} \leq F_i \leq F_i^{upper}, i = 1, 2, \dots, K.$

A feasible region is then defined by equality and inequality constraint functions. The equality constraints are given by system of Eqs (5). The inequality constraints respect on the real muscle behaviour:

- $F_i^{lower} = 0$, i.e., muscles can not push,
- $F_i^{upper} = F_i^{max}$, i.e., muscles can not generate force greater than their maximal.

Each human task is controlled by the central nervous system using a particular criterion or a set of criteria such as pain, physical endurance of an individual, time of activity, fatigue, etc. Thus, so many cost functions are described in the literature. The cost function should be able to reflect the inherent physical activity or pathology as well as it should be able to include relevant physiological characteristics and functional properties such as maximum force or activity [10]. Some of the most commonly used non-linear cost function presented in [9] are used and then compared:

- sum of the square of the individual muscle forces: $F_0 = \sum_{k=1}^K (F^k)^2$,
- sum of the cube of the individual average muscle stresses: $F_0 = \sum_{k=1}^K (\sigma^k)^3$,
- sum of the square of the individual normalized muscle forces: $F_0 = \sum_{k=1}^K \left(\frac{F^k}{F_{max}^k} \right)^2$.

The optimization problem is solved by Python optimize package providing several commonly used optimization algorithms. Namely Sequential Least Squares Programming (SLSQP) algorithm is used. The SLSQP optimizer uses a slightly modified version of Lawson and Henson’s nonlinear least-squares solver [3].

3. Results and discussion

3.1. Comparison of the objective functions

The optimization method is used to solve an overdetermined problem of the upper limb muscular balance. Three objective functions are used. Their convergence and number of iteration are compared. One model case is presented — the arm hangs vertically along the body, the upper arm and the forearm form the right angle, the palm is turned to the body. The external load is situated vertically down to the palm and its size is 30 N. The results are summarized in Table 1, muscle forces are shown for the biceps brachii muscle.

Table 1. Comparison of used objective functions

| Objective function | Number of iterations | Force of the biceps brachii muscle [N] |
|--|----------------------|--|
| $F_{01} = \sum_{k=1}^K (F^k)^2$ | 4 | 138.708 108 |
| $F_{02} = \sum_{k=1}^K (\sigma^k)^3$ | 2 | 138.707 543 |
| $F_{03} = \sum_{k=1}^K \left(\frac{F^k}{F_{max}^k} \right)^2$ | 20 | 135.844 856 |

The first and the second objective functions have the comparable resulting values. The second one has in addition the low number of iterations. Therefore this function is used for the following computation.

3.2. Muscle forces in various positions of the upper limb

Tested positions of the upper limb model are described in Table 2 and shown in Figs. 7–10. The model is located in the global cartesian coordinate system. The all modeled segments (represented by black lines), the considered major muscles (represented by gray lines), the all joints (represented by black dots) and all muscle origins and insertions (represented by gray dots) are shown. The gray square represents the H-point located in the center of the global coordinate system.

Table 2. Tested position of the upper limb model

| Position number | Arm rotation [rad] | Forearm rotation [rad] | Hand rotation [rad] |
|-----------------|--------------------|------------------------|---------------------|
| 1 | 0 | 0 | 0 |
| 2 | 0 | $3/4 \pi$ | $\pi/2$ |
| 3 | 0 | $\pi/2$ | $\pi/2$ |
| 4 | $\pi/2$ | $\pi/2$ | $\pi/2$ |

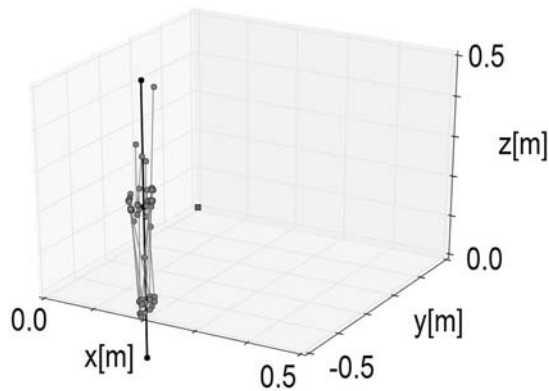


Fig. 7. Position number 1

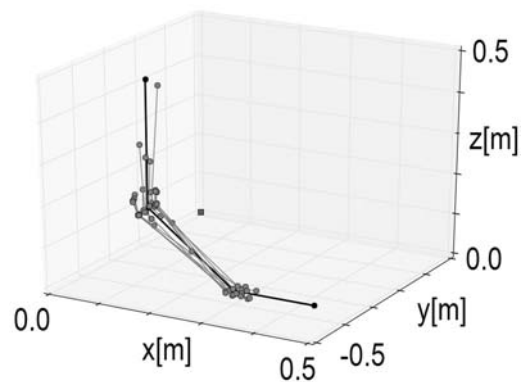


Fig. 8. Position number 2

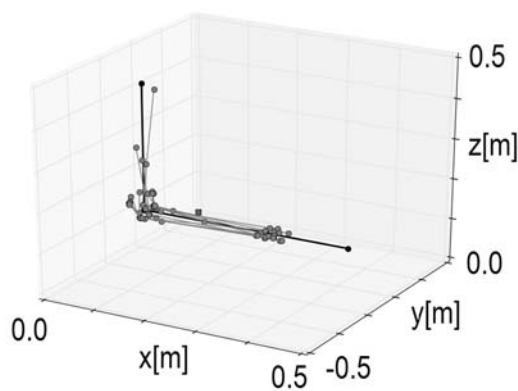


Fig. 9. Position number 3

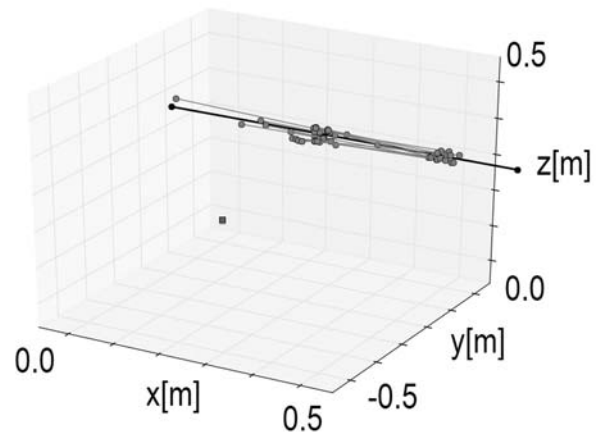


Fig. 10. Position number 4

Discussed positions differ in segments positions defined by arm rotation, forearm rotation and hand rotation.

The upper limb is loaded by the weight of 30 N. The load is applied to the end of the hand. Its orientation is vertically down for each discussed case. Table 3 summarizes a comparison of resulting forces generated by the biceps brachii muscle to keep the four discussed static positions.

Table 3. Resulting forces generated by the biceps brachii muscle for different static position of the upper limb model

| ID | Muscle name | Muscle force [N] | | | |
|----|--------------------------------|------------------|------------|------------|------------|
| | | Position 1 | Position 2 | Position 3 | Position 4 |
| 1 | biceps brachii (LH) | 0.0 | 125.981 | 138.706 | 139.191 |
| 2 | brachioradialis | 0.0 | 169.028 | 215.142 | 215.892 |
| 3 | triceps brachii | 0.0 | 0.0 | 0.0 | 0.0 |
| 4 | triceps brachii | 0.0 | 0.0 | 0.0 | 0.0 |
| 5 | brachialis | 0.0 | 59.334 | 74.492 | 75.500 |
| 6 | anconeus | 0.0 | 0.0 | 0.0 | 0.0 |
| 7 | pronator teres | 0.0 | 273.811 | 351.334 | 354.255 |
| 8 | extensor carpi radialis longus | 0.0 | 0.0 | 0.0 | 0.0 |
| 9 | extensor carpi radialis brevis | 0.0 | 0.0 | 0.0 | 0.0 |
| 10 | extensor dig. com. | 0.0 | 0.0 | 0.0 | 0.0 |
| 11 | extensor car. ul. | 0.0 | 0.0 | 0.0 | 0.0 |
| 12 | flexor car. ul. | 0.0 | 55.929 | 73.162 | 74.357 |
| 13 | flexor carpi radialis | 0.0 | 42.636 | 57.842 | 58.786 |
| 14 | flexor pollicis longus | 0.0 | 68.861 | 93.420 | 94.945 |
| 15 | flexor dig. sub. | 0.0 | 14.896 | 20.209 | 23.200 |

Obviously, the muscles don't generate any activation to keep the first static position (Table 3). The comparison shows that the muscle activation grows up for the next static positions. Several muscles such as triceps brachii muscle generate the zero activation all the time. These muscles are antagonists for discussed positions.

4. Model verification using biceps brachii muscle

4.1. Experiment set up

Presented upper limb model is verified by previously measured data using a clinical method called electromyography (EMG) [11]. EMG is a standard measuring method based on an analysis of electrical signals closely associated with muscle activations [5].

The test is performed by measuring the EMG activity on the biceps brachii muscle when carrying a given increasing load. The EMG signal is measured using surface electrodes.

The volunteer holds his right upper limb flexed in the elbow so that the arm and the forearm form the right angle. The palm is turned to the body. The weight is hung on the hand so that



Fig. 11. Measurement of biceps brachii EMG signal

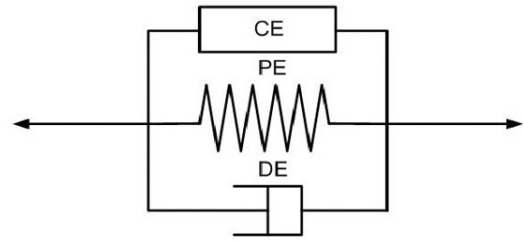


Fig. 12. Hill-type muscle model

the load is applied on the supposed center of gravity of the hand. The electrodes are stuck on the skin. The experiment set up is shown in Fig. 11. The weight is subsequently increased from 0.5 kg to 5 kg with the step 0.5 kg. For each load the EMG signal of biceps brachii muscle is monitored.

As EMG output the EMG activity-time relation is obtained. The raw signal is then filtered, rectified and normalized as is published in detail in [11].

4.2. Hill-type muscle model implementation

EMG measurement is useful for verification of muscular forces determination. Up to now the particular total muscle force of the upper limb model were determined. Obviously the normalized EMG signal is compared to the active part of the total muscle force. Hence the Hill-type muscle model respecting the active and passive muscle properties [12] is implemented.

The Hill-type muscle model consists in general of three elements as depicted in Fig. 12. The contractile element (CE) represents an active muscle part, the parallel elastic element (PE) represents a passive part and the parallel dumping element (DE) substitutes viscous muscle properties. The parallel elements represent a collagen and elastin network of muscle. The resulting muscle force is then calculated with the following equations:

$$F_{muscle}(t, l, v) = N_a(t)F_{CE}(l, v) + F_{PE}(l) + F_{DE}(v), \quad (8)$$

$$F_{CE}(l, v) = F_l(l)F_v(v), \quad (9)$$

where the total muscle force, F_{muscle} , is obtained from the presented upper limb model, $N_a(t)$ is a function determining the muscle active state, $F_{CE}(l, v)$ is a force generated by contractile element, $F_{PE}(l)$ is given by parallel elastic element, $F_{DE}(v)$ is the force in viscous element, $F_l(l)$ represents an active force-length characteristic, $F_v(v)$ is an active force-velocity characteristic.

Mathematical expressions of mentioned characteristics, F_l , F_v , F_{PE} , and their parameters are described by equations in [12]. The active muscle force-length relation, $F_l(l)$, is given by the following formula:

$$F_l(l) = F_{max} \exp \left(- \left(\frac{\frac{l}{l_{opt}} - 1}{C_{sh}} \right)^2 \right), \quad (10)$$

where F_{max} substitutes the maximal muscle force, l is the instantaneous muscle length, l_{opt} is the optimal muscle length when the muscle can generate the maximal force and C_{sh} is the shape parameter determining concavity of muscle force-length characteristic.

The force-velocity characteristic, $F_v(v)$, is defined as the relation between the maximum muscle force and instantaneous rate of the muscle length change when a muscle is fully activated, described in [12]. In considered loaded static position of the upper arm, the muscles generate forces without changing their length. In this case of isometric contraction, the force-velocity characteristic is constant of value one ($F_v(v) = 1$).

The parallel elastic element, F_{PE} , is calculated as:

$$F_{PE} = \frac{F_{max}}{\exp(C_{PE}) - 1} \left\{ \exp \left[\frac{C_{PE}}{PE_{max}} \left(\frac{l}{l_{opt}} - 1 \right) \right] - 1 \right\}, \quad (11)$$

where C_{PE} and PE_{max} are the shape parameters of defined dependencies. According to [12] they are set as follows:

$$C_{sh} = 0.4, \quad C_{PE} = 5.0, \quad PE_{max} = 0.6.$$

The force in the parallel damper element, F_{DE} , is given by the formula:

$$F_{DE} = k_{DE}v, \quad (12)$$

where k_{DE} is the damping coefficient, v is the muscle contraction velocity. Due to presented case of isometric contraction, the force of dumper element is zero ($F_{DE} = 0$).

Substituting (9)–(12) into (8) the relation for the muscle activation, N_a , is obtained:

$$N_a(t) = \frac{F_{muscle} - F_{PE}(l)}{F_{CE}(l)}. \quad (13)$$

The external loading of the upper arm model is constant for each mentioned position. Thus, the generated muscle force is constant too and the muscle activity is than constant in time.

4.3. Comparison of muscle activation using biceps brachii muscle

For the upper limb model verification it is necessary to normalize obtained muscle activations. The activations are normalized by the maximal activations that can be generated by the muscle [1, 2]. However, determination of the maximal muscle activation is usually not flawless in practice. Thus, instead of the maximal activation the activation generated with 5 kg of load ($N_{a_{max}}$) is considered to normalize the results:

$$N_{a_{norm}} = \frac{N_{a_i}}{N_{a_{max}}}. \quad (14)$$

Table 4 shows the comparison of the measured and the computed muscle activations of biceps brachii muscle using three presented objective function. These values are computed and measured, for ten different loads. Obtained values determine muscle activities that are exerted by the biceps brachii muscle to keep the limb position loaded from 0.5 kg to 5 kg. All cases are normalized by the case with 5 kg of load.

Fig. 13 shows the positioned upper limb model.

The values of normalized muscle activations, as well as differences, are the same using the first and the second objective function. The third function has much different resulting values. Moreover, the second one has the low number of iterations. Thus, this function is considered to be the most suitable of all discussed functions.

The values of measured muscle activation of biceps brachii muscle are comparable to the computed values. The upper limb is verified.

Table 4. The comparison of the measured and computed activation of the biceps brachii muscle normalized by case with 5 kg of load, where F_{01} , F_{02} , F_{03} are the used objective function

| Loading [kg] | Normalized muscle activations [-] | | | | Difference [%] | | |
|--------------|-----------------------------------|------------|----------|----------|----------------|----------|----------|
| | Model | Experiment | | | F_{01} | F_{02} | F_{03} |
| | | F_{01} | F_{02} | F_{03} | | | |
| 0.5 | 0.180 | 0.209 | 0.209 | 0.203 | 13.06 | 13.6 | 16.1 |
| 1.0 | 0.271 | 0.297 | 0.297 | 0.294 | 12.6 | 12.6 | 13.5 |
| 1.5 | 0.362 | 0.385 | 0.385 | 0.383 | 8.6 | 8.6 | 9.1 |
| 2.0 | 0.454 | 0.473 | 0.473 | 0.411 | 10.1 | 10.1 | 21.9 |
| 2.5 | 0.545 | 0.561 | 0.561 | 0.488 | 2.9 | 2.9 | 15.6 |
| 3.0 | 0.636 | 0.648 | 0.648 | 0.564 | 9.6 | 9.6 | 4.5 |
| 3.5 | 0.727 | 0.736 | 0.736 | 0.735 | 11.3 | 11.3 | 11.2 |
| 4.0 | 0.818 | 0.824 | 0.824 | 0.823 | 3.3 | 3.3 | 3.1 |
| 4.5 | 0.909 | 0.912 | 0.912 | 0.912 | 13.3 | 13.3 | 13.3 |
| 5.0 | 1.000 | 1.000 | 1.000 | 1.000 | 0.00 | 0.0 | 0.0 |

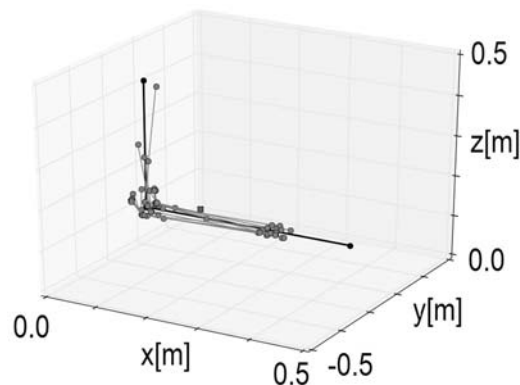


Fig. 13. Resulting model of the upper limb

5. Conclusion

The main aim of proposed study is to compose a computational upper limb model of a human body. The model computes muscle forces generated to keep a given loaded static upper limb position. The upper limb model is implemented in a programming language called Python.

Model consists of three rigid segments representing the arm, the forearm and the hand. These segments are connected by rotational joints. Joint movements are restricted to a planar. The whole upper limb model is fixed to a frame in the shoulder joint. Muscles are modeled by strings fixed to the segments according to the real anatomy.

A static balance problem is solved by a principle of virtual work. The musculoskeletal system is overdetermined. It means that more muscles contribute to keep a given static position than it is necessary. Thus, this problem is solved by an optimization method searching the least energetically-consuming solution. Three objective functions are used and then compared. The normalized muscle activations, as well as differences, are the same using the first and the second function. The third function has different resulting values. The second function has in addition

the lower number of iterations. Thus, the second objective function is considered to be the most suitable.

The model is verified by EMG measurement. The normalized measured and computed muscle activations of the biceps brachii muscle are compared. The analysis shows that the values are comparable.

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