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Problematics of large batch winding of technical fabrics

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When weaving technical textiles we often encounter problems that are not known from weaving ordinary textiles. This is due to the significantly different mechanical properties of the fibres forming the fabrics. At the same time, productivity pressures cause additional complications, especially in the marginal areas of weaving, whether snipping or just wrapping the resulting product. As the resulting pack becomes larger, it becomes corrugated and consequently damages the fabric. This problem has increased in our case when weaving 3D fabric with higher thickness and lower transversal stiffness. We were therefore faced with the task of solving the problem of behaviours of such a batch. Due to the complexity of the problem we had to use finite elements method. It allowed us to cope with the anisotropic character of the bath and with the non linearities of mechanical properties.

Fabric large batch is essentially a strip of fabric spirally wound onto a rigid central tube (mandrel), Fig. 1. It can be produced in two fundamentally different ways, namely by winding on the driven central mandrel or by rolling the perimeter of the batch on two driven rolls. In the second method, the driving torque is exerted by friction between the fabric surface layer and the driving roll, whereby the contact force can be exerted by the actual weight of the wrap or by the controlled roller pressure against the central non driven tube. In both cases, the controlled variable is the tension in the wound fabric. During winding, only the surface layer has the

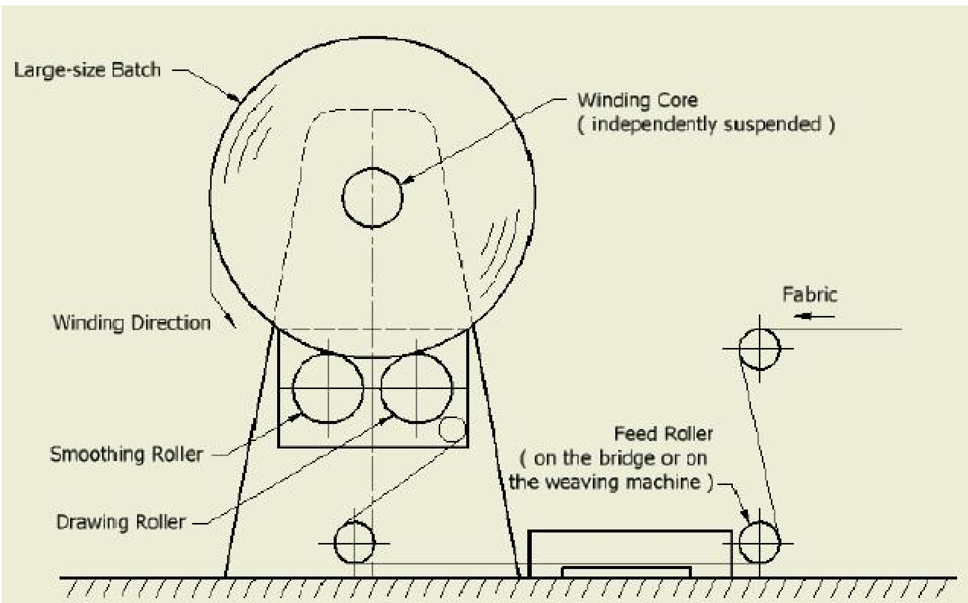

Figure 1. Schema of a large batch winder; courtesy of CEDIMA

required tension, in the inner layers, this pre-stressing decreases due to their suppleness. This phenomenon is the stronger the more supple the individual layers are in the radial direction. When the inner layers are compressed by the outer layers, this tangential stress may even vanish. These layers then become wavy in a certain radius range of batch. This is frequent especially in the gravity package, where the winding tension is limited to a certain extent by the weight of the wound fabric. Due to the three-dimensional nature of the batch, this phenomenon occurs differently in the middle of its width, where the state of stress has the character of a plane deformation, and near its free ends, where the state approaches the plane stresses. Together with the inherently different properties of the fabric in the middle of the woven width and at its edges, this leads to a double twist of the batch and a shear deformation of the wound fabric.

The first step, which is the subject of this work, was to determine the properties of batch as a rotationally symmetrical body and compare them with the experiment. The strongly anisotropic character given by the structure of the fabric layers was taken into account, but neither the varying load on the circumference of the batch from the driving rolls, nor the different properties along the rotation axis, were taken into account.

The basic equation describing the behaviour of such an object has the following form:

$$Q \cdot \frac{d}{dr}\left(r \cdot \left(\frac{du}{dr} + \nu_{tr} \cdot \frac{u}{r}\right)\right) - \left(Q \cdot \nu_{tr} \cdot \frac{du}{dr} + \frac{u}{r}\right) = f_t \cdot \frac{(1 - \nu_{tr}^2 \cdot Q)}{E_t} ,$$

where $Q = \frac{E_r}{E_t} = \frac{\nu_{rt}}{\nu_{tr}}$ is the ratio between elastic modulus in the radial and tangential direction and right hand side member of the governing differential equation is a function of f_t – force per unit of width and per unit of radius (or better, unit of thickness) applied on the fabric during the winding (essentially a tangential stress; for the purposes of this work we consider it constant).

The analytic solution of this problem is possible for linear material behaviour and for $Q \neq 1$:

$$u = C_1 \cdot r^{\frac{1}{\sqrt{Q}}} + C_2 \cdot r^{-\frac{1}{\sqrt{Q}}} - f_t \cdot \frac{r}{E_t} \cdot \frac{1 - \nu_{tr}^2 \cdot Q}{1 - Q} .$$

While loosely wound batches satisfy well this equation, batches with higher tension of winding are beginning to show non-linearity in the material properties of the batch if perceived as a continuum. This is caused by the compression of the individual layers of the fabric, which increases the contact area between the layers and together, the overall batch compacting grows. While the properties in the tangential direction change minimally, the modulus of elasticity in the radial direction starts to increase strongly. This in turn increases the value of the Q ratio.

There may be several models for expressing the dependence of the elastic modulus on radial compression, but with the exception of the simplest models, expressing the function $Q = f(\varepsilon_r)$ prevents any analytical solution of the fundamental differential equation.

To cope with this problem we had to use finite elements method. One-dimensional finite elements with cubic approximation of radial displacements were chosen for modelling the batch as a rotationally symmetric continuum. Nodal variables are radial displacements and the first derivative of these displacements (essentially the value of ε_r strain). Consequently, their continuity ensures the continuity of tangential and radial stresses too.

When deriving the stiffness matrix, we chose the standard procedure of FEM method [1]

$$\mathbf{K_e} = \int_{V_e} |\mathbf{B}|^T \times |\mathbf{E}| \times |\mathbf{B}| \cdot dV ,$$

where the matrix $\mathbf{B}$ takes the form

$$\mathbf{B} = \begin{vmatrix} \frac{d\mathbf{N}}{dr} \\ \frac{1}{r} \cdot \mathbf{N} \end{vmatrix}$$

and $\mathbf{E}$ has the following form

$$\mathbf{E} = \frac{E_t}{1 - Q \cdot \nu_{tr}^2} \cdot \begin{vmatrix} Q & Q \cdot \nu_{tr} \\ Q \cdot \nu_{tr} & 1 \end{vmatrix} \; .$$

Although the resulting functions can be integrated analytically, the presence of $\ln\left(\frac{r_i + L_e}{r_i}\right)$ terms in the resulting functions, which tend to zero, makes them numerically difficult to evaluate. Therefore, a four-point Gaussian numerical integration was chosen, which proved to be faster than the repeated enumeration of logarithms. Moreover, for sake of simplicity integration on real elements was carried out instead of reference elements.

In the absence of other loads, the nodal forces are calculated from the tangent stress values f_t using functions of approximation

$$[F_e] = \int_{L_e} f_t \cdot \frac{1}{\rho} \cdot |N| \cdot \rho \cdot d\rho \, .$$

Seemingly, f_t has a character of volumetric forces but actually it is that of surface stress on the symmetry planes, hence the use of $(f_t \cdot \varepsilon_t \cdot dV)$ specific energy integration. The boundary conditions were expressed both by the prescribed displacement at the inner batch radius (i.e., at one free edge of the discretization) and by the prescribed radial stress at the free surface, i.e., at the other free end of the discretization. It is therefore a problem with mixed boundary conditions. While the expression of Dirichlet's type BC on the inner radius is obvious, the free surface condition

$$\sigma_r\big|_{r=R_{ext}} = \frac{Q \cdot E_t}{1 - Q \cdot \nu_{tr}^2} \cdot (\varepsilon_r + \nu \cdot \varepsilon_t) = f\,(Q) \cdot \left(u_{n-1} + \frac{\nu_{tr}}{r_N} \cdot u_n\right) = 0 \, ,$$

where u_i, u_{i-1} and r_N are corresponding nodal values and $f\,(Q) > 0$ may be expressed as transformation of nodal variables on the concerned element in the form [1]

$$\begin{aligned} \mathbf{K_e'} &= \mathbf{R}^T \times \mathbf{K_e} \times \mathbf{R} && \ldots \text{ for transformation of matrix of rigidity,} \\ \mathbf{F_e'} &= \mathbf{R}^T \times \mathbf{F_e} && \ldots \text{ for transformation of generalized nodal forces.} \end{aligned}$$

The transformation matrix $\mathbf{R}$ takes the form (for $\sigma_r\big|_{r=R_{ext}} = 0$):

$$\mathbf{R} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\frac{\rho_4}{\nu_{tr}} & \frac{\rho_4}{\nu_{tr}} \end{vmatrix} \, .$$

Of course, the calculated nodal values of displacements must be re-transformed using R backwards into the natural displacements after calculation.

With regard to the assumed cause of the mechanical properties evolution, the dependence of the elasticity modulus in the radial direction (in terms of our approach, the values of the parameter Q, as the properties in the tangent direction are assumed constant) is chosen so as to simulate a normal distribution of contact surface size evolution between layers. Since its distribution function is not explicitly described, we have chosen a replacement model in the form:

$$\frac{dQ}{d\varepsilon_r}\,(\varepsilon_{r,0}, A) = \frac{2 \cdot A}{\pi \cdot \left(A^2 \cdot (\varepsilon_r - \varepsilon_{r,0})^2 + 1\right)^2} \, (= f\,(\mu, \sigma)) \, ,$$

where $A = \sqrt{\frac{\pi}{8}} \cdot \frac{1}{\sigma}$. Values of $\varepsilon_{r,0}$ and σ correspond to mean value and standard deviation, respectively, of a normal distribution. This function satisfies condition $\int_{-\infty}^{\infty} f = 1$. Its integral form takes the following form

$$Q\left(\varepsilon_r\right) = \frac{1}{2} + \frac{A}{\pi} \cdot \left(\frac{\operatorname{atan}\left(\left(\varepsilon_r - \varepsilon_{r,0}\right) \cdot A\right)}{A} + \frac{\left(\varepsilon_r - \varepsilon_{r,0}\right)}{A^2 \cdot \left(\varepsilon_r - \varepsilon_{r,0}\right)^2 + 1} \right)$$

with A defined above.

The solution of the system of non-linear equations was carried out using the substitution method, where $\{X^i\}$ are i-th approximation of column-vectors of residua R, nodal dsiplacements U and nodal forces F, respectively, [1]

$$\begin{aligned} \left\{R^i\right\} = \left\{R\left(U^{i-1}\right)\right\} &= \{F\} - \mathbf{K}\left(U^{i-1}\right)\left\{U^{i-1}\right\} \ , \\ \mathbf{K}\left(U^{i-1}\right)\left\{\Delta U^i\right\} &= \left\{R^i\right\} \ , \\ \left\{U^i\right\} &= \left\{U^{i-1}\right\} + \left\{\Delta U^i\right\} \ . \end{aligned}$$

Fig. 2 shows an example of the solution for radial stresses for a relatively small batch. A batch of this size was subject of our experiments; even here the difference is notable and it grows with the batch size.

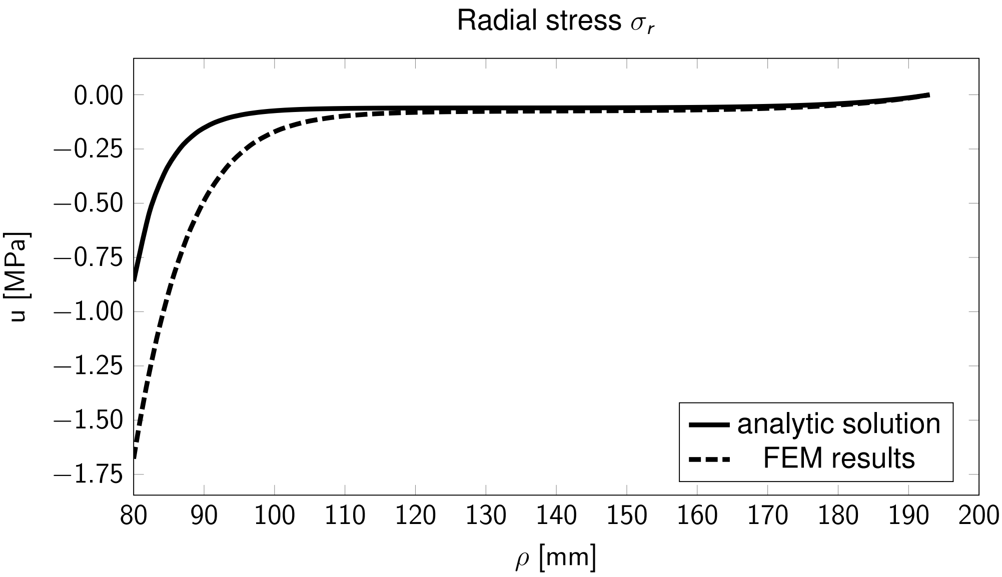

Figure 2. Difference between linear material and FEM solution using non-linear material

The use of FEM has allowed us to explain some of the discrepancies in the behaviour of large batches identified in experiments. In the future we expect to use the same material model for 2-D and 3-D FEM modelling of the batch.

References

[1] Dhatt, G., Touzot, G., A presentation of the finite element method, Maloine S.A., Paris, 1984. (in French)