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Boundary effects of a nonconcentric semipermeable sphere using Happel and Kuwabara cell models

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Abstract

The effect of a closed boundary on the hydrodynamic drag of a nonconcentric semipermeable sphere in an incompressible viscous fluid is investigated. Darcy's law holds in the permeable region and Stokes flow used inside the spherical cavity. Suitable boundary conditions are used on the surface of a semipermeable sphere and spherical cavity. Two spherical coordinate systems are used to solve the problem. By superposition principle, a general solution is constructed from the solutions based on the semipermeable sphere and spherical cavity. Numerical results for the hydrodynamic drag force exerted on the particle is obtained with good convergence for various values of the relative distance between the centers of the inner sphere and spherical cavity, permeability parameter and the separation parameter. The numerical values of the hydrodynamic drag force generalize the results obtained for an eccentric solid sphere.

Keywords: semipermeable sphere, Stokes flow, Darcy's law, nonconcentric, drag

1. Introduction

The low velocity flow around an object like solid, semipermeable, permeable or composite particles has been interesting subject in several scientific and engineering applications exemplified pellets as catalysts, fluid-solid disperse systems, sedimentation etc. Many investigators over the world carried out researches on numerous problems concerning flow through porous bodies. Leonov [16] examined the porous sphere by a uniform stream of an incompressible viscous fluid. Joseph and Tao [9] discussed the coupled problem of a viscous fluid past a permeable sphere. Feng and Michaelides [3] obtained the drag force exerted on permeable sphere at finite but small Reynolds numbers. Srinivasacharya [18] studied the Stokes flow past a porous approximate spherical shell using Darcy's law. Shapovalov [17] discussed the highly viscous flow past a partially permeable spherical particle. Vereshchagin and Dolgushev [20] studied the filtration flow through the spherical shell. Yadav et al. [21, 22] investigated the hydrodynamic permeability of membranes built up by porous spherical shells, and porous deformed spheroidal particles using particle-in-cell method.

Gluckman et al. [6] proposed boundary collocation method for slow viscous flow past a finite assemblage of particles. Using this multipole truncation technique, many researchers solved multi-particle interaction, particle-wall boundaries, and non-concentrated problems (Leichtberg et al. [15], Ganatos et al. [4, 5], Dagan [1], Keh and Lee [10]). Faltas and Saad [2] investigated the slow motion of slip sphere in an eccentric cell using Happel and Kuwabara boundary conditions [7, 14]. Srivastava et al. [19] studied the flow of an incompressible viscous fluid

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past a porous sphere in presence of transverse applied uniform magnetic field using particle-in-cell method. Recently, Krishna Prasad and Bucha [13] studied the flow past a semipermeable sphere under magnetic effect. Krishna Prasad [12] investigated the micropolar fluid past a semipermeable sphere using different cell models. Yadav et al. [23] studied the hydrodynamic permeability of a porous spheroidal particles. Khanukaeva et al. [11] examined the creeping flow of micropolar fluid parallel to the axis of porous cylindrical cells. Yadav et al. [24] investigated motion of a non-homogeneous porous spherical particle in a spherical container.

In present paper, we have extended the work done by Shapovalov [17] to an eccentric semipermeable sphere in cell models. The hydrodynamic resisting force acting on the particle is obtained. Numerical results for the case of motion of solid and semipermeable particle in a concentric of closed boundary are included.

2. Mathematical statement

Consider the slow motion of a semipermeable spherically symmetric particle with radius a in an incompressible viscous fluid, surrounded by a non-concentric spherical cavity with radius b, as depicted in Fig. 1. Let $\eta = \frac{a}{b}$ be the separation parameter. The spherical cavity is frictionless. The fluid comes near the cavity surface and past a semipermeable sphere translating at a constant velocity U from the negative z-axis. Here, (r, θ, ϕ) and (ρ, ϕ, z) denote the spherical coordinate and circular cylindrical systems, respectively. The origin is fixed at the centre of spherical cavity. The centre of the semipermeable sphere is placed at a distance d from the centre of the cavity. Let (r_1, θ_1, ϕ) and (r_2, θ_2, ϕ) be the spherical coordinates based on the centre of a semipermeable sphere and spherical cavity, respectively. The relation between the radii of semipermeable sphere r_1 and spherical cavity r_2 is given by $r_1^2 = r_2^2 + d^2 - 2r_2 d \cos\theta_2$ or $r_2^2 = r_1^2 + d^2 + 2r_1 d \cos\theta_1$.

The field equations governing the flow outside the semipermeable sphere are the continuity and Stokes equations

$$\nabla \cdot \vec{v}^{(1)} = 0, \tag{1}$$

$$\nabla p^{(1)} + \mu \nabla \times \nabla \times \vec{v}^{(1)} = 0. \tag{2}$$

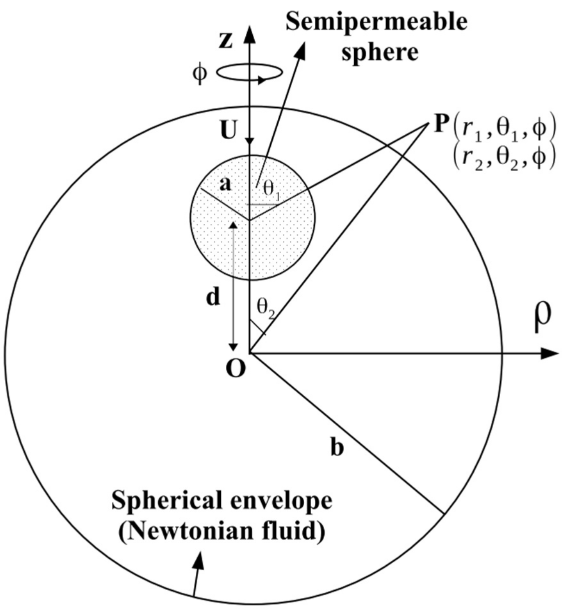

Fig. 1. Geometry of the problem

For the flow field inside the semipermeable sphere, there are the continuity equation and the Darcy's law

$$\nabla \cdot \vec{v}^{\,(2)} = 0, \tag{3}$$

$$\vec{v}^{\,(2)} = -\frac{k}{\mu}\,\nabla p^{(2)}, \tag{4}$$

where $\vec{v}^{\,(i)}$ and $p^{(i)}$, $i = 1, 2$ denote the velocity vector and the fluid pressure at any point, μ is the coefficient of viscosity, and k is the permeability of the porous medium.

Since the considered problem is axially symmetric, all the flow quantities are independent of ϕ. Thus, one can take the velocity vectors in the cylindrical coordinates as

$$\vec{v}^{\,(i)} = v_\rho^{(i)}\,\vec{e}_\rho + v_z^{(i)}\,\vec{e}_z, \quad i = 1, 2. \tag{5}$$

Since $\nabla \cdot \vec{v}^{\,(i)} = 0$, one can represent the velocity components $v_\rho^{(i)}$ and $v_z^{(i)}$ in terms of the Stokes stream function as

$$v_\rho^{(i)} = \frac{1}{\rho}\frac{\partial \psi^{(i)}}{\partial z}, \qquad v_z^{(i)} = -\frac{1}{\rho}\frac{\partial \psi^{(i)}}{\partial \rho}, \quad i = 1, 2. \tag{6}$$

The fourth order and second linear partial differential equations for the stream functions $\psi^{(i)}$, $i = 1, 2$ are obtained by eliminating the pressures from (2) and (4)

$$E^4 \psi^{(1)} = 0, \tag{7}$$

$$E^2 \psi^{(2)} = 0, \tag{8}$$

where $E^2 = \frac{\partial^2}{\partial \rho^2} - \frac{1}{\rho}\frac{\partial}{\partial \rho} + \frac{\partial^2}{\partial z^2}$ is the Stokesian operator.

3. Boundary conditions

To determine the flow velocity components outside and inside the semipermeable sphere, continuity of pressure, normal velocity, zero tangential velocity at the interface are used, while at the cavity surface, continuity of radial velocity, Happel and Kuwabara boundary conditions are assumed [9, 12, 13, 17]. Therefore, the following relations can be obtained

on $r_1 = a$:

$$v_\rho^{(1)} \tan\theta_1 + v_z^{(1)} = v_\rho^{(2)} \tan\theta_1 + v_z^{(2)}, \tag{9}$$

$$v_\rho^{(1)} \cot\theta_1 - v_z^{(1)} = 0, \tag{10}$$

$$p^{(1)} = p^{(2)}, \tag{11}$$

on $r_2 = b$:

$$v_\rho^{(1)} \tan\theta_2 + v_z^{(1)} = -U, \tag{12}$$

(a) Happel model

$$t^{(1)}_{r_2\theta_2} = 0, \tag{13}$$

(b) Kuwabara model

$$E^2 \psi^{(1)} = 0. \tag{14}$$

4. Solution of the problem

Using the principle of superposition, the stream functions $\psi^{(i)}$ of the fluid flow, which satisfy the boundary conditions on the spherical surfaces in the spherical coordinates [2,4,5,8,10], are obtained by solving (7) and (8)

$$\psi^{(1)} = \sum_{n=2}^{\infty} \left[\left(A_n\, r_1^{-n+1} + B_n\, r_1^{-n+3} \right) \vartheta_n(\xi_1) + \left(C_n\, r_2^{n} + D_n\, r_2^{n+2} \right) \vartheta_n(\xi_2) \right], \tag{15}$$

$$\psi^{(2)} = \sum_{n=2}^{\infty} \left(e_n\, r_1^{n} + f_n\, r_1^{-n+1} \right) \vartheta_n(\xi_1). \tag{16}$$

The stream function $\psi^{(2)}$ is finite at the center of the semipermeable sphere and imposes the condition $f_n = 0$. In (15) and (16), $\vartheta_n(\cdot)$ denotes the Gegenbauer function of the first kind of order n and degree $-\frac{1}{2}$ and $\xi_1 = \cos\theta_1$, $\xi_2 = \cos\theta_2$. The unknown coefficients A_n, B_n, C_n, and D_n can be determined using the boundary conditions (9)–(14).

The axial $v_\rho^{(i)}$ and radial $v_z^{(i)}$ velocity components, the pressure $p^{(i)}$, $i = 1, 2$, and the tangential stresses $t_{r_2\theta_2}^{(1)}$ are given by

$$v_\rho^{(1)} = \sum_{n=2}^{\infty} \left[A_n\, \mathbf{A}_{1n}(\zeta_1) + B_n\, \mathbf{B}_{1n}(\zeta_1) + C_n\, \mathbf{C}_{1n}(\zeta_2) + D_n\, \mathbf{D}_{1n}(\zeta_2) \right], \tag{17}$$

$$v_z^{(1)} = \sum_{n=2}^{\infty} \left[A_n\, \mathbf{A}_{2n}(\zeta_1) + B_n\, \mathbf{B}_{2n}(\zeta_1) + C_n\, \mathbf{C}_{2n}(\zeta_2) + D_n\, \mathbf{D}_{2n}(\zeta_2) \right], \tag{18}$$

$$p^{(1)} = \sum_{n=2}^{\infty} \left[B_n\, \mathbf{B}_{3n}(\zeta_1) + D_n\, \mathbf{D}_{3n}(\zeta_2) \right], \tag{19}$$

$$t_{r_2\theta_2}^{(1)} = \sum_{n=2}^{\infty} \left[A_n\, \mathbf{A}_{4n}(\zeta_1) + B_n\, \mathbf{B}_{4n}(\zeta_1) + C_n\, \mathbf{C}_{4n}(\zeta_2) + D_n\, \mathbf{D}_{4n}(\zeta_2) \right], \tag{20}$$

$$E^2\psi^{(1)} = \sum_{n=2}^{\infty} \left[B_n\, \mathbf{B}_{7n}(\zeta_1) + D_n\, \mathbf{D}_{7n}(\zeta_2) \right], \tag{21}$$

$$v_\rho^{(2)} = \sum_{n=2}^{\infty} E_n\, \mathbf{E}_{1n}(\zeta_1), \tag{22}$$

$$v_z^{(2)} = \sum_{n=2}^{\infty} E_n\, \mathbf{E}_{2n}(\zeta_1), \tag{23}$$

$$p^{(2)} = \sum_{n=2}^{\infty} E_n\, \mathbf{E}_{3n}(\zeta_1), \tag{24}$$

where $\zeta_1 = (r_1, \theta_1)$, $\zeta_2 = (r_2, \theta_2)$ and

$$\sum_{n=2}^{\infty} \left[A_n\, \mathbf{A}_{5n}(\zeta_3) + B_n\, \mathbf{B}_{5n}(\zeta_3) - E_n\, \mathbf{E}_{5n}(\zeta_3) + C_n\, \mathbf{C}_{5n}(\zeta_4) + D_n\, \mathbf{D}_{5n}(\zeta_4) \right] = 0, \tag{25}$$

$$\sum_{n=2}^{\infty} \left[A_n\, \mathbf{A}_{6n}(\zeta_3) + B_n\, \mathbf{B}_{6n}(\zeta_3) + C_n\, \mathbf{C}_{6n}(\zeta_4) + D_n\, \mathbf{D}_{6n}(\zeta_4) \right] = 0, \tag{26}$$

$$\sum_{n=2}^{\infty} \left[B_n\, \mathbf{B}_{3n}(\zeta_3) - E_n\, \mathbf{E}_{3n}(\zeta_3) + D_n\, \mathbf{D}_{3n}(\zeta_4) \right] = 0, \tag{27}$$

$$\sum_{n=2}^{\infty} [A_n\,\mathbf{A}_{5n}(\zeta_5) + B_n\,\mathbf{B}_{5n}(\zeta_5) + C_n\,\mathbf{C}_{5n}(\zeta_6) + D_n\,\mathbf{D}_{5n}(\zeta_6)] = -1, \quad (28)$$

$$\sum_{n=2}^{\infty} [A_n\,\mathbf{A}_{4n}(\zeta_5) + B_n\,\mathbf{B}_{4n}(\zeta_5) + C_n\,\mathbf{C}_{4n}(\zeta_6) + D_n\,\mathbf{D}_{4n}(\zeta_6)] = 0, \quad (29)$$

$$\sum_{n=2}^{\infty} [B_n\,\mathbf{B}_{7n}(\zeta_5) + D_n\,\mathbf{D}_{7n}(\zeta_6)] = 0, \quad (30)$$

where $\zeta_3 = (1, \theta_1)$, $\zeta_4 = [(r_2, \theta_2)]_{r_1=1}$, $\zeta_5 = [(r_1, \theta_1)]_{r_1=\frac{1}{\eta}}$, $\zeta_6 = (\frac{1}{\eta}, \theta_1)$, and the functions $\mathbf{A}_{pn}$, $\mathbf{B}_{pn}$, $\mathbf{C}_{pn}$, $\mathbf{D}_{pn}$, and $\mathbf{E}_{pn}$ with $p = 1, \ldots, 7$ are given in Appendix A.

5. Drag on the semipermeable sphere

The drag force can be calculated by using the formula

$$F = \pi\,\mu\,a \int_0^{\pi} r^3 \sin^3\theta \frac{\partial}{\partial r}\left(\frac{E^2\psi^{(1)}}{r^2\sin^2\theta}\right) r \bigg|_{r=1} \mathrm{d}\theta = -4\,\pi\,\mu\,a\,U\,B_2, \quad (31)$$

which indicates that B_2 contributes to the hydrodynamic force experienced by the semipermeable sphere. The expression B_2 is the lowest order coefficient. The value of B_2 is the most precise and quickest convergent result.

The exact solution for the case of concentric semipermeable sphere, using the Happel and Kuwabara models, is given as

$$F_{Hp} = -4\,\pi\,\mu\,a\,U \left[\frac{2(\beta^2-10)\eta^5 + 3\beta^2}{2(\beta^2-10)\eta^6 + 3(2-\beta^2)\eta^5 + \beta^2(3\eta-2) - 1}\right], \quad (32)$$

$$F_{Ku} = -4\,\pi\,\mu\,a\,U \left[\frac{15\beta^2}{2(\beta^2-10)\eta^6 - 10(2+\beta^2)\eta^3 + 2\beta^2(9\eta-5) - 5}\right]. \quad (33)$$

If there is no cavity surface, i.e., $a/(b-d) = 0$, then the fluid is unbounded. The drag acting on a semipermeable sphere in a viscous fluid is given as [17]

$$F_\infty = -4\,\pi\,\mu\,a\,U \left[\frac{3\beta^2}{2\beta^2+1}\right]. \quad (34)$$

If $\beta \to \infty$, the drag exerted on a rigid sphere is obtained [8].

The wall correction factor W is calculated as the ratio of the drag force acting on the semipermeable sphere in the closed boundary to the semipermeable sphere in the free medium. With the aid of (31) and (34), we get

$$W = \frac{F}{F_\infty}. \quad (35)$$

6. Results and discussion

To obtain the numerical solution of the drag force exerted on a semipermeable sphere translating within a spherical cavity, let us choose boundary collocation points along the half-circular generating arcs of the semipermeable sphere and spherical cavity. The initial point $\theta_i = 0$ and the terminal point $\theta_i = \pi$ are chosen along with the point $\theta_i = \pi/2$ on both the arcs. If these three points are used in the system of equations (25)–(28) and in (29) and (30) for the Happel and Kuwabara models, respectively, then the coefficient matrix becomes a singular matrix. To avoid this difficulty and to achieve good accuracy, it is possible to use the method recommended in the literature [1, 2, 4–6, 10]. The adjacent points of $\theta_i = 0, \pi/2, \pi$ are $\theta_i = \epsilon$, $\theta_i = \pi/2 - \epsilon$, $\theta_i = \pi/2 + \epsilon$, $\theta_i = \pi - \epsilon$, where ϵ is a specified value ($\epsilon = 0.01°$) chosen so that the coefficient matrix is nonsingular. Let us divide the two quarter-circular arcs into equal segments so that additional points are selected as mirror-image pairs about $\theta = \pi/2$. Choosing of finite number of discrete points on the arcs results in a system of linear equations. This system is solved by using the Gaussian elimination method.

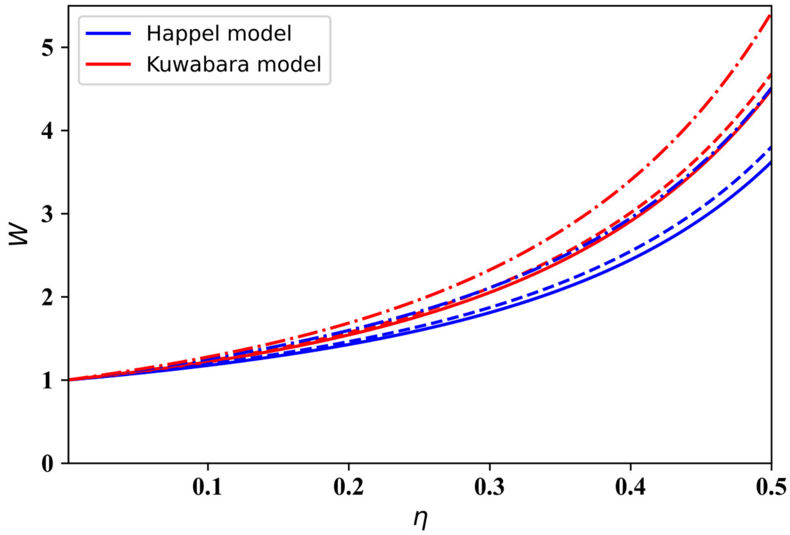

Fig. 2. Variation of W versus η for different values of δ (solid line $\delta \to 0$, dashed line $\delta = 0.25$, and dash-dotted line $\delta = 0.5$) and fixed $k_1 = 0.001$

The numerical solution of the wall correction factor W exerted on the semipermeable sphere in the nonconcentric cavity containing viscous fluid for both the aforementioned cases is presented in Fig. 2 and Tables 1 and 2. The obtained results converge to at least six decimal places. The following parameters are present in the problem:

Table 1. Convergence of the wall correction factor W_{Hp} for different values of δ, η, and k_1

wall correction factor W_{Hp} (**Happel's model**)					
N	$k_1 \to 0$ $\eta = 0.5$ $\delta = 10^{-5}$	N	$k_1 = 0.001$ $\eta = 0.5$ $\delta = 0.25$	N	$k_1 = 0.001$ $\eta = 0.5$ $\delta = 0.5$
6	3.629 62	6	3.814 08	6	4.781 45
8	3.629 62	12	3.803 14	12	4.546 66
	–	18	3.803 41	18	4.518 16
	–	20	3.803 43	24	4.513 36
	–	22	3.803 43	30	4.513 10
	–		–	36	4.513 04
	–		–	38	4.513 03
	–		–	40	4.513 03
	–		–		–

Table 2. Convergence of the wall correction factor W_{Ku} for different values of δ, η, and k_1

wall correction factor W_{Ku} (**Kuwabara's model**)					
N	$k_1 \to 0$ $\eta = 0.5$ $\delta = 10^{-5}$	N	$k_1 = 0.001$ $\eta = 0.5$ $\delta = 0.25$	N	$k_1 = 0.001$ $\eta = 0.5$ $\delta = 0.5$
6	4.507 04	6	4.701 13	6	5.930 37
8	4.507 04	12	4.682 11	12	5.432 78
	–	18	4.682 53	18	5.422 03
	–	24	4.682 51	24	5.419 23
	–	30	4.682 49	30	5.418 50
	–	32	4.682 49	36	5.418 32
	–		–	38	5.418 28
	–		–	40	5.418 28
	–		–		–

Table 3. Wall correction factor W_{Hp} for different values of k_1, η, and δ

		W		
δ	η	$k_1 \to 0$	$k_1 = 0.001$	$k_1 = 0.003$
0.000 01	0.001	1.001 50	1.001 50	1.001 50
	0.010	1.015 23	1.015 22	1.015 21
	0.100	1.176 46	1.176 36	1.176 15
	0.200	1.428 03	1.427 72	1.427 11
	0.300	1.811 52	1.810 76	1.809 24
	0.400	2.448 13	2.446 15	2.442 21
	0.500	3.629 63	3.623 46	3.611 18
0.25	0.001	1.001 64	1.001 64	1.001 64
	0.010	1.016 64	1.016 63	1.016 61
	0.100	1.191 88	1.191 77	1.191 54
	0.200	1.462 73	1.462 38	1.461 69
	0.300	1.872 58	1.871 70	1.869 95
	0.400	2.550 82	2.548 49	2.543 86
	0.500	3.810 83	3.803 43	3.788 73
0.5	0.001	1.002 19	1.002 19	1.002 18
	0.010	1.022 15	1.022 14	1.022 12
	0.100	1.252 30	1.252 14	1.251 81
	0.200	1.598 05	1.597 53	1.596 48
	0.300	2.110 16	2.108 75	2.105 92
	0.400	2.952 11	2.948 11	2.940 14
	0.500	4.526 49	4.513 02	4.486 40

1. The separation parameter: $\eta = a/b$ $(0 < \eta < 0.5)$.
2. The parameter $\delta = \frac{d}{b-a}$ is the normalized deviation distance of the center of inner sphere from the center of spherical container: If $\delta \to 0$, the center of inner sphere and spherical container coincide and the problem is reduced to translation of a semipermeable sphere in a concentric spherical container.

Table 4. Wall correction factor W_{Ku} for different values of k_1, η, and δ

δ	η	W		
		$k_1 \to 0$	$k_1 = 0.001$	$k_1 = 0.003$
0.000 01	0.001	1.001 80	1.001 80	1.001 80
	0.010	1.018 33	1.018 32	1.018 30
	0.100	1.218 03	1.217 89	1.217 62
	0.200	1.543 24	1.542 78	1.541 87
	0.300	2.054 00	2.052 69	2.050 07
	0.400	2.913 92	2.909 98	2.902 15
	0.500	4.507 04	4.493 47	4.466 62
0.25	0.001	1.001 91	1.001 91	1.001 90
	0.010	1.019 37	1.019 36	1.019 34
	0.100	1.230 13	1.229 98	1.229 69
	0.200	1.572 55	1.572 06	1.571 08
	0.300	2.109 92	2.108 50	2.105 66
	0.400	3.015 56	3.011 27	3.002 74
	0.500	4.697 41	4.682 48	4.652 93
0.5	0.001	1.002 32	1.002 31	1.002 31
	0.010	1.023 52	1.023 51	1.023 49
	0.100	1.277 53	1.277 34	1.276 98
	0.200	1.686 11	1.685 48	1.684 20
	0.300	2.325 83	2.323 97	2.320 25
	0.400	3.408 95	3.403 27	3.391 92
	0.500	5.438 68	5.418 28	5.377 59

3. The permeability parameter $\beta = a/\sqrt{k}$: If $\beta \to \infty$, the special case of translation of a solid sphere in an eccentric spherical container is obtained.

Fig. 2 depicts the variation of the wall correction factor W with the separation parameter η for different values of δ and for the fixed value of the permeability parameter $k_1 (= 1/\beta = \sqrt{k}/a)$. It is observed that W is a monotonic increasing function of η. Tables 1 and 2 indicate that W decreases with an increase in nonzero permeability parameter k_1 and increases with an increase in the separation parameter η. It is found that the drag force on an eccentric semipermeable sphere is less than the drag force exerted on an eccentric solid sphere in a bounded spherical medium. Collocation results of W in Table 1 for the concentric case $(\delta \to 0)$ agree with the exact solutions and for a nonconcentric particle $(\delta \neq 0)$ of solid case, $k_1 \to 0$ matches with the results of Faltas and Saad [2].

7. Conclusions

In this work, the numerical solution for the slow motion of a semipermeable sphere in a nonconcentric spherical cavity was studied. The wall correction factor acting on the particle was calculated for different parameter values and the results showed that the solution procedure converges rapidly. It was observed that the wall correction factor is an increasing function of separation parameter and is a decreasing function of the permeability parameter. The distance between the centers of the particle and the container played a significant role in evaluating the values of the wall correction factor. It was noted that the wall correction factor is minimal when the particle is at concentric position and increases monotonically with the relative distance between the centers of the particle and the container. Also, it was found that the drag force on an eccentric semipermeable sphere is less than the drag force exerted on an eccentric solid sphere in a bounded spherical medium.

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Appendix A

The functions appearing in (25)–(30) are defined as

$$\mathbf{A}_{1n}(\zeta) = -r^{-n-1}(n+1)\,\vartheta_{n+1}(\xi)(1-\xi^2)^{-1/2}, \tag{36}$$

$$\mathbf{B}_{1n}(\zeta) = -r^{-n+1}\left[(n+1)\,\vartheta_{n+1}(\xi) - 2\,\xi\,\vartheta_n(\xi)\right](1-\xi^2)^{-1/2}, \tag{37}$$

$$\mathbf{C}_{1n}(\zeta) = -r^{n-2}\left[(n+1)\,\vartheta_{n+1}(\xi) - (2n-1)\,\xi\,\vartheta_n(\xi)\right](1-\xi^2)^{-1/2} = \mathbf{E}_{1n}(\zeta), \tag{38}$$

$$\mathbf{D}_{1n}(\zeta) = -r^{n}\left[(n+1)\,\vartheta_{n+1}(\xi) - (2n+1)\,\xi\,\vartheta_n(\xi)\right](1-\xi^2)^{-1/2}, \tag{39}$$

$$\mathbf{A}_{2n}(\zeta) = -r^{-n-1}P_n(\xi), \tag{40}$$

$$\mathbf{B}_{2n}(\zeta) = -r^{-n+1}\left[2\,\vartheta_n(\xi) + P_n(\xi)\right], \tag{41}$$

$$\mathbf{C}_{2n}(\zeta) = -r^{n-2}\left[(2n-1)\,\vartheta_n(\xi) + P_n(\xi)\right] = \mathbf{E}_{2n}(\zeta), \tag{42}$$

$$\mathbf{D}_{2n}(\zeta) = -r^{n}\left[(2n+1)\,\vartheta_n(\xi) + P_n(\xi)\right], \tag{43}$$

$$\mathbf{B}_{3n}(\zeta) = -\frac{2(2n-3)}{n}r^{-n}P_{n-1}(\xi), \tag{44}$$

$$\mathbf{D}_{3n}(\zeta) = -\frac{2(2n+1)}{(n-1)}r^{n-1}P_{n-1}(\xi), \tag{45}$$

$$\mathbf{E}_{3n}(\zeta) = \beta^2\frac{1}{(n-1)}r^{n-1}P_{n-1}(\xi), \tag{46}$$

$$\mathbf{A}_{4n}(\zeta) = 2\,(n^2-1)r^{-n-2}\,\vartheta_n(\xi)(1-\xi^2)^{-1/2}, \tag{47}$$

$$\mathbf{B}_{4n}(\zeta) = 2\,n(n-2)r^{-n}\,\vartheta_n(\xi)(1-\xi^2)^{-1/2}, \tag{48}$$

$$\mathbf{C}_{4n}(\zeta) = 2\,n(n-2)r^{n-3}\,\vartheta_n(\xi)(1-\xi^2)^{-1/2}, \tag{49}$$

$$= \mathbf{E}_{4n}(\zeta), \tag{50}$$

$$\mathbf{D}_{4n}(\zeta) = 2\,(n^2-1)r^{n-1}\,\vartheta_n(\xi)(1-\xi^2)^{-1/2}, \tag{51}$$

$$\mathbf{A}_{5n}(\zeta) = \mathbf{A}_{1n}(\xi)\xi^{-1}(1-\xi^2)^{1/2} + \mathbf{A}_{2n}(\xi), \tag{52}$$

$$\mathbf{B}_{5n}(\zeta) = \mathbf{B}_{1n}(\xi)\xi^{-1}(1-\xi^2)^{1/2} + \mathbf{B}_{2n}(\xi), \tag{53}$$

$$\mathbf{C}_{5n}(\zeta) = \mathbf{C}_{1n}(\xi)\xi^{-1}(1-\xi^2)^{1/2} + \mathbf{C}_{2n}(\xi), \tag{54}$$

$$\mathbf{D}_{5n}(\zeta) = \mathbf{D}_{1n}(\xi)\xi^{-1}(1-\xi^2)^{1/2} + \mathbf{D}_{2n}(\xi), \tag{55}$$

$$\mathbf{A}_{6n}(\zeta) = \mathbf{A}_{1n}(\xi)\xi(1-\xi^2)^{-1/2} - \mathbf{A}_{2n}(\xi), \tag{56}$$

$$\mathbf{B}_{6n}(\zeta) = \mathbf{B}_{1n}(\xi)\xi(1-\xi^2)^{-1/2} - \mathbf{B}_{2n}(\xi), \tag{57}$$

$$\mathbf{C}_{6n}(\zeta) = \mathbf{C}_{1n}(\xi)\xi(1-\xi^2)^{-1/2} - \mathbf{C}_{2n}(\xi), \tag{58}$$

$$\mathbf{D}_{6n}(\zeta) = \mathbf{D}_{1n}(\xi)\xi(1-\xi^2)^{-1/2} - \mathbf{D}_{2n}(\xi), \tag{59}$$

$$\mathbf{B}_{7n}(\zeta) = -2\,(2n-3)r^{-n+1}\,\vartheta_n(\xi), \tag{60}$$

$$\mathbf{D}_{7n}(\zeta) = 2\,(2n+1)r^{n}\,\vartheta_n(\xi). \tag{61}$$

Additionally, the article utilises the following relations between the spherical (r, θ, ϕ) and cylindrical (ρ, ϕ, z) coordinate systems:

$$r_1 = [\rho^2 + (z-d)^2]^{1/2}, \ \cos\theta_1 = \frac{z-d}{r_1}, \ \sin\theta_1 = \frac{\rho}{r_1},$$
$$r_2 = [\rho^2 + z^2]^{1/2}, \qquad \cos\theta_2 = \frac{z}{r_2}, \qquad \sin\theta_2 = \frac{\rho}{r_2}.$$