QUANTITATIVE METHODS IN ECONOMICS

Multiple Criteria Decision Making XX

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QUEUING SYSTEM WITH IMPATIENT CUSTOMERS - APPLYING SIMULATION TECHNIQUES TO SOLVE A SMALL BUSINESS PROBLEM

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Abstract
The study deals with an analysis and optimization of a queuing system in the service sector. The company under review is a traditional men's hairdresser (barber) whose business model is based on limited offer of services (dry hair cut) with the emphasis on short waiting time and no orders in advance. The analyzed queuing system is characterized by impatient customers in the system. The aim of the study is to design an optimal operating system, which will balance the interests of operator (service owner), service staff, and service customers. Random distributions of system input parameters are modeled on the basis of known information about the number of arriving customers and the time of service. The resulting optimal parameter values were obtained by simulation procedure. The simulation was performed by MS Excel using VBA (Visual Basic for Applications).

Keywords: *queuing system, impatience of customers, optimization, simulation methods*

JEL Classification: C630, L840
AMS Classification: 68U20

1 INTRODUCTION

Queuing theory is a frequently used discipline for modelling and determining important parameters in processes where units (requirements) occur entering the system and waiting until they are served. The occurrence of these units is random. Due to limited number of service lines (service channels) there may be a problem consisting in accumulation of requirements and formation of queues. Such processes may be modelled by means of queuing models. Usually, we need to find such a state of a given model as seen from the point of the number of service lines in which the requirements for the service do not wait too long and, at the same time, the service lines are sufficiently busy.

Some simpler models of queuing systems can be solved analytically. This means that based on known mathematical relations it is possible to calculate the output characteristics that describe the behaviour of such a model. Analytical approaches to the systems of queuing can be found, for example, in Ross (2014). For complex models or models described by means of less common random distributions it is necessary to estimate the output characteristics on the basis of a simulated passage of units through the modelled system. Some selected simulation techniques used in the queuing systems are presented, for example, by Stewart (2009).

The topic related to the application of queuing models for solving diverse issues has been addressed by a number of studies. E.g. De Boer (2000) uses the application of queuing model in the field of telecommunication networks, and, similarly, Madani (2013) in banking. Some studies use queuing models as a tool for solving issues related to improving customer service in the field of retailing (Xing et al. 2015), healthcare (Wang et al. 2019) or general public services (Xian et al. 2016).

The aim of this study is to build on previous work on this topic (Svoboda et al. 2018) and to design an optimal service system in a small business operating in services sector (Vacek et al. 2011), respecting the customer's impatience factor.

Customer impatience manifests itself in queuing systems in two ways. The first way is that the customer does not enter the system if there are already a certain number of customers in the system (so-called a priori impatience). The second way is characterized by the fact that the customer enters the system, queues up and after a certain critical waiting time he/she resigns to be served and leaves the system (so-called a posterior impatience). Basic approaches for customer impatience modeling are presented e.g. in a monograph (Lukáš 2009).

2 CHARACTERISTICS OF THE ANALYZED CASE

The analyzed case is a traditional men´s hairdresser´s with a long history in the centre of town. This location brings both, several advantages as well as some disadvantages. The main advantage is a large number of potential customers moving in its proximity on a daily basis. There are a lot of stops of numerous lines of municipal transport that can also be found near the hairdresser´s. The disadvantage is, on the contrary, the lack of car parking lots in the close vicinity and related high parking rates. The business strategy is based on these factors. The business model is based on a limited offer of services (dry hair cut), on the system of no orders in advance, lower prices and, especially, short waiting times. The opening hours of the hairdresser´s are from 7 a.m. to 8 p.m. only on weekdays, which is once more associated with the location of the establishment.

2.1 Characteristic of the service

There are five service places (hairdressing chairs) in the establishment. The working hours are seven hours, with 6.5 hours of actual operation, and the remaining time is evenly divided into 15 minutes before the start of operation and 15 minutes after the end of the operation. A total of 10 workers (hairdressers) in two-shift operation are required to ensure maximum intensity of service, each with five shift workers, with the first shift serving from 7 a.m. to 1:30 p.m. and the second from 1:30 p.m. to 8 p.m.

The time of serving one customer is understood as the total time which a hairdresser devotes to that particular customer. The time consists of the following activities: calling on a customer whose turn it is, accommodating the customer in the chair, protecting customer´s clothes, agreeing on the type of haircut, the haircut itself and potential corrections in case the customer is not satisfied, removing the protection of clothes and removing the cut off hairs (brushing down), issuing the receipt and receiving the payment (only cash payments are received) and sweeping the work area. The time of serving one customer depends on a lot of factors: the chosen haircut, the density and length of hair, the technique of cutting (scissors or electric trimmer), the speed of the particular hairdresser, corrections of the haircut and possibly also on additional requirements, for example beard trimming. The time of serving one customer is most often 12 minutes, the shortest service time is 9 minutes and the maximum service time is 18 minutes (according the service provider estimates).

2.2 Customers´ entries into the system

The exact times of customers´ entries into the establishment are not known. There is no electronic system in use that would allocate the customer an order number. A daily number of the served customers is known from the accounting documents. The number of customers served in one day may vary from 140 to 200, the average number being around 170 customers.

The distribution of the number of customers during the day is not even. There are three local peak periods during the day: the first one approximately between 7:30 a.m. and 8:30 a.m., the second one between 11:30 a.m. and 12:30 p.m., and the third one between 3 p.m. and 5 p.m. These local peaks can be explained by customers working hours. The first peak represents customers visiting the hairdresser in the morning on the way to work, the second peak is formed by customers who use the lunch break, and the last peak consists of customers after leaving work. For this reason, we can consider the customers as impatient. They enter the hairdresser only if there is no big queue. On the basis of the empirically gained data, the average numbers of entering units in each individual 30 minute interval are displayed on the Figure 1 below.

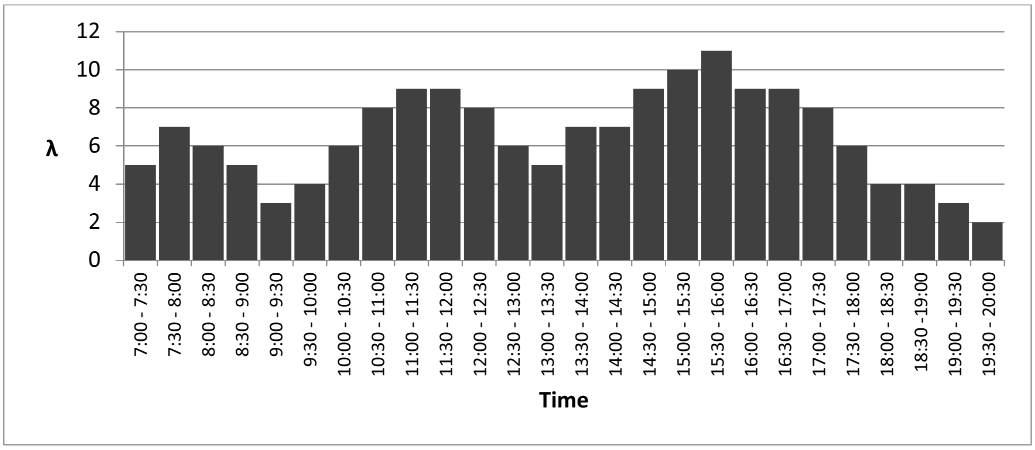

Figure 1: Intensity of customers' entries
Source: author's calculations

3 DESIGN OF THE MODEL

A triangle division is used to model the operating time based on the provided information. The intervals between the arrivals of two customers are modelled by means of exponential division with a variable mean value, which is a standard way of modelling in similar cases. Both the divisions, including their characteristics, are described, for example in (Dlouhý et al. 2007). The triangle division *Tri(a,b,c)* with parameters *a, b, c* is recommended in cases where there are no particular data available but we know the minimum value (parameter *a*), the most frequent value (parameter *b*) and the maximum value (parameter *c*). In our case we have a division with parameters *Tri*(9, 12, 18). The mean value of the triangle division is given by the relation (1):

$$E(X) = \frac{a+b+c}{3} \tag{1}$$

By substituting to relation (1) we will gain the mean value of customer service time which equals 13 min. One operating channel thus serves 4.6 customers in one hour on average. It can be easily calculated that one operating channel could serve, on average, 59.8 customers in a thirteen hours long shift. The exponential division has the only parameter λ which is interpreted as the intensity of entries, i.e. the number of entries over a time unit. The reciprocal value $1/\lambda$ is the mean value of this division, i.e. the average time between the arrivals of two customers. The value λ is considered as a variable dependent on time in our model, according to Figure 1.

The impatience of the customers entering the hairdresser is rather a priori, i.e. the probability of the customer entering the system decreases from a certain number of queued customers. According to information from the service provider, the impatience begins to show when there are already three customers in the queue, and as the queue grows, the probability of entering another customer decreases relatively quickly. The amount of queued customers rarely exceeds the number 6. Based on this information, we used the relationship (2) to model the probability of customer entry into the system:

$$\begin{aligned} q \le 2 \quad & P(q) = 1 \\ q > 2 \quad & P(q) = e^{-\alpha \cdot q}, \qquad \alpha = 0.7 \end{aligned} \tag{2}$$

where

- q is the number of customers in the queue,
- $P(q)$ is the probability of entering the system of a new incoming customer if there are exactly q customers in the queue, and
- α is an optional parameter that affects the impatience.

In our model, α = 0.7. The Table 1 shows the probability a new customer coming into the hairdresser stays here (he/she queues) when q customers are queued already.

Table 1: Probability a new customer queues when q other customers are queued

q	0	1	2	3	4	5	6	7	8
$P(q)$	1.000	1.000	1.000	0.497	0.247	0.122	0.061	0.030	0.015

Source: author's calculations

It is necessary to know the cost of service and the revenue for serving the customer for economic evaluation of the model. The basic and most used service is hair cutting, whether by scissors or electric shears, for a single price of CZK 139. Service costs consist of labor costs per worker (hairdresser) that consist of the hairdresser's salary (gross wage), and health and social insurance contributions counted in our model count as 0.35 times the gross wage. The gross wage of the hairdresser consists of a fixed component of CZK 80 per hour (i.e. CZK 108 in labor costs) and CZK 40 (CZK 54 in labor costs) for each served customer. Hairdressers usually get tips from the vast majority of customers, which they keep, usually CZK 11 or CZK 21. We calculate with a conservative estimate of CZK 11 in our model.

Owing to the above mentioned characteristic of the variability of the intensity of inputs we will solve the task by means of simulation. The simulation model was programmed in the environment of MS Excel by means of VBA language. One simulation run models (simulates) the course of one working day in the establishment. This simulation has a finite time horizon. The simulation run is started at 7 a.m. and finished at 8 p.m.

4 COMPUTATIONAL RESULTS

Overall 6 different models of the above described system of queuing were analyzed. The first model is a model of maximal service, i.e. all service points (channels) are used throughout the working hours. Such a service system requires 10 workers (hairdressers). In each subsequent model, one worker is gradually reduced. As mentioned earlier, each worker works 7 hours. Decrease in staff by one worker reduces the operator's labor costs by 756, - CZK (7x108). On the other hand, the operator loses CZK 85 (hair cutting price minus hairdresser´s bonus for served customer) with every customer who does not enter the hairdresser because of the large queue.

A total of 200 simulations were performed for each model. We are interested in the following characteristics of the realized simulation: number of entering customers, number of non-entering (impatient) customers, average waiting time for a queued customer, and total waiting time of all service lines for a customer. From these characteristics we can easily calculate:

service line vacancy (in %), total labor costs, one employee's wage, and operator's income. The operator's income is represented by revenues reduced by staff costs. The operator's income is not the profit, the operator must pay rent, energies, and other operating and administrative costs that are fixed and cannot be changed in our model. Only the number of workers can be changed and thus labor costs influenced. The number of open service lines for each model is shown in Table 2. Operating times were determined with respect to customers' entries intensity as shown in Figure 1.

Table 2: Operating times on individual channels (service lines)

	n	channel 1	channel 2	channel 3	channel 4	channel 5
Model 1	10	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00
Model 2	9	07:00-20:00	07:00-20:00	07:00-20:00	07:00-20:00	11:00-17:30
Model 3	8	07:00-20:00	07:00-20:00	07:00-20:00	10:30-17:00	11:00-17:30
Model 4	7	07:00-20:00	07:00-20:00	10:00-16:30	10:30-17:00	11:00-17:30
Model 5	6	07:00-20:00	07:00-20:00	10:30-17:00	11:00-17:30	x
Model 6	5	07:00-20:00	07:00-20:00	11:00-17:30	x	x

Source: author's calculations

The results for each model (mean values) are shown in Table 3. In addition to the above variant, the simulation for Model 6 was also realized in case the first channel was available throughout the working hours, the second channel was available from 10:00 to 16:30, the third from 10:30 to 17:00, and the fourth from 11:00 to 17:30. However, the characteristics of this variant showed worse results than the original one and therefore we do not list them.

Table 3: Simulation results for individual models (one-day average values)

	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6
Number of entrants [persons]	166.7	166.2	163.7	156.8	150.9	136.0
Number of non-entrants [persons]	3.3	4.0	6.0	12.7	18.4	33.8
Average waiting time [hour:min:s]	0:01:01	0:01:18	0:02:04	0:04:20	0:05:47	0:09:09
Total time of service lines vacancy	27:09:42	21:31:08	16:09:01	11:33:49	6:19:58	3:00:05
Service lines vacancy [%]	41.8%	36.8%	31.1%	25.4%	16.2%	9.2%
Labor costs [CZK]	17,102.-	16,265.-	15,321.-	14,138.-	13,008.-	11,396.-
Worker's wage [CZK/worker]	1,266.-	1,338.-	1,418.-	1,496.-	1,605.-	1,620.-
Operator's income [CZK]	**6,070.-**	**6,838.-**	**7,436.-**	**7,659.-**	**7,965.-**	**7,513.-**

Source: author's calculations

The results show that, up to model 5, the operator's income increases with service lines (channels) reductions, and wage savings outweigh the losses of the non-entering customers. In Model 6, the losses from non-entering customers outweighed the labor cost savings.

In terms of operator's income, the least profitable is Model 1. In this model, the daily takings are on average CZK 23,171; after deduction of labor costs, the operator remains CZK 6,070, i.e. CZK 121,390 per month (at 20 working days per month). This amount is large enough to cover the monthly rent, energies, operating and administrative costs, annual remunerations, and reasonable profit for the operator. From the labor costs after deduction of insurance contributions, we calculate an employee's gross daily income of CZK 1,266 for seven-hour working time, which corresponds to a gross monthly wage of CZK 25,336. The tip can be added to this wage that can be estimated to about CZK 3,674.- per month net (at 16.7 customers served per day on average). The model is very favorable to customers, service lines vacancy is 41.8%, the number of non-entrants is only 3.3 per day on average, and the customer’s average waiting time is only 1:01 (1.016 minutes). The calculation of the above characteristics for other models is left to the reader. A typical time frame with non-entering (impatient) customers for each model is shown in Figure 2.

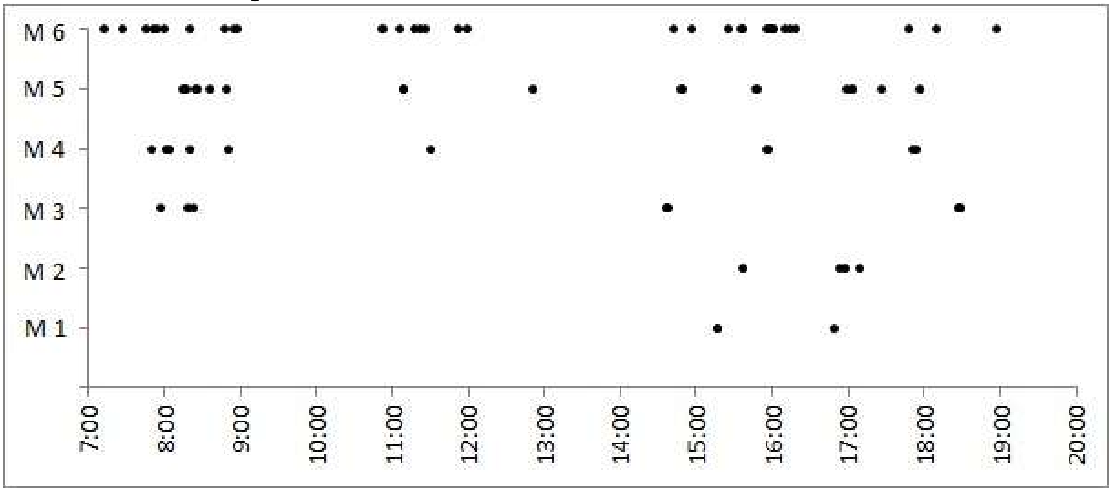

Figure 2: Time frame with non-entering customers
Source: author's calculations

5 CONCLUSIONS

All simulated models are profitable for the operator. It depends on the marketing strategy whether the operator wants to maximize the profit or customer's comfort. From a long-term point of view and from the perspective of all three entities: customers, workers and operator, it seems best to have 9 workers. Workers may be ill, take leave, etc. Taking into account that a worker is entitled to 4 weeks of leave, i.e. 36 weeks in total for 9 workers, plus illness or care for sick children, it can be assumed that total absence at the workplace can easily reach 52 weeks, i.e. there will be 8 workers a day in the workplace on average. If the operator employs 9 workers, he/she can react very flexibly to the absence of the worker at the workplace and switch from Model 2 (all workers present) to Model 3 (1 worker absent) or Model 4 (2 workers absent). Assuming, these three models will be used evenly, the operator's monthly income would be CZK 146,219. In such a case, the average gross monthly wage of employees is CZK 28,355 and tipping is CZK 4,497. On average, customers wait in the queue for 2 minutes and 34 seconds and the total number of non-entering customers per day is 7.6. These parameters can be considered balanced and the entire service system sustainable.

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