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Numerical analysis of progressive collapse for reinforced concrete frames

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Abstract

This research investigates numerically the potential failure of different framed structures after removal of a corner column. This study is directed towards the role of the joint above the removed column in terms of framed structures stability and internal forces redistribution utilizing linear static, nonlinear static and nonlinear dynamic analysis procedures. First, the preliminary linear static analysis as a part of more detailed nonlinear static and dynamic analysis to model reinforced concrete frames is performed. For nonlinear numerical models, three different moment-rotation capacity curves are considered for the joint above the removed column. These curves are taken according to FEMA 356 regulation according to the reinforcement condition at the joint location to represent normal joint, anchorage deficient joint and shear deficient joint. In linear static models, the results confirmed that, the more moment the joint can resist at the removed column location, the less moment is generated at the beam's other end. In nonlinear static analysis, the model with normal plastic hinge properties recorded a higher carrying capacity than the model with both shear and anchorage plastic hinges. Nonlinear dynamic analysis with less assumptions offer the more representative modeling for the considered case study which is obvious from its closer perdition of internal forces and deflection compared to the reference findings.

Keywords: progressive collapse, structures, beam column joint, numerical analysis, plastic hinge

1. Introduction

For structural safety, it is essential that the joints of reinforced concrete (RC) frames have an adequate level of integrity that offers a ductile behavior under the action of different loads. Under the action of lateral loads on a framed building, plastic hinges are formed in beams ends and columns' bases to mitigate the effects of the resulted high straining actions and postpone structures collapse [12], as shown in Fig. 1. FEMA 356 [5] guidelines proposed the general properties of these plastic hinges.

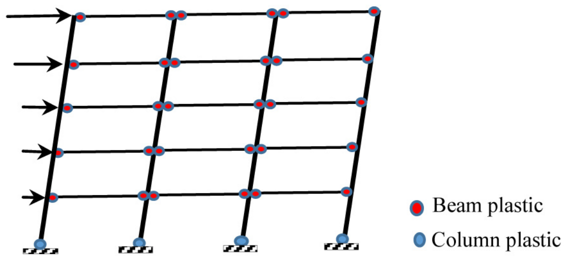

Fig. 1. The expected plastic hinges in framed structures [12]

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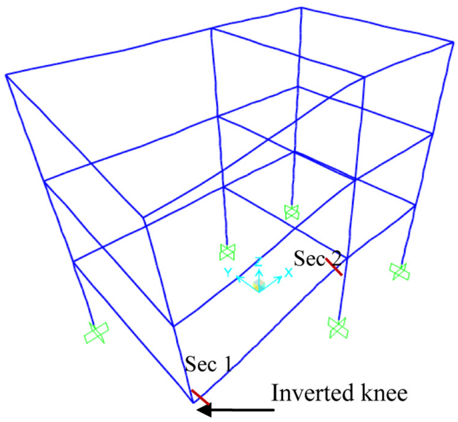
Similarly, it is important to evaluate the ultimate load carrying capacity of a RC frame after scenarios of ground column loss by considering the possibility of limited deformability of joints near to the lost column based on the joints reinforcement condition.

The stability analysis of a multi-storey frame after a column removal scenario should be represented as a nonlinear dynamic phenomenon, because significant geometric and material nonlinearity is at stake. Building progressive collapse mitigation guidelines [3, 6] allow the use of different numerical approaches that start from a simplified two dimensional linear elastic static analysis, and reach more advanced three dimensional nonlinear dynamic analysis. Nonlinear dynamic analysis takes into account damping effects, materials softening and changes in different elements stiffness [9]. Many studies concluded that factor of 2 for the dynamic increase factor overestimated the dynamic effects in both steel and RC buildings if seismic detailing requirements were implemented [7, 9, 10, 13].

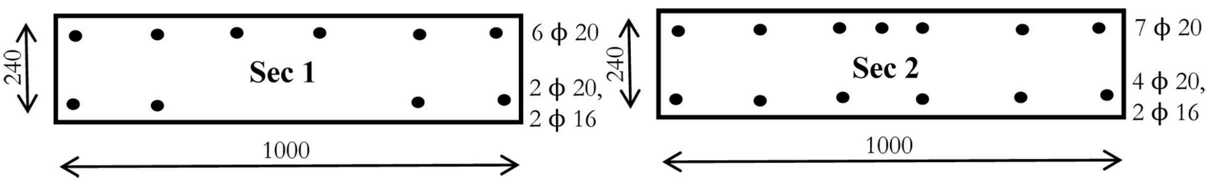
Kim et al. [8] used viscous dampers to improve structure's resistance against progressive collapse using non-linear dynamic analysis. The previous modelling efforts for progressive collapse simulation considered many design guidelines and column removal scenarios (interior or exterior column), but they assumed intact status for the connections above the removed column. With this assumption, the behavior and damage extent will not be properly estimated for some buildings. For that, the role of the resulted improper joints after column removal will be handled in this study. The linear static, nonlinear static and dynamic analyses are performed using software SAP2000 [2]. This study presents approaches for evaluating the progressive collapse potential of a RC frame that has been tested at the ELSA laboratory in Italy by Kokot et al. [9]. On the other side, the RC frame studied by Santafé et al. [11] is used as a reference case study for both nonlinear static and dynamic models.

An inverted knee joint is formed after the loss of a ground corner column at exterior beam-column joint location as shown in Fig. 2; consequently, stress concentration and cracks are expected at that location. Two reinforcement defects in such joints are noticed; leakage of vertical transverse reinforcement and improper column bar anchorage.

a) Real view of tested building [9]



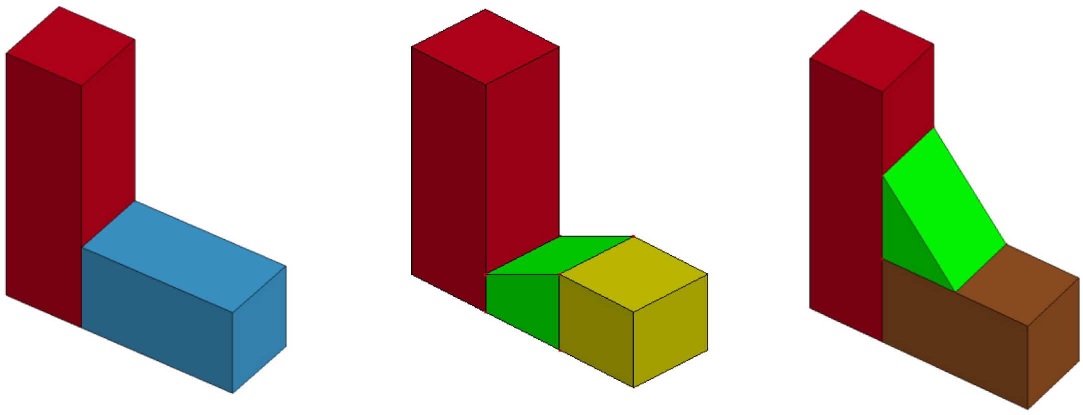
b) 3D model for the real building



c) Beam cross sections details

Fig. 2. RC frame before and after column loss

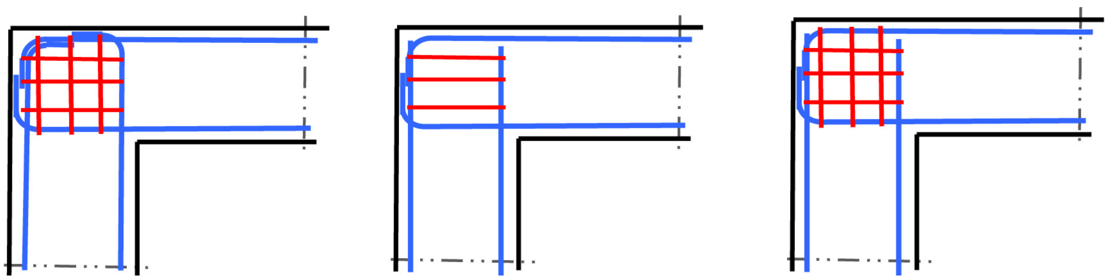
Beam-column joints play a main role in connecting the building beams and columns and assure member's integrity [1]. In linear static analysis, the reduction in the elements stiffness during loading is not available and to overcome this limitation, different models are proposed with previously defined reductions in joint stiffness. These models represent the most expected status for the joint from intact joint assumption to full damage case. A schematic representation for that is illustrated in Fig. 3, on the other side, improved joint stiffness is also examined.



a) Normal joint b) Joint with reduced thickness c) Improved joint

Fig. 3. Different probable cases for the beam-column joint above the removed column

During different loading stages, it is essential to reproduce corresponding stiffness and strength for each structure component, as the nonlinear behavior of RC elements is governed by their reinforcement's details. In nonlinear analysis, beam-column joint without proper column bars' anchorage conditions nor vertical stirrups is considered to simulate the resulted knee joint real condition after a column loss as in Fig. 4b. Providing joint with vertical stirrups is considered as well to limit such joint stiffness degradation as in Fig. 4c.



a) Normal joint b) Shear deficient c) Anchorage deficient

Fig. 4. Beam column knee joint with proper and improper reinforcement details

SAP2000 program is used in this study to simulate 2-D and 3-D RC building case studies. These buildings include a group of regular frames with different number of stories and bays under a corner column loss scenario. The research main objective is to investigate the performance of existing buildings after a ground corner column loss using linear static, nonlinear static and nonlinear dynamic numerical analyses in order to control displacements and avoid damage spreading in the remaining structures. Well designed and proper detailed building elements have the ability to resist high deformations by developing plastic hinges at their most critical locations.

2. Case studies

For validation purposes, the numerical models are used to reproduce the response of previously studied RC frames; first reference frame is selected because of the limited level of deformations as the frame is only subjected to its own weight (see Fig. 2a); the second reference frame is selected because of two reasons: first, the availability of more detailed nonlinear analysis results by the layered beam approach; second, the leakage in bar anchorage or joint shear reinforcement will have an obvious impact on frame response as the frame has dropped beams (beams with thicknesses bigger than the slab thickness). Then parametric studies are conducted to determine the variation in different models response based on various plastic hinges properties established according to joints reinforcement conditions. The first reference frame for linear static simulation part is a three storey two bay RC frame connected with a slab of thickness 0.24 m (see Fig. 2). The main beams dimensions are 1.0×0.24 m. The slab has the same height (0.20 m block and 0.04 m solid slab topping) as the beams. The frames columns have a cross section of 0.4×0.4 m; the interior ground column has six bars of 20 mm diameter; the exterior ones have six bars of 16 mm diameter. The structure was designed for medium seismicity which corresponds to 0.25 g peak ground acceleration with accordance to EC-8 [4]. The materials used in this building were C25/30 concrete and S500 steel with 200 GPa elastic modulus.

The RC frame considered for nonlinear simulations is a 5 storey 8 bay building with a 0.45×0.60 m beams and 0.45×0.45 m columns. The main beams were reinforced with top and bottom continuous 2 bars of 32 mm diameter and additional bar with the same diameter at columns locations, beams reinforcement ratios on both top and bottom sides ranging from 0.9 to 0.6%. The columns reinforcements were 4 bars of 32 mm diameter. This frame was designed according to EC-8 [4]. The reinforcement details are presented in Fig. 6. The materials used were concrete of strength C25/30 concrete and steel S500 steel with 200 GPa elastic modulus.

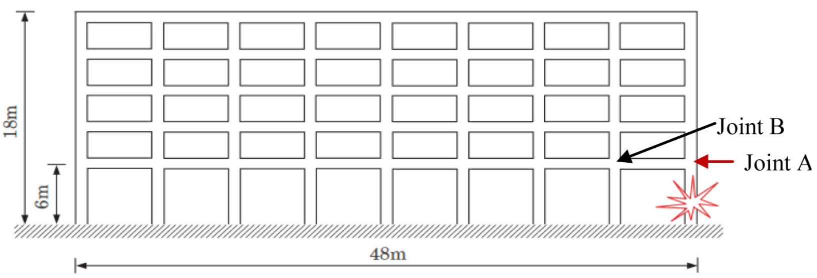


Fig. 5. Elevation view for multi-storey frame case study [11]

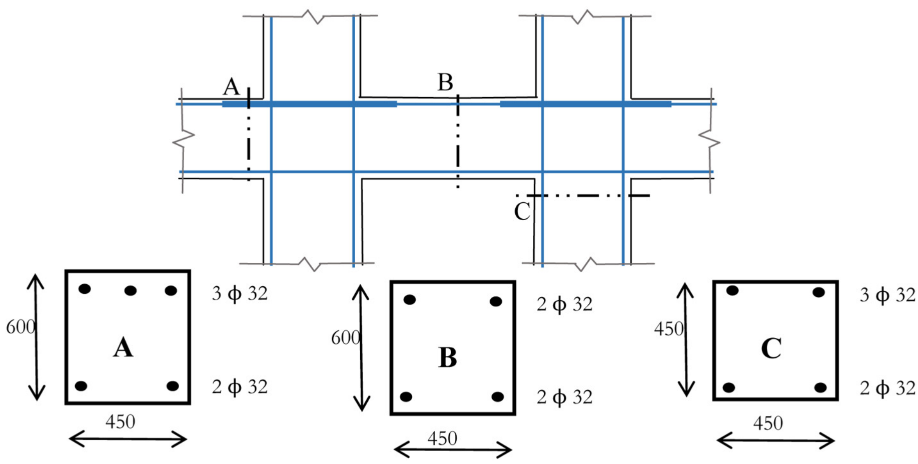


Fig. 6. Reinforcement details in elevation and cross section views in mm [11]

3. Finite element models development

All beams and columns are modelled by frame elements, each frame element has two nodes and each node has three degrees of freedom (two transitional and one rotational). Different frame elements sizes are used (1, 0.5 and 0.25 m) and convergent results are obtained. With linear static analysis in SAP2000, element refinement is found to have insignificant effects; this is because of the limited deformation in different frame elements as a result of light applied loads for that, findings of models with 1 m element size are presented. A gravity vertical uniform load with a value of 17.33 kN/m is applied to all beam frame elements in order to represent frame self-weight. The ground columns are modelled with fixed bottom supports to represent the real buildings condition as shown in Fig. 2. Following that, the model analysis was run in a two dimensional linear static mode.

Nonlinear analysis considers the materials inelastic behavior and geometric nonlinearity as a result of the developed large deformations. At a certain load and deflection level, the most heavily stressed section in the structure becomes completely plastic. With further loading, the bending moment (B.M.) at the formed plastic hinge location is kept constant and more plastic hinges are formed in other locations. The degree of the structure's indeterminacy therefore decreases one by one unit, until there is no more structure's resistance to further loading. At this moment, a mechanism has been reached. The nonlinear static analysis in this study is performed for a RC frame with SAP2000. First, one should build a model of the frame similar to the one used for linear static analysis, then using the option of automatic plastic hinge definition in parallel with defining all reinforcement characteristics (bars sizes, numbers and yield strength). Second, assign these plastic hinges to their expected location (at the start and end of each frame element). For special cases, plastic hinge properties can be defined manually according to the anchorage perfection of the section reinforcement. The building performances are assessed with the capacity curve generated in each case. Performance levels are used to describe the limiting damage condition. The performance levels are expressed in terms of target displacement and defined deformation of structural elements. The three performance levels considered are Immediate Occupancy (IO), Life Safety (LS) and Collapse Prevention (CP) as presented in Fig. 7.

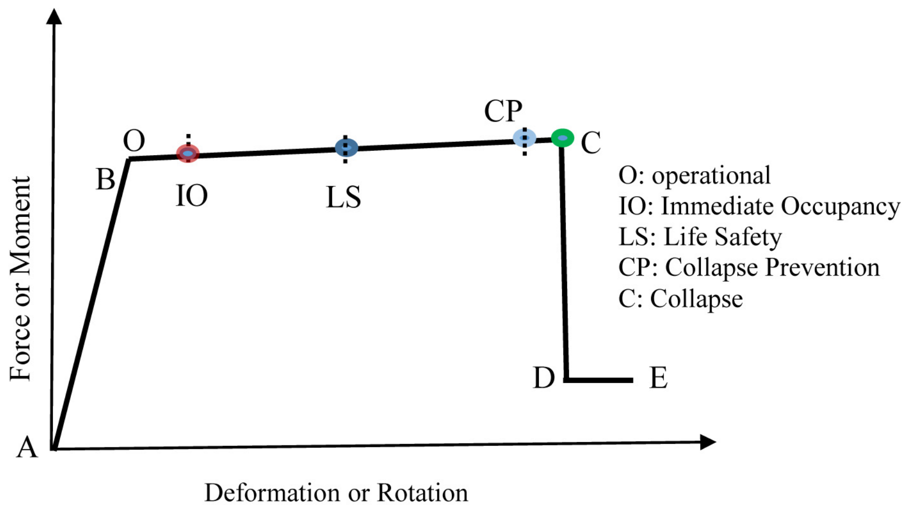
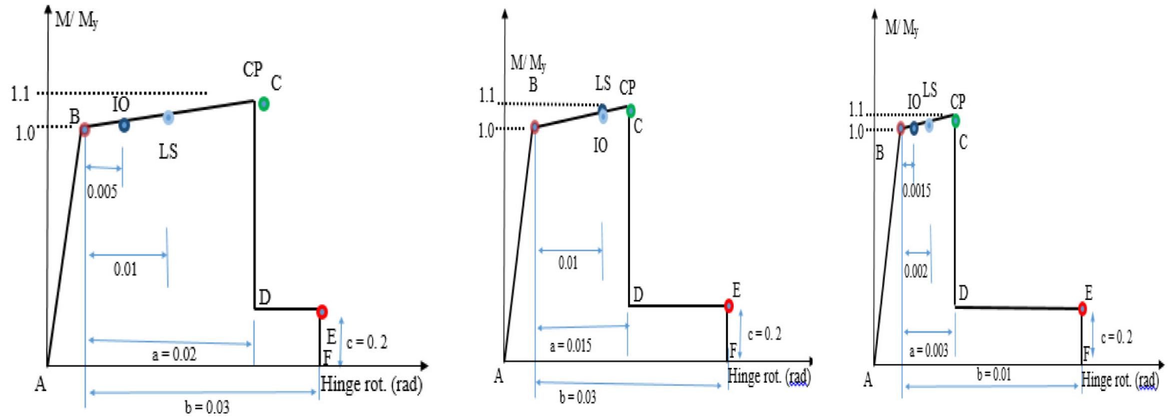


Fig. 7. Hinge behavior curve according CSI [3]

Based on Fig. 7, elastic deformation is represented until point B, then the plastic deformation starts to take place after point B until point C. In this plateau, the plastic deformation domain exists with milder slope for the strain hardening till reaching plastic hinge maximum capacity. This is followed by a sudden drop in plastic hinge capacity at point D to represent the plastic hinge residual capacity. Point E represents the plastic hinge ultimate deformation. During this study, three different cases are made to model the frame in different possibilities according to different configuration of reinforcement as shown in Fig. 8. First, the case 1 which assumes that all plastic hinges are in normal case based on FEMA 356 [5] assumption (the reference case) and using default plastic hinge characteristics by SAP2000. The second case 2 assumes that all plastic hinges are in normal case except the one at joint above the lost column which considers softer response to simulate joint with deficient anchorage situation. Lastly, the case 3 assumes that all plastic hinges are in normal case except the one at joint above the lost column which considers softer response to simulate joint with deficient shear resistance as shown in Fig. 8c.



a) Normal plastic hinge b) Anchorage deficient c) Shear deficient

Fig. 8. Plastic hinge characteristics for different cases using FEMA 356 assumption

4. Results and discussion

The current study focuses on scenario of a corner column loss, because of the lack of surrounding elements that could help to redistribute the axial force initially resisted by the damaged corner column. Different numerical analyses are performed to assess the role of the joint above the lost column on the entire frame resistance to progressive collapse.

4.1. Linear simulations

In Fig. 9, a positive moment is formed at location ‘1’ due to the Vierendeel action. The obtained result matches to some extent with the ones in Kokot et al. [9]; yet, the bending moment B.M. is found higher at interior joints due to the resistance of transverse beams used in the 3D simulation in the reference study. B.M., axial forces and shear forces are presented in Fig. 9. The internal forces values are shown and compared to reference ones in Table 1 for frame span ‘a’.

Numerical results for the intact joint at location ‘2’ (normal case, B.M., axial compression force and extension distance for the negative B.M. are 239 kNm, 55.77 kN and 2.6 m, respectively, and presented in Fig. 10) are compared with the results for other joints conditions as shown in Fig. 11.

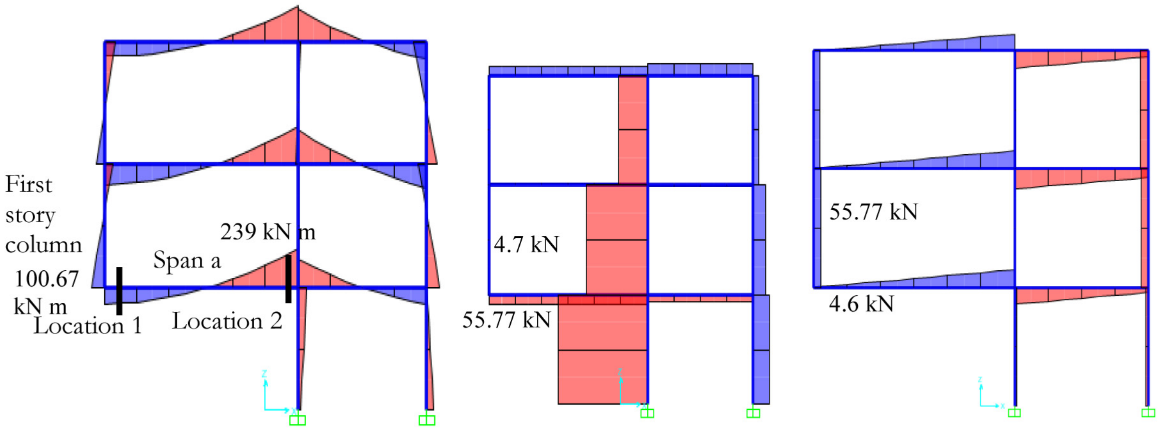


Fig. 9. Schematic drawing for B.M., axial forces and shear forces

Table 1. B.M. in kNm, axial force N and shear force V in kN at location '1', Negative values stand to compression force

Joint	Column side			Beam side		
	M	N	V	M	N	V
Proper joint	100.67	−4.7	55.77	100.67	−55.77	≈ 4.6
Reduced joint	61.60	2.7	44.30	61.11	−44.30	2.6
Improper joint	0.00	≈ 2.9	17.34	0.00	−17.34	0.0
Improved joint	128.60	−9.1	63.28	128.60	−63.28	9.2

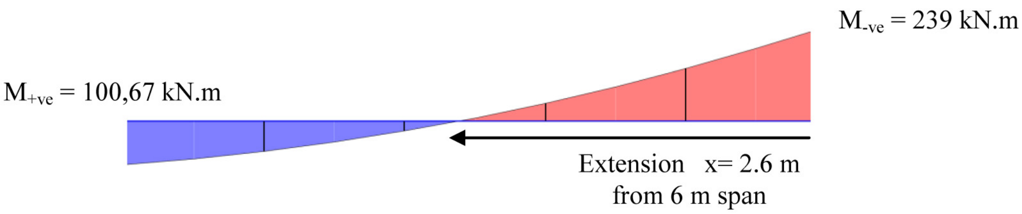


Fig. 10. B.M. D for first storey beam and location of counter flexure point

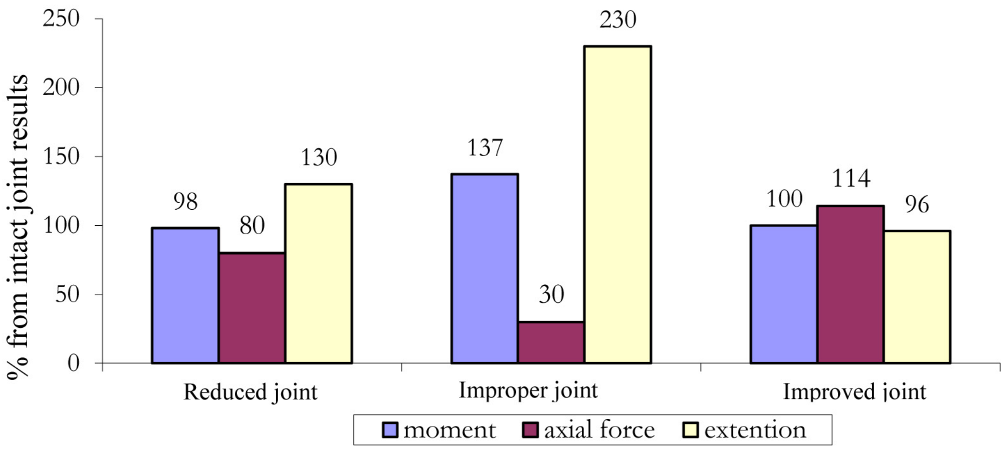


Fig. 11. B.M., axial force and negative moment extension at location '2'

The axial force in the first storey column above the removed one gives an indication on how the panels above that removed column contribute to vertical loads redistribution.

Fig. 12. First storey column axial force variation for different joint stiffnesses

Fig. 12 presents the normal force in the column of the first storey above the lost one for different cases of joint stiffness.

The normal force sign and value of the first storey column illustrate the interrelationship between the first and second floor beam. In the usual case (intact joint), the column normal force is 4.6 kN (compression); this means that a small vertical force is transferred from the second to the first storey beam as a result of the effectiveness of the joint above the lost column to transfer these loads to the adjacent column. With joint stiffness degradation, the column normal force is changed and its value is reduced, this force is turned into a positive one (tension) and this tension force increases with an increasing joint damage degree until the full damaged state is reached. This means that the first storey beam is not able to transfer its loads to the adjacent column and it hangs its loads upward to the storey beam above. This spreads the damage from the first storey to the upper stories, as is clear from the distribution of B.M. for the second storey beam (see Fig. 13).

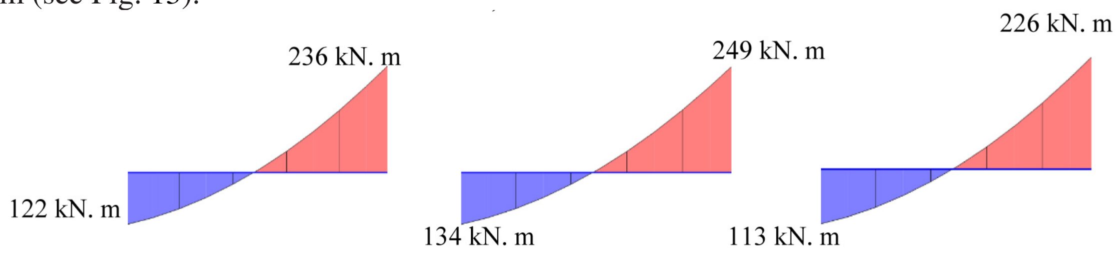


Fig. 13. Beam B.M. at the second storey level at different joint stiffness cases

On the other side, by increasing the stiffness of the joint above the lost column, the first storey column normal force returns into a compression force with a considerable value as indicated in Fig. 12c; this means that the first storey beam is still able to work and transfers more loads to the adjacent column. In addition to that, the damage can be controlled and prevented from spreading out to more areas as was the case before; the B.M distribution in the second storey beam supports that, as presented in Fig. 13.

A RC frame with deficient joints is found more critical because of high resulted moments at different joints locations. The intact joint assumption is not appropriate when assessing the behavior of an existing RC frame with expected damage at one of its connections. This procedure is limited to be used only for low rise buildings with light loads, as recommended by different design guidelines [3, 6]. This linear static analysis cannot account for neither load redistribution after yielding at critical sections, nor nonlinear material properties.

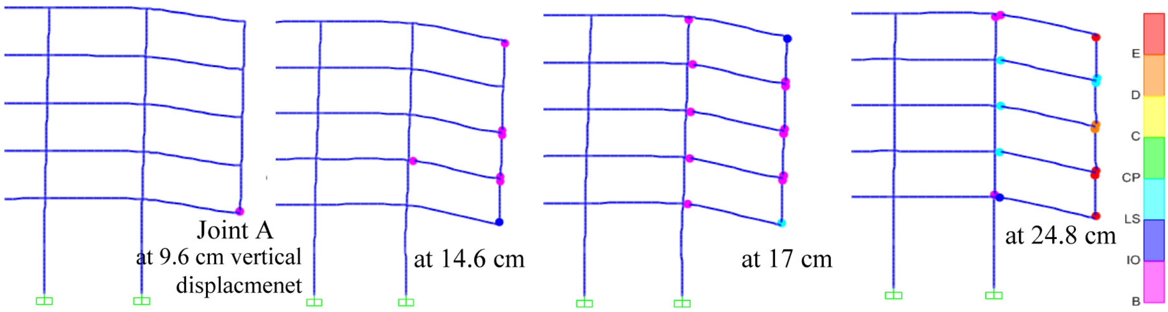
A statically indeterminate RC structure such as a multi-storey frame, does not fail directly when one of its critical sections reaches its ultimate capacity. If a structure is redundant with properly detailed joints, more moment redistribution can take place in the flexural members by developing plastic hinges at critical sections such as in location ‘2’ in Fig. 9. The moment redistribution reduces the values of negative moments at the support locations, but on the other side, increases the values of positive moments along the beam span.

4.2. Nonlinear numerical simulation

The formed plastic hinges in the considered RC frame are presented and compared for different cases. The following figures show these plastic hinges at different loading levels to have an indication on how the stories above the removed column interact and deform till reach the maximum specified vertical displacement.

The different hinging stages for the frames with user defined plastic hinges are presented as well. Each figure represents location, number and status of each plastic hinge for the different cases 1, 2 and 3, respectively. Similar pattern of the formed plastic hinges for the case study frame in both cases ‘2’ and ‘3’ are noticed at the different loading levels, especially at yielding point.

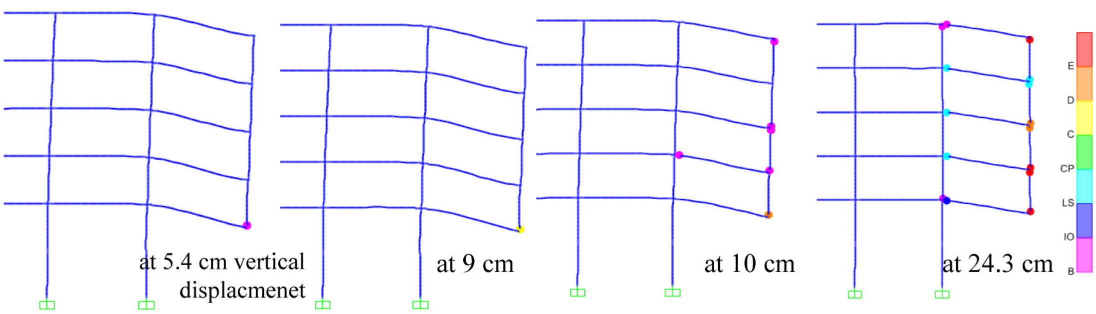
In Fig. 14, plastic hinge formation starts at joint ‘A’, and then propagates to upper stories then to interior joints followed by changing the plastic hinge status at joint ‘A’ to become at the LS limit. With increased vertical deflection, all interior joint were yielded and joint ‘A’ reached CP limit. The final shape of plastic hinges formed was similar for different considered cases. It is noticed that plastic hinges with high plastic rotation were found at location ‘A’ and roof knee joint, which give an indication about vulnerability of these joint at high deformation level.



a) First hinge stage b) LS stage c) CP stage d) Final stage

Fig. 14. Plastic hinges developments for case ‘1’

In Fig. 15, plastic hinge formation starts at joint ‘A’ at deflection with smaller values than in case ‘1’, then instead of propagating to upper stories and interior joints, its status turned directly and at smaller deflection to CP limit with high plastic rotation. With increased vertical deflection, all interior joint were yielded while joint ‘A’ reached its maximum plastic rotation. However, there are significant similarities in hinging patterns for both cases ‘2’ and ‘3’, there is more softening rate in plastic hinges rotation at joint ‘A’ due to more brittle nature of shear deficient joints than anchorage deficient one at the same corresponding vertical deflection.



a) First hinge stage b) Softening at knee joint c) other joints at IO level d) Final stage

Fig. 15. Plastic hinges developments for case ‘2’

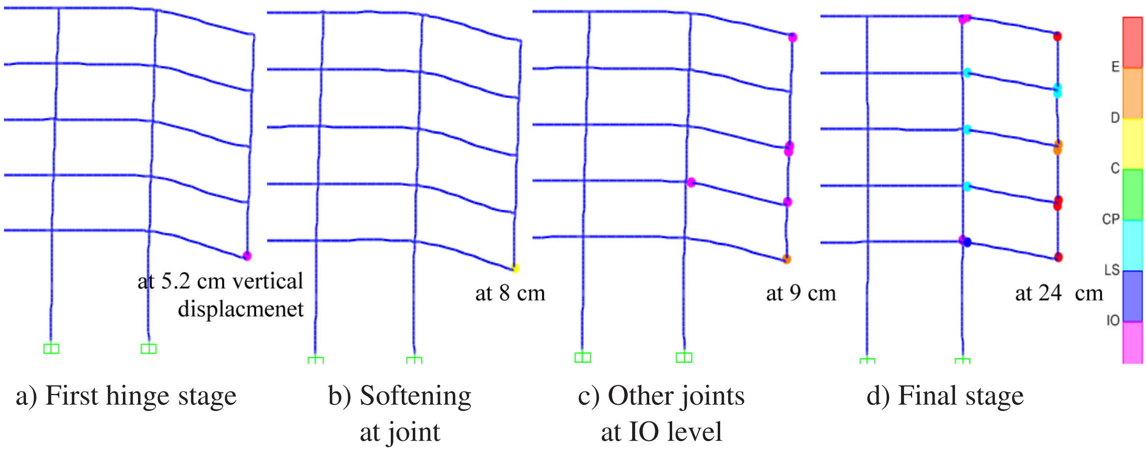


Fig. 16. Plastic hinges developments for case '3'

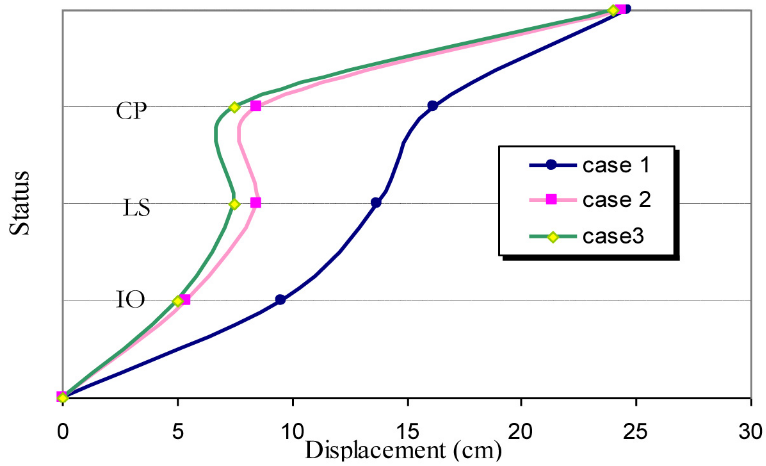


Fig. 17. Push down displacement versus plastic hinge status

In Fig. 17, the pushdown displacement versus plastic hinge status of the studied RC frame with different cases at joint 'A' are presented. Bullets on each curve represent the status of plastic hinge (P.h.) formed; IO (Immediate occupancy), LS (Live safety) and CP (Collapse prevention). According to this figure, as the shear or anchorage deficiencies exist, plastic hinge at smaller deflection exceeds many acceptance criteria which show the structure capacity degradation after a corner column loss. Table 2 provides the sequences of plastic hinges formation for each case together with the associated vertical displacement at joint 'A'.

Table 2. Plastic hinges in different cases vs vertical deflection (cm) at joint 'A'

	First P.h.	P.h. Int. joint	Mechanism
Case 1	9.6	13.7	24.8
Case 2	5.4	13.6	24.4
Case 3	5.2	13.6	24.0

4.3. Plastic hinges status

Figs. 18–21 present the results for the formed plastic hinges at joints 'A' and 'B' in the beam above the removed column. The plastic hinge at left-end at joint B in RC frame in case '1' reaches LS limit and according to Fig. 18, the plastic hinge ends with residual moment capacity of 265 kNm which is higher than the residual moment capacity in both cases '2' and '3' which was 150 kNm as shown in Fig. 19. However, this joint in case '1' has a plastic rotation of 9.5×10^{-3} radians which is closer to the other two cases, the structure at this case can resist 7 560 kN which is more than the 7 000 kN, the maximum resistance in both cases '2' or '3' as illustrated in Fig. 22.

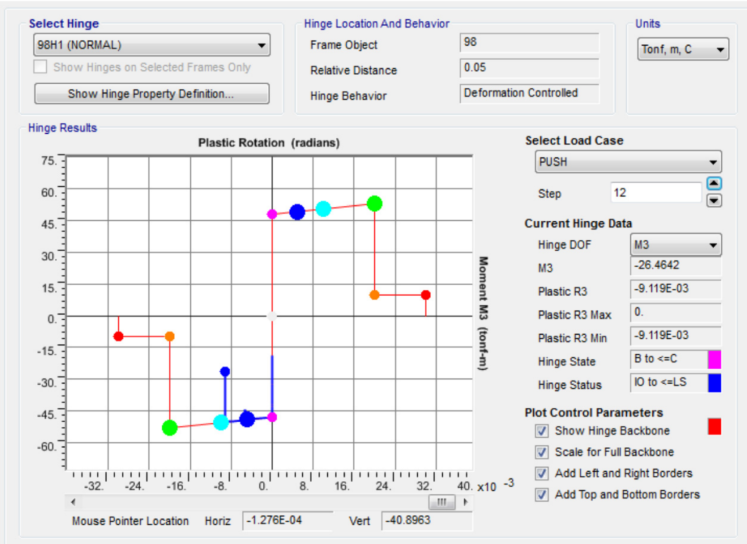


Fig. 18. Moment-Plastic hinge rotation for the plastic hinge at joint 'B' in case '1'

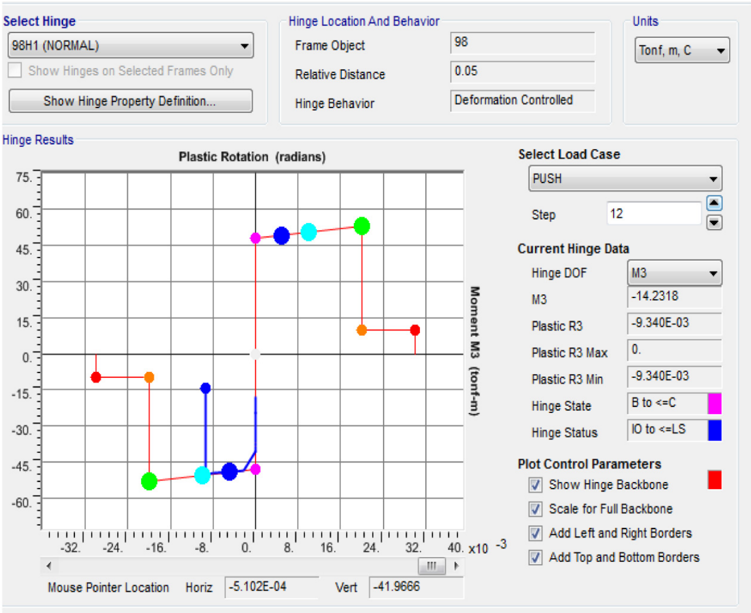


Fig. 19. Moment-Plastic hinge rotation for the plastic hinge at joint 'B' for both cases '2' and '3'

Figs. 20 and 21 illustrate the response of the first formed plastic hinge in the frame which took place at joint 'A' location. In both cases '2' and '3', the resulted moment-plastic rotation is noticed matched to some extent with previously define hinge backbone curve. On the other side, post-peak deviation is noticed between the resulted moment-plastic rotation and the defined backbone curve in case '1' as clear in Fig. 20 in the blue line after the green bolt.

This deviation may be attributed to the formation of higher normal forces in case '1' than in the other two cases. Which give an indication of case '1' distinction over cases '2' and '3' in increasing global structure carrying capacity, and locally by allowing the formation of higher compression forces in the beam than in the other two case, which improve its ultimate resistance.

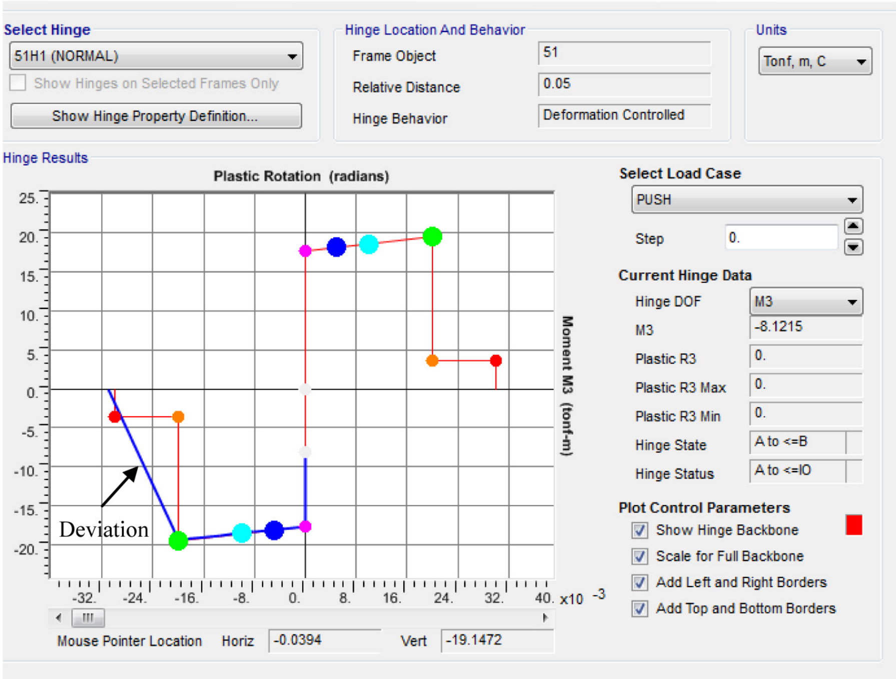


Fig. 20. Moment-Plastic hinge rotation for the plastic hinge at joint ‘A’ in case ‘1’

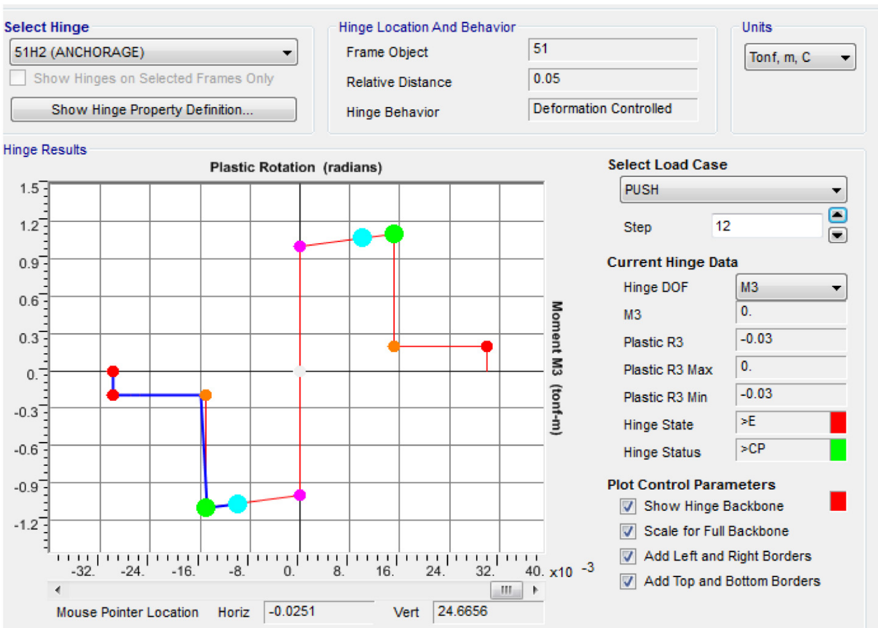


Fig. 21. Moment-Plastic hinge rotation for the plastic hinge at joint ‘A’ for cases ‘2’ and ‘3’

4.4. *Supports reaction force*

At each step of analysis, the total base reaction is plotted against joint ‘A’ vertical displacement, which provides information about the ductility capacity of the structure system. Figs. 22a-b present the capacity curves for the studied RC frame for different considered cases at joint ‘A’. Fig. 22a shows the resulted reaction forces at all supports during loading the frame with increased displacement as an indication for structure carrying capacity, the results of the reaction force for the normal case is 7 560 kN and showed higher carrying capacity than for the both cases ‘2’ and ‘3’ with about 8% as shown in Fig. 22b. It is worth noting that even though case ‘3’ has two sources of deficiency (anchorage and shear at column and beam sides, respectively) and case '2' has only one (anchorage), they have the same capacity curves, and this can be referred to beam section higher ultimate flexural capacity compared to column cross section. This is obvious from plastic hinge sequencing in Figs. 14–16, as the plastic hinge at joint 'A' was usually formed in the column and not in the beam side.

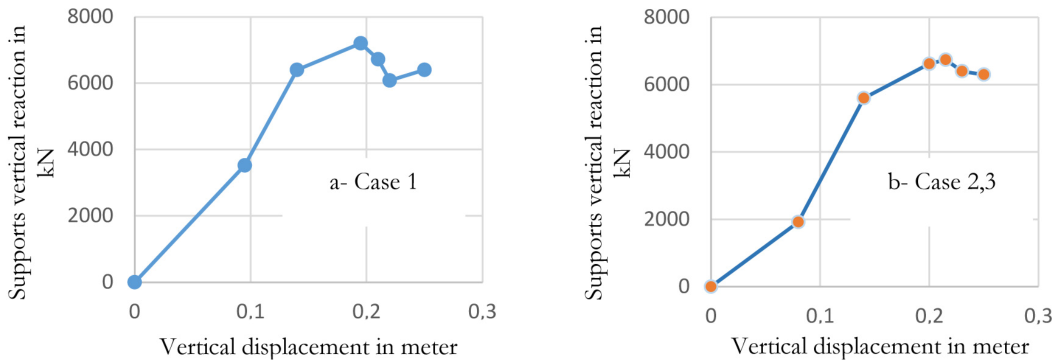


Fig. 22. Supports reaction force vs joint a vertical displacement for different cases

4.5. *Dynamic nonlinear analysis*

This analysis type explicitly includes nonlinearity (nonlinearity was included in the model by using "Plastic hinges" at both ends of every beam and column elements). The important modeling parameters included the damping ratio, time step, column removal time are taken according to [3]. The distribution of B.M. and normal force in the beam above the lost column is found to be similar to the results obtained by Santafé et al. [11] as shown in Table 3.

Table 3. Values of B.M. in kNm and axial normal force in kN for the beam above the lost column, two values are reported: the first one is obtained in the current analysis and the second one is taken from Santafé et al. [11]

	M at Joint B. kNm	M at Joint A kNm	Max. comp. force in kN
Nonlinear dynamic	639/600	283/300	165/170
Nonlinear static	580/600	254/300	147/170
Linear static	562/600	261/300	152/170

In addition to that, the vertical displacement at joint 'A' above the removed column is found 0.267 m and 0.31 m for the current nonlinear dynamic study and Santafé findings respectively. The corresponding vertical displacement for nonlinear static analysis was 0.248 m, which is further from the reference results than the nonlinear dynamic analysis results.

5. Conclusion

In this study, finite element software package SAP2000 was used to discuss the response and performance of two RC frame based on the stiffness and damage level in the beam-column joint above the damaged corner column. The results are summarized as follows:

- Beam-column joints with proper stiffness and perfect details are able to prevent and minimize damage propagation in the column above and consequently reduce deformation after event of losing ground corner column;
- Improved joints are able to reduce and limit the spreading of damage after scenario of column loss;

- Shear and anchorage deficient joints reduce overall RC frame ultimate carrying capacity with about 8% and initiate the formation of more plastic hinges with severe damage indications;
- In this specific case, column bars anchorage deficiency effects consequences are more obvious than beam shear deficiency.

This paper presented the differences in RC frames responses based on the defined properties of their expected plastic hinges utilizing pushdown loading procedure.

Some future work is still needed to enable SAP 2000 numerical models to consider other values for the hinge moment capacity rather than the yield moment. Further numerical beam-column joints models are needed to simulate phenomena with brittle failure; bond failure which was likely to occur at small displacement and hinder the section to reach its yield limit. This will lead to better anchorage deficiency representation in such joints.

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