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Transient and Sensitivity Analysis at Semi-Discrete Multiconductor Transmission Line Models

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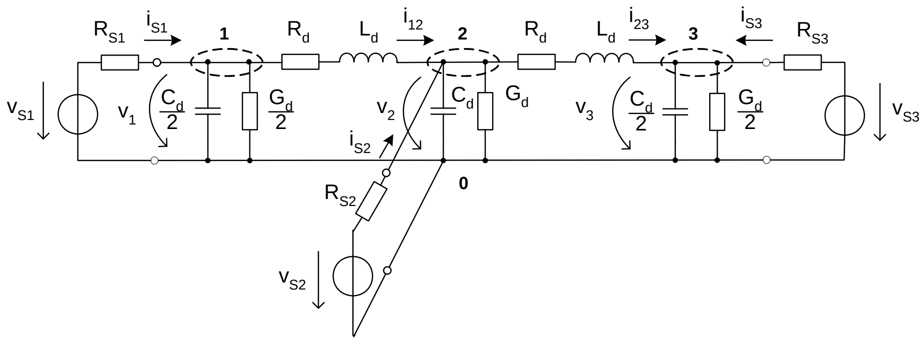
Abstract:
The paper deals with methods for the solution of transient phenomena and sensitivities at semi-discrete models of multiconductor transmission lines (MTL). The models are formed by a cascade connection of generalized lumped-parameter Π networks, they can be fed in arbitrary nodes, nonzero initial conditions can also be taken into account. The solutions shown here are formulated following the state-variable method. First the equations enabling to solve voltage and current distributions along the MTL wires are derived, both in the time domain and in the operational domain. Second the formulae for sensitivities of voltage and current distributions are derived, based on the *s*-domain solution. The sensitivities can be stated with respect to parameters of per-unit-length matrices, the MTL length and lumped parameters of the external circuits as well. In this case the time-domain solutions are obtained by using a numerical inversion of Laplace transforms. The matrices occuring in all the formulae are sparse which enables to take into account large numbers of Π sections in the models while using common PC for the computation. The Matlab language environment is used with advantages for this purpose.

INTRODUCTION

The issues connected with a propagation of voltages and currents on multiconductor transmission lines (MTL) play important role in a design of modern high-speed circuits [1]. Besides they also find their place at simulation of power lines behaviour [2]. The sensitivity analysis helps to find critical components affecting behaviour of the system and to perform its optimal design. There are various approaches how to simulate behaviour of the MTLs computer-aided. Beside a solution of the matrix telegraph equations, based either on Laplace transform approaches [3], [4] (MTL continuous models) or on FDTD approaches [4], [5] (MTL discrete models), the solution can also be based on the models formed by lumped-parameter structures. These are usually in the form of a cascade connection of generalized Π or T networks. As only a geometrical coordinate is discretized here, the models are refered as semi-discrete ones [1], [4]. In this case well-known methods of the circuit theory can be used to find the solution. Although this approach may be less accurate, depending on properties of the model itself, it is easier to configure it to consider various inhomogeneities (e.g MTL imperfections), or to drive arbitrary model nodes to simulate strokes of lightning at power lines, for example. In this paper the model is composed of a cascade connection of generalized Π networks, while its solution is based on the state-variable method. Specially a direct solution in the time domain based on the matrix convolution integral evaluation and the Laplace transform technique are compared as for their computational efficiency. Further, formulae for determination of sensitivities are derived following the *s*-domain solution. The sensitivities with respect to elements of MTL per-unit-length matrices, a MTL length and lumped parameters of the external circuits can be stated. To get the solution in the time domain a method for the numerical inversion of Laplace transforms (NILT) is applied. In this paper the NILT method based on the FFT in conjunction with the quotient-difference algorithm is applied [6]. To enable the solution on a common PC a sparse matrix notation is considered to save the computer RAM and CPU time considerably. Such a computation can very effectively be done e.g. in the Matlab language environment.

MTL LUMPED-PARAMETER MODEL

We start with a lumped-parameter model of a single, two-conductor, uniform transmission line (TL) in Fig. 1, which is here reduced to only two Π sections in cascade. All the nodes can be fed from an external circuits described by their Thévenin equivalents, with nonzero internal resistances. It is correct presumption if real feeding circuits are considered. Besides initial capacitor voltages and inductor currents can exist modeling initial voltage and current distributions along the wires of the original TL. By applying mixed cut-set and loop analysis the state equations in a matrix form can be written by (1). In this example, 2 sections led to 5 state variables. Generally, m sections result in $2m$+1 state variables, m+1 capacitor voltages and m inductor currents. If we denote l as a length and L_0, R_0, C_0 and G_0 as per-unit-length parameters of the TL, the lumped parameters of the model in Fig. 1 are defined as $L_d = L_0 l/m$, $R_d = R_0 l/m$, $C_d = C_0 l/m$ and $G_d = G_0 l/m$. The source currents can be determined by $i_{Sk} = (v_{Sk} - v_k)/R_{Sk}$, as internal resistances are nonzero.



Obr. 1: Reduced Π-network model of single uniform transmission line

$$\left[\begin{array}{ccc|cc} C_d/2 & 0 & 0 & 0 & 0 \\ 0 & C_d & 0 & 0 & 0 \\ 0 & 0 & C_d/2 & 0 & 0 \\ \hline 0 & 0 & 0 & L_d & 0 \\ 0 & 0 & 0 & 0 & L_d \end{array}\right] \cdot \frac{d}{dt} \left[\begin{array}{c} v_1(t) \\ v_2(t) \\ v_3(t) \\ \hline i_{12}(t) \\ i_{23}(t) \end{array}\right] = -\left(\left[\begin{array}{ccc|cc} G_d/2 & 0 & 0 & 1 & 0 \\ 0 & G_d & 0 & -1 & 1 \\ 0 & 0 & G_d/2 & 0 & -1 \\ \hline -1 & 1 & 0 & R_d & 0 \\ 0 & -1 & 1 & 0 & R_d \end{array}\right] + \left[\begin{array}{ccc|cc} G_{S1} & 0 & 0 & 0 & 0 \\ 0 & G_{S2} & 0 & 0 & 0 \\ 0 & 0 & G_{S3} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \right) \cdot \left[\begin{array}{c} v_1(t) \\ v_2(t) \\ v_3(t) \\ \hline i_{12}(t) \\ i_{23}(t) \end{array}\right] + \left[\begin{array}{ccc|cc} G_{S1} & 0 & 0 & 0 & 0 \\ 0 & G_{S2} & 0 & 0 & 0 \\ 0 & 0 & G_{S3} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \left[\begin{array}{c} v_{S1}(t) \\ v_{S2}(t) \\ v_{S3}(t) \\ \hline 0 \\ 0 \end{array}\right] \quad (1)$$

MTL *m*-Sectional Model Formulation

In case of an $(n+1)$-conductor transmission line we consider $n \times n$ per-unit-length matrices $\mathbf{L}_0$, $\mathbf{R}_0$, $\mathbf{C}_0$ and $\mathbf{G}_0$ to describe its properties. In Fig. 2 there is a two wire segment of the Π-network model of such a MTL. As a result each component in (1) becomes an $n \times n$ square matrix, or an $n \times 1$ column vector as for voltages and currents. Then a general description in a formal matrix form can be given as [7]

$$\mathbf{M}\frac{d\mathbf{x}(t)}{dt} = -\left(\mathbf{H}+\mathbf{P}\right)\mathbf{x}(t) + \mathbf{P}\mathbf{u}(t) \quad (2)$$

The terms stated above are as follows. The vector

$$\mathbf{x}(t) = \left[\mathbf{v}_C(t) \quad \mathbf{i}_L(t)\right]^T \quad (3)$$

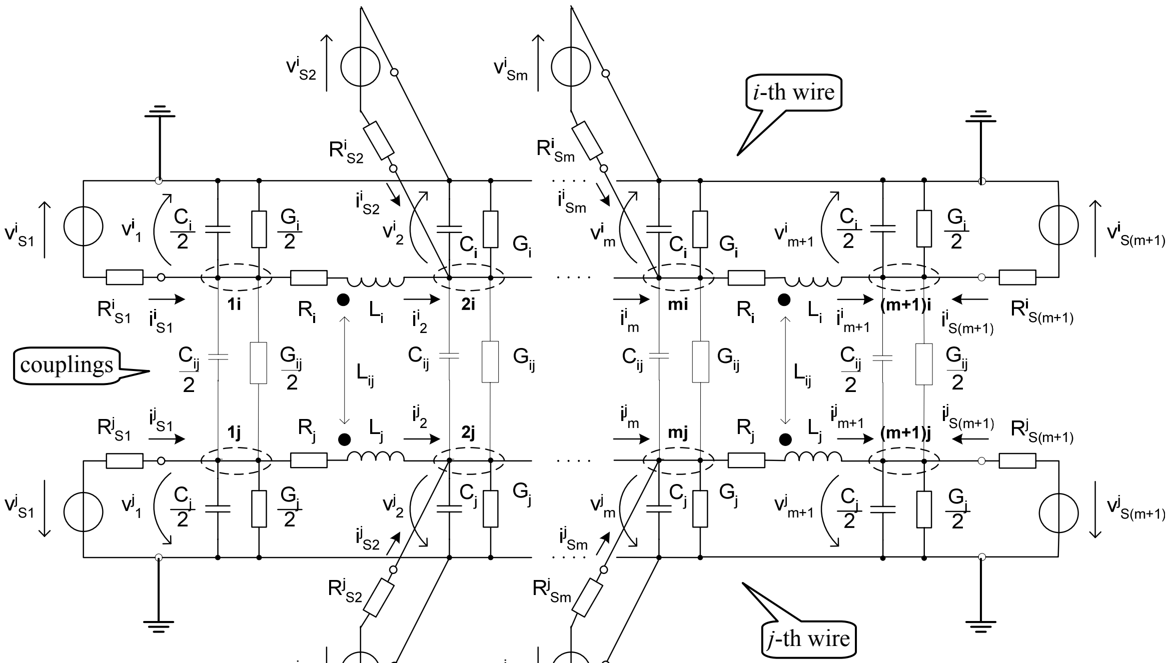
consists of the unknown state variables. Generally, for an m-sectional model, we get $n(2m+1)$ elements inside $\mathbf{x}(t)$, grouped into $n \times 1$ column vectors, namely $\mathbf{v}_C(t)$ holds $m+1$ vectors of capacitor voltages and $\mathbf{v}_L(t)$ holds m vectors of inductor currents. The memory matrix

$$\mathbf{M} = \begin{bmatrix} \mathbf{C} & \mathbf{0} \\ \mathbf{0} & \mathbf{L} \end{bmatrix} \quad (4)$$

can be created via matrices

$$\mathbf{C} = \mathbf{I}_{m+1} \otimes \mathbf{C}_d \quad \text{and} \quad \mathbf{L} = \mathbf{I}_m \otimes \mathbf{L}_d \ , \quad (5)$$

where $\mathbf{I}_{m+1}$ and $\mathbf{I}_m$ are identity matrices of indexed orders, ⊗ denotes a Kronecker tensor product of matrices, $\mathbf{C}_d = \mathbf{C}_0 l/m$ and $\mathbf{L}_d = \mathbf{L}_0 l/m$.



Obr. 2: Two wire segment of MTL *m*-sectional Π-network model excited from external sources

The memoryless matrix

$$\mathbf{H} = \begin{bmatrix} \mathbf{G} & \mathbf{E} \\ -\mathbf{E}^T & \mathbf{R} \end{bmatrix} \tag{6}$$

can be created via matrices

$$\mathbf{G} = \mathbf{I}_{m+1} \otimes \mathbf{G}_d \quad \text{and} \quad \mathbf{R} = \mathbf{I}_m \otimes \mathbf{R}_d \ , \tag{7}$$

where $\mathbf{G}_d = \mathbf{G}_0 l/m$ and $\mathbf{R}_d = \mathbf{R}_0 l/m$. The matrix $\mathbf{E}$ has the structure corresponding to (1) when ±1 and 0 elements are replaced by $\pm\mathbf{I}_n$ (identity) and $\mathbf{0}_n$ (zero) matrices. The matrix

$$\mathbf{P} = \begin{bmatrix} \mathbf{Y}_S & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \tag{8}$$

contains a square submatrix $\mathbf{Y}_S$ depending on external circuits. The circuit description supposes the feeding sources having their regular generalized Thévenin equivalents, when inverse internal matrices exist. The source current vectors are then $\mathbf{i}_{Sk} = \mathbf{R}_{Sk}^{-1}(\mathbf{v}_{Sk} - \mathbf{v}_k)$, and the inverse matrices $\mathbf{R}_{Sk}^{-1}$ form a block diagonal of $\mathbf{Y}_S$. Finally a column vector

$$\mathbf{u}(t) = \begin{bmatrix} \mathbf{v}_S(t) \\ \mathbf{0} \end{bmatrix} , \tag{9}$$

where $\mathbf{v}_S(t)$ consists of the internal voltage vectors of generalized Thévenin equivalents.

MTL MODEL SOLUTIONS

Solution in the Time Domain

The first-order ordinary differential equation (2) has its time-domain solution as

$$\mathbf{x}(t) = e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})t}\mathbf{x}(0) + \int_0^t e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})(t-\tau)}\mathbf{M}^{-1}\mathbf{P}\mathbf{u}(\tau)d\tau \tag{10}$$

when choosing $t = 0$ as the initial instant of time. To evaluate this formula two key parts must be resolved: the matrix exponential function and the convolution integral evaluation. Both are becoming harder to perform due to large order of matrices used. From computational point of view it is therefore more advantageous to write the recursive formula

$$\mathbf{x}(t_k) = e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})(t_k - t_{k-1})}\mathbf{x}(t_{k-1}) + \\ + \int_{t_{k-1}}^{t_k} e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})(t_k-\tau)}\mathbf{M}^{-1}\mathbf{P}\mathbf{u}(\tau)d\tau \tag{11}$$

for $k = 1, 2, 3, \ldots$, and $t_0 = 0$, as a finite set of time points $\{t_k\}$ is always considered in practice. If an equidistant time division $\Delta t = t_k - t_{k-1}$, $\forall k$, is chosen the CPU time can notably be saved. Namely the matrix exponential function can only once be evaluated in this case being highly time-consuming operation for high-order matrices. Generally, in case when the convolution integral has to be evaluated, both proper Δt and a method of numerical integration must deliberately be chosen. For the simplest rectangular rule of the integration the approximate recursive formula can be applied as (derivation based on the theory in [8])

$$\mathbf{x}_k = e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})\Delta t}\mathbf{x}_{k-1} + (\mathbf{I} - e^{-\mathbf{M}^{-1}(\mathbf{H}+\mathbf{P})\Delta t})(\mathbf{H}+\mathbf{P})^{-1}\mathbf{P}\mathbf{u}_k \ , \tag{12}$$

where $\mathbf{x}_k \approx \mathbf{x}(t_k)$, $\mathbf{u}_k = \mathbf{u}(t_k)$ and $\mathbf{I}$ denotes the identity matrix. In case $\mathbf{u}(t) = 0$, i.e. when finding response to the initial condition $\mathbf{x}(0) \neq 0$ only, the recursive formula leads to an exact solution, i.e. $\mathbf{x}_k = \mathbf{x}(t_k)$, independently on a Δt choice. The matrix exponential function can be gained via a Taylor series expansion, eigenvalues/eigenvectors decomposition or method of Padé approximation, for example.

Solution in the *s* Domain

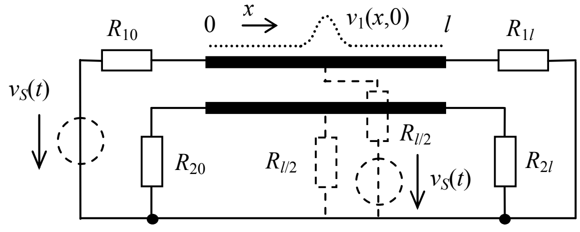
Applying Laplace transform onto (2) and doing some arrangements we get an *s*-domain solution

$$\mathbf{x}(s) = \left(\mathbf{H} + \mathbf{P} + s\mathbf{M}\right)^{-1}\left(\mathbf{M}\mathbf{x}_0 + \mathbf{P}\mathbf{u}(s)\right) \ , \tag{13}$$

where $\mathbf{x}(s) = Ł\{\mathbf{x}(t)\}$ and $\mathbf{u}(s) = Ł\{\mathbf{u}(t)\}$ denote Laplace transforms of time-dependent variables, and $\mathbf{x}_0 = \mathbf{x}(t)|_{t=0}$ is the vector of initial conditions defined by (3). Note that the *s*-domain solution can be generalized towards MTLs driven/terminated by memory-element circuits, via the matrix $\mathbf{P} \equiv \mathbf{P}(s)$. Similarly, possible frequency dependences of the MTL per-unit-length matrices could be incorporated, resulting in $\mathbf{M} \equiv \mathbf{M}(s)$ and $\mathbf{H} \equiv \mathbf{H}(s)$. There are a few approaches how to get the time-domain solution $\mathbf{x}(t) = Ł^{-1}\{\mathbf{x}(s)\}$ [1]. Here the direct application of the NILT algorithm [6] is utilized to obtain a set of approximate solutions $\{\mathbf{x}_k\}$ from (13).

Examples of MTL Waves Distribution

Practical experiments have been performed in the Matlab language environment. Because of high orders of matrices in view they were exclusively treated in the sparse notation. The (2+1)-conductor uniform transmission line system is shown in Fig. 3.



Obr. 3: MTL excited from external or wire initial voltage

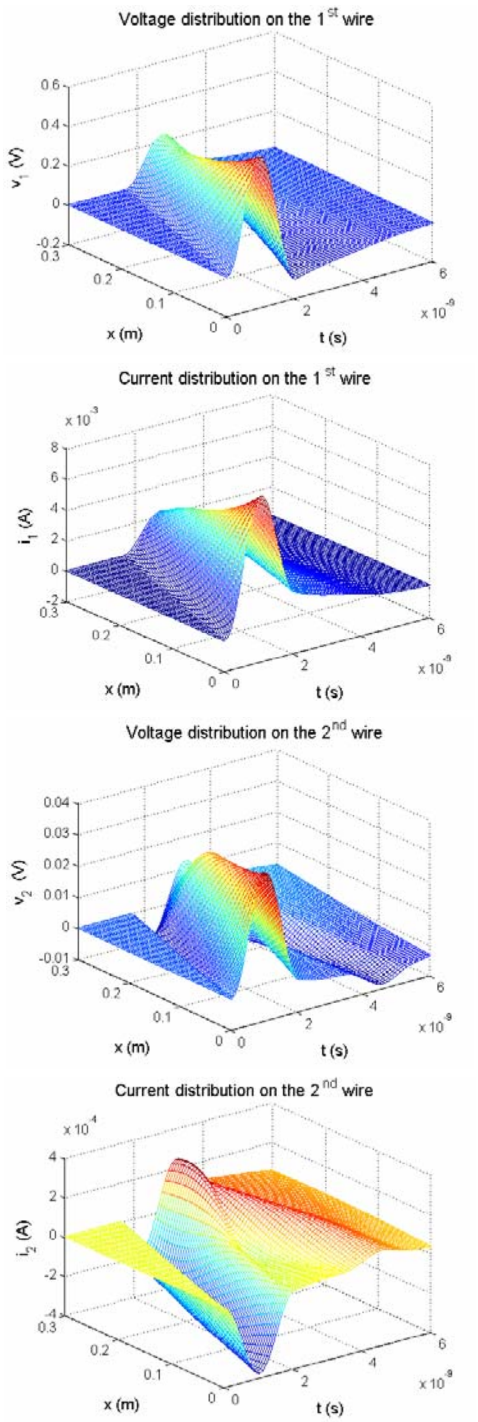
The line is of the length $l = 0.3m$, its per-unit-length matrices are [7]

$$\mathbf{L}_0 = \begin{bmatrix} 494.6 & 63.3 \\ 63.3 & 494.6 \end{bmatrix}\frac{nH}{m}, \ \mathbf{R}_0 = \begin{bmatrix} 0.1 & 0.02 \\ 0.02 & 0.1 \end{bmatrix}\frac{\Omega}{m} \ , \\ \mathbf{C}_0 = \begin{bmatrix} 62.8 & -4.9 \\ -4.9 & 62.8 \end{bmatrix}\frac{pF}{m}, \quad \mathbf{G}_0 = \begin{bmatrix} 0.1 & -0.01 \\ -0.01 & 0.1 \end{bmatrix}\frac{S}{m} \ . \tag{14}$$

The external components are equal $R_{10} = R_{2l} = 50\Omega$, $R_{1l} = R_{20} = 100\Omega$ and $R_{l/2} = 75\Omega$. It results in the Thévenin internal resistance matrices

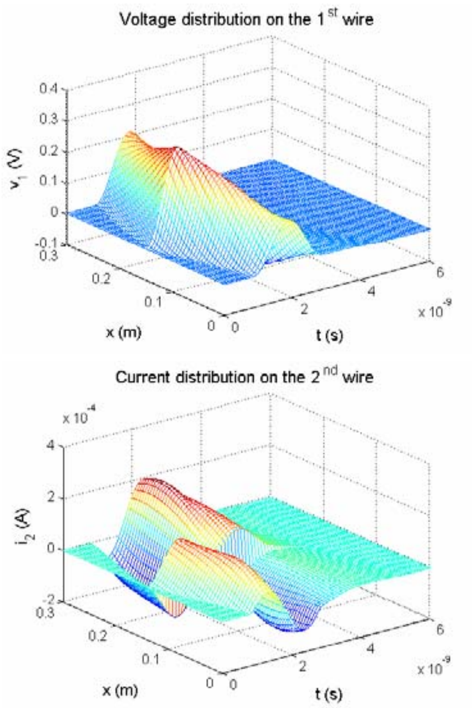
$$\mathbf{R}_{S0} = \begin{bmatrix} 50 & 0 \\ 0 & 100 \end{bmatrix}, \ \mathbf{R}_{Sl} = \begin{bmatrix} 100 & 0 \\ 0 & 50 \end{bmatrix}, \ \mathbf{R}_{S(l/2)} = \begin{bmatrix} 75 & 0 \\ 0 & 75 \end{bmatrix}. \quad (15)$$

The voltage/current distributions when exciting the first wire at $x = 0$, and without $R_{l/2}$ connected, are in Fig. 4. The feeding voltage is $v_S(t) = \sin^2(\pi t/2\cdot 10^{-9})$ if $0 \le t \le 2\cdot 10^{-9}$, and $v_S(t) = 0$ othervise.



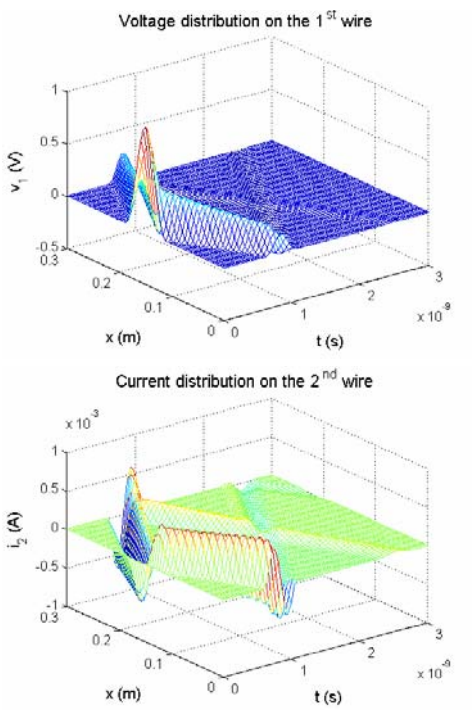
Obr. 4: Voltage/current distributions (MTL fed at $x = 0$)

The voltage/current distributions when feeding the first wire at its middle, $x = l/2$, are shown in Fig. 5. In this case all three internal matrices (15) take effect as complete six resistors are connected to the MTL.



Obr. 5: Voltage/current distributions (MTL fed at $x = l/2$)

The response to initial voltage distribution on the first wire $v_1(x,0) = \sin^2\left(\pi(4x/l - 3/2)\right)$ if $3l/8 \le x \le 5l/8$, and $v_1(x,0) = 0$ otherwise, without $R_{l/2}$ connected, is finally shown in Fig. 6.



Obr. 6: Voltage/current distributions (wire nonzero voltage)

MTL MODEL SENSITIVITY

Herein formulae for the s-domain sensitivities with respect to either MTL parameters or external circuits parameters will be stated. Namely, a differentiation of the s-domain solution (13) with respect to a parameter γ and some arrangements lead to a formula

$$\frac{\partial \mathbf{x}(s)}{\partial \gamma} = -\left(\mathbf{H}+\mathbf{P}+s\mathbf{M}\right)^{-1} \times \\ \times \left(\frac{\partial \mathbf{H}}{\partial \gamma}\mathbf{x}(s) + \frac{\partial \mathbf{M}}{\partial \gamma}\left(s\mathbf{x}(s)-\mathbf{x}_0\right) + \frac{\partial \mathbf{P}}{\partial \gamma}\left(\mathbf{x}(s)-\mathbf{u}(s)\right)\right) . \quad (16)$$

In case the zero initial contidions are only considered, $\mathbf{x}_0 = \mathbf{0}$, the above formula could slightly be simplified, see [9]. The further solution will be split according to a type of γ.

Distributed Parameter Sensitivity

In this case the parameter γ is an element of either one of the per-unit-length matrix $\mathbf{L}_0$, $\mathbf{R}_0$, $\mathbf{C}_0$ or $\mathbf{G}_0$, or the length l. Consequently $\partial\mathbf{P}/\partial\gamma = \mathbf{0}$ and from (16) we can directly write

$$\frac{\partial \mathbf{x}(s)}{\partial \gamma} = -\left(\mathbf{H}+\mathbf{P}+s\mathbf{M}\right)^{-1} \times \\ \times \left(\frac{\partial \mathbf{H}}{\partial \gamma}\mathbf{x}(s) + \frac{\partial \mathbf{M}}{\partial \gamma}\left(s\mathbf{x}(s)-\mathbf{x}_0\right)\right) , \quad (17)$$

where, according to (4) and (6), the derivatives are

$$\frac{\partial \mathbf{M}}{\partial \gamma} = \begin{bmatrix} \frac{\partial \mathbf{C}}{\partial \gamma} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{L}}{\partial \gamma} \end{bmatrix} \quad \text{and} \quad \frac{\partial \mathbf{H}}{\partial \gamma} = \begin{bmatrix} \frac{\partial \mathbf{G}}{\partial \gamma} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{R}}{\partial \gamma} \end{bmatrix} . \quad (18)$$

More detailed specifications are shown in Tab. 1.

Tab. 1: Derivatives of **M** and **H** with respect to γ

Parameter γ	$\frac{\partial \mathbf{M}}{\partial \gamma}$	$\frac{\partial \mathbf{H}}{\partial \gamma}$	$\frac{\partial \mathbf{A}}{\partial A_{ij}}$
$\gamma \in \mathbf{C}_0$	$\begin{bmatrix} \frac{\partial \mathbf{C}}{\partial C_{ij}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\frac{\partial \mathbf{A}}{\partial A_{ij}} = \mathbf{I} \otimes \frac{\partial \mathbf{A}_0}{\partial A_{ij}} \frac{l}{m}$
$\gamma \in \mathbf{L}_0$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{L}}{\partial L_{ij}} \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	
$\gamma \in \mathbf{G}_0$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial \mathbf{G}}{\partial G_{ij}} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	
$\gamma \in \mathbf{R}_0$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$	$\begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{R}}{\partial R_{ij}} \end{bmatrix}$	
$\gamma \equiv l$	$\begin{bmatrix} \frac{\partial \mathbf{C}}{\partial l} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{L}}{\partial l} \end{bmatrix}$	$\begin{bmatrix} \frac{\partial \mathbf{G}}{\partial l} & \mathbf{0} \\ \mathbf{0} & \frac{\partial \mathbf{R}}{\partial l} \end{bmatrix}$	$\frac{\partial \mathbf{A}}{\partial l} = \mathbf{I} \otimes \frac{\mathbf{A}_0}{m}$

Particular derivatives of the matrices **C**, **L** and **G**, **R** can be expressed following (5) and (7), respectively. In the right column of Tab. 1, **A** denotes any of this matrix while $\mathbf{A}_0$ means the corresponding per-unit-length matrix. Finally, the matrices $\mathbf{I} \equiv \mathbf{I}_{m+1}$ or $\mathbf{I} \equiv \mathbf{I}_m$ are identity, just in compliance with (5) and (7).

Lumped Parameter Sensitivity

In this case a parameter γ is an element of $\mathbf{Y}_S$ defining a structure of external circuits, and thus it influences **P**. Consequently $\partial\mathbf{M}/\partial\gamma = \partial\mathbf{H}/\partial\gamma = \mathbf{0}$ and (16) leads to

$$\frac{\partial \mathbf{x}(s)}{\partial \gamma} = -\left(\mathbf{H}+\mathbf{P}+s\mathbf{M}\right)^{-1} \frac{\partial \mathbf{P}}{\partial \gamma}\left(\mathbf{x}(s)-\mathbf{u}(s)\right) . \quad (19)$$

where, in compliance with (8), the derivative is

$$\frac{\partial \mathbf{P}}{\partial \gamma} = \begin{bmatrix} \frac{\partial \mathbf{Y}_S}{\partial \gamma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} . \quad (20)$$

If $\gamma \equiv R_S$ denotes an resistance contained in some Thévenin internal matrix $\mathbf{R}_S$ we have a matrix

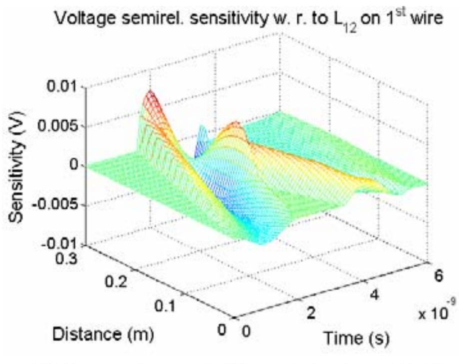
$$\frac{\partial \mathbf{R}_S^{-1}}{\partial R_S} = -\mathbf{R}_S^{-1} \frac{\partial \mathbf{R}_S}{\partial R_S} \mathbf{R}_S^{-1} \quad (21)$$

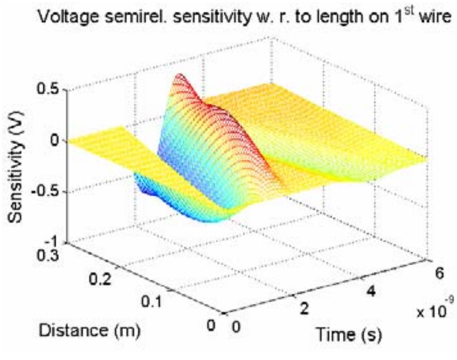
as a submatrix in corresponding diagonal position in $\partial\mathbf{Y}_S/\partial R_S$, with zeros elsewhere.

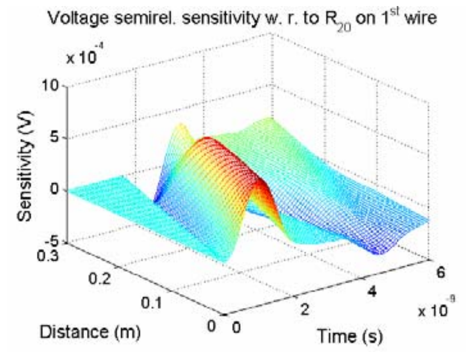
Examples of MTL Sensitivities Distribution

Let us again consider the MTL in Fig. 3, fed at $x = 0$. Semirelative sensitivities shown in Figs. 7 & 8 were computed via NILT method [6], by a formula

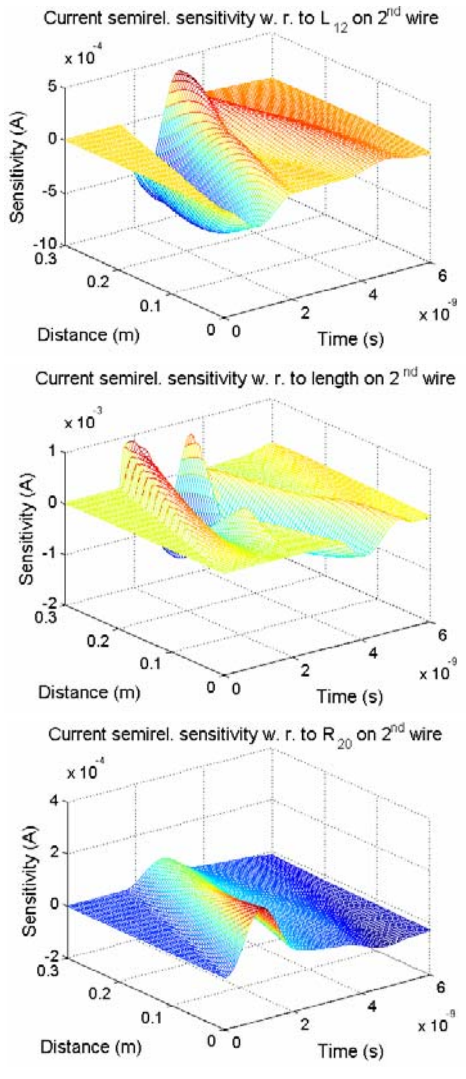
$$\mathbf{S}_\gamma\left(\mathbf{x}(t),\gamma\right) = \gamma \cdot \mathbb{L}^{-1}\left\{\frac{\partial \mathbf{x}(s)}{\partial \gamma}\right\} . \quad (22)$$







Obr. 7: Voltage semirelative sensitivities (MTL first wire)



Obr. 8: Current semirelative sensitivities (MTL second wire)

CONCLUSION

Both discussed techniques, the direct time-domain approach and the Laplace transform approach, led to the same results from practical point of view. They agree very well with the results obtained by solving the continuous MTL model through matrix telegraph equations [3]. Here m was chosen from 128 to 512 which resulted in solving 514 to 2050 equations for the (2+1)-conductor transmission line. Therefore the sparse matrices were utilized in maximal measure for the computation. In case of the NILT method applied to (13) the CPU times were ranging from 2.5 to 13 seconds running on a PC with 2 GHz processor and 2 GB RAM. In case of a direct time domain solution according to (12) the main difficulty was to evaluate the matrix exponential function accurately enough. The CPU times were ranging from 6 to 300 seconds, however, some stability problems for $m > 256$ arose out. More detailed error analysis has been performed in [9] where the Thomson cable was considered, with known analytical solution. The experiments led to the relative errors from roughly 10^{-5} to 10^{-8}, for numbers of Π sections from $m = 64$ to 4096. The methods are easily usable for nonuniform MTLs when (5) and (7) are modified to develop respective system matrices.

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