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A ballistic missile shutdown point estimation method based on double fitting corrections

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ABSTRACT

For long-range infrared systems, a new method is proposed in this paper to estimate the shutdown point of ballistic missile. In order to reduce the effect of model error and positioning error of observation point on estimation accuracy, two successive fitting corrections are used in a three-dimensional observation space and a two-dimensional characteristic space respectively. Firstly, the three-dimensional observation data points are fitted by a trajectory plane model, and these points are projected to the trajectory plane for the first correction. Then, a characteristic space l-β is set up to describe these projective points in a two-dimensional polar coordinates and the projective points are fitted and corrected by polynomial curves along two axes respectively. Finally, the motion characters of these projective points are converted to the first three-dimensional observation space and the motion state of shutdown point are estimated. The simulation results show that our method is feasible and valid.

Keywords

ballistic missile, shutdown point, trajectory plane, characteristic space, double corrections.

1. INTRODUCTION

As ballistic missiles have been the main component element in long-distance guidance weapons[1]. Ballistic missile tracking in time is one of the key technologies in the missile defense system. Many related work about ballistic missile tracking have been done. Xu Zhang [1] proposed a method for ballistic missile trajectory prediction based on predictor-corrector method considering the radar's measurement noise. Zhang Feng [2] investigated the method of ballistic missile tracking using dynamic model. Dong Gwan Lee [3] proposed a method to predict ballistic missile trajectory based on the Kepler's law for the missile defense system using cueing information. Robert L. Cooperman [4] developed a tactical ballistic missile tracker with an interacting multiple models framework to solve the problem of tracking a ballistic missile by the varying target dynamics in the boost, exo-atmospheric and endo-atmospheric phases of flight. Bennavoli[5] proposed an approach which combines a nonlinear batch estimator with a recursive multiple model particle filter in order to estimate the launch and impact points of a ballistic target.

In order to predict position and velocity vector of shutdown point, this paper proposes a shutdown point estimation method based on double fitting corrections.

2. PROBLEM & CLASSIC METHOD

All observation points are from a three-dimensional space which can be represented by the Inner earth space rectangular coordinates $O-XYZ$. The estimated parameters of shutdown point need to be represented in the same space. It is assumed that the launch time of the missile is t_0 , the first time of observation data points is t_m , the shutdown time is t_n ($0 \le m < n$). Target position and velocity vector when the shutdown time can be estimated from three-dimensional observation data points. In the post-boost phase, the ballistic missile targets are mainly acted upon by gravitational force and follow a Keplerian orbit, so fall point depends mainly on parameters of shutdown point. In addition, because of 6 parameters, it is not simple and intuitive to measure the estimation accuracy of shutdown point. So, fall point error(FPE) is used to measure the shutdown point estimation accuracy in this paper.

Classic method estimated the position and velocity vector using polynomial fitting in x , y , z directions respectively. Shutdown point error of classic method relatively large because it is the sum of three directions. If a trajectory plane can be determined , these three-dimensional points can be projected to the trajectory plane and estimation of shutdown point can be processed on a 2-D space. The FPE would become the sum of errors in 2 directions. In theory, the sum of errors in 2 directions is smaller than that in 3 directions.

3. PROPOSED METHOD

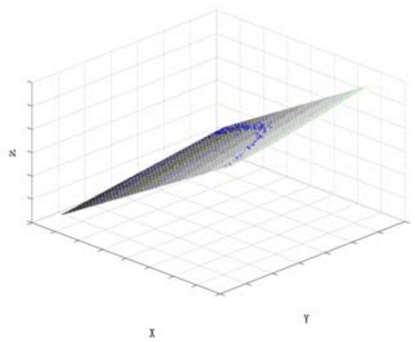

Figure 1 Trajectory plane and projective points

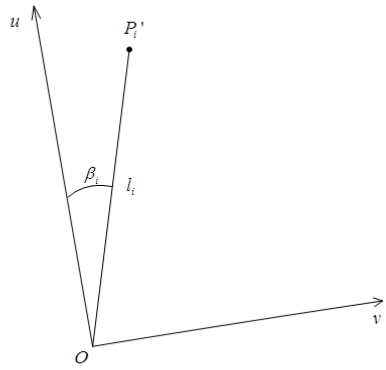

Figure 2 Relationship between $l-\beta$ and $u-v$

The specific steps are as follows:

S1. Determining the trajectory plane:

The trajectory plane: $Ax + By + Cz + D = 0$ is obtained by linear regression with some three-dimensional observation points.

S2. Projecting observation points to plane:

According the projecting method of point to plane, we can get the projective point $P_i'(P_{Xi}', P_{Yi}', P_{Zi}')$ of the observation point $P_i(P_{Xi}, P_{Yi}, P_{Zi})$ at the time t_i, $t_k \le t_i \le t_n$. Trajectory plane can be seen in Figure 1.

S3.Calculating l and β :

The first-correction vector at time t_i is defined as

$$\overrightarrow{OP_i}'(OP_{Xi}', OP_{Yi}', OP_{Zi}') \\ = P' - O = (P_{Xi}' - O_X, P_{Yi}' - O_Y, P_{Zi}' - O_Z) \quad (1)$$

Where (O_X, O_Y, O_Z) is the projective point of the center of earth.

The initial vector is defined as the first-correction vector at $t_m : \overrightarrow{OP_m}'(OP_{Xm}', OP_{Ym}', OP_{Zm}')$, then it is also defined as the polar axis;

The length l_i and the angle β_i of the first-correction vector at time t_i are

$$l_i = \left|\overrightarrow{OP_i'}\right| \quad (2)$$

$$\beta_i = a\cos(\overrightarrow{k_i} \bullet \overrightarrow{k_m}) \quad (3)$$

where $\overrightarrow{k_i}$ is the unit vector of $\overrightarrow{OP_i'}$, and $\overrightarrow{k_m}$ is the unit vector of $\overrightarrow{OP_m'}$.

S4.Transforming characteristic vectors in Coordinates $l-\beta$ to $u-v$:

Build a rectangular coordinates $u-v$, its origin coincide with the origin of coordinates $l-\beta$, and u axis coincide with the polar axis, and v axis is perpendicular to u axis. Transform projective point $P_i'(l_i, \beta_i)$ to $u-v$ coordinates, the result is $P_i'(P_{Ui}, P_{Vi})$. The transformation formulas are:

$$\begin{cases} P_{Ui} = l_i \times \cos\beta_i \\ P_{Vi} = l_i \times \sin\beta_i \end{cases} \quad (4)$$

The relationship between $l-\beta$ and $u-v$ can be seen in figure 2.

S5.Second correction in coordinates $u-v$:

$$u(t) = a_1 t^3 + b_1 t^2 + c_1 t + d_1 \quad (5-1)$$

$$v(t) = a_2 t^3 + b_2 t^2 + c_2 t + d_2 \quad (5-2)$$

In coordinates $u-v$, $P_i'(P_{Ui}, P_{Vi})$ ($k \le i \le n$) can be fitted by the above curves model (here, we take 3 as the fitting order). $P_i''(P_{Ui}', P_{Vi}')$ is the corrected point of $P_i'(P_{Ui}, P_{Vi})$. Then take derivative of $(10-1)$, we obtain:

$$u'(t) = 3a_1 t^2 + 2b_1 t + c \quad (6)$$

The velocity in u direction can be get as $V_{Ui} = u'(t_i)$. The velocity in v direction, V_{Vi} can also be obtained.

S6.Calculating position and velocity of shutdown point in observation space:

$P_n''(P_{Un}', P_{Vn}')$ and $\overrightarrow{V_n}(V_{Un}, V_{Vn})$ are the position and the velocity of shutdown point in $u-v$ coordinates.

Assume that point $P_n''(P_{Xn}'', P_{Yn}'', P_{Zn}'')$ is the estimated position of shutdown point in observation space, let the corresponding second-correction vector

$$\begin{aligned} \overrightarrow{OP_n''} &= P_n'' - O \\ &= (P_{Xn}'' - O_X, P_{Yn}'' - O_Y, P_{Zn}'' - O_Z) \end{aligned}.$$

The position of P_n'' can be obtained by solving:

$$\begin{cases} A \bullet P_{Xn}'' + B \bullet P_{Yn}'' + C \bullet P_{Zn}'' + D = 0 \\ l_n' = \sqrt{P_{Un}'^2 + P_{Vn}'^2} \\ \beta_n' = \arctan(\dfrac{P_{Un}'}{P_{Vn}'}) \end{cases} \quad (7)$$

Where $l_n' = \left|\overrightarrow{OP_n''}\right|$, and $\beta_n' = \arccos(\overrightarrow{k_n'} \bullet \overrightarrow{k_m})$, and $\overrightarrow{k_n'}$ is the unit vector of $\overrightarrow{OP_n''}$.

It is obvious that the following equations can be obtained according to geometric characteristics:

$$\begin{cases} \overrightarrow{k_m}(1)\left(P_{Xi}'' - O_X\right) + \overrightarrow{k_m}(2)\left(P_{Yi}'' - O_Y\right) + \overrightarrow{k_m}(3)\left(P_{Zi}'' - O_Z\right) = \cos(a\tan(P_{Ui}'/P_{Vi}')) * \sqrt{P_{Ui}'^2 + P_{Vi}'^2} \\ A * P_{Xi}'' + B * P_{Yi}'' + C * P_{Zi}'' + D = 0 \\ \left(P_{Xi}'' - O_X\right)^2 + \left(P_{Yi}'' - O_Y\right)^2 + \left(P_{Zi}'' - O_Z\right)^2 = P_{Ui}'^2 + P_{Vi}'^2 \end{cases} \quad (8)$$

Take derivative of Formula (8), we can obtain the following equations:

$$\begin{cases} \overrightarrow{k_m}(1)*V_{Xi}+\overrightarrow{k_m}(2)*V_{Yi}+\overrightarrow{k_m}(3)*V_{Zi}=-\sin(\arctan(P_{Ui}'/P_{Vi}'))*\dfrac{V_{Ui}*P_{Vi}'-V_{Vi}*P_{Ui}'}{P_{Ui}'^2+P_{Vi}'^2} \\ *\sqrt{P_{Ui}'^2+P_{Vi}'^2}+\cos(\arctan(P_{Ui}'/P_{Vi}'))*\dfrac{V_{Ui}*P_{Ui}'+V_{Vi}*P_{Vi}'}{\sqrt{P_{Ui}'^2+P_{Vi}'^2}} \\ A*V_{Xi}+B*V_{Yi}+C*V_{Zi}=0 \\ \left(P_{Xi}''-O_X\right)*V_{Xi}+\left(P_{Yi}''-O_Y\right)*V_{Yi}+\left(P_{Zi}''-O_Z\right)*V_{Zi}=V_{Ui}*P_{Ui}'+V_{Vi}*P_{Vi}' \end{cases} \tag{9}$$

Velocity vector $\overrightarrow{V_n}(V_{Xn},V_{Yn},V_{Zn})$ can be obtained by Formula (9).

4. RESULTS

The performance of the classic method and our proposed method are evaluated by simulation experiments.

3-D observation data points of a ballistic missile trajectory are simulated with positioning error of $\alpha(\alpha \neq 0)$. Take the fitting order to 3, number of fitting points to 150-450, some simulation experiments are carried out. Figure 3 illustrates FPE curves along with the number of fitting points in different methods. The number of points used to determine the trajectory plane in the experiment is 500, and it is the first optimal value obtained by simulation experiments.

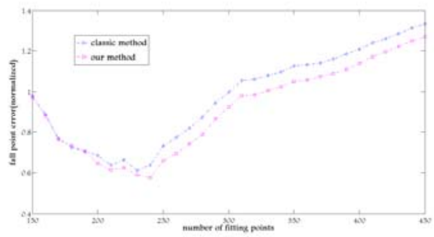

Figure 3 Normalized FPE

It is shown that the predicted FPE by our proposed method is smaller than that by the classic method.

5. CONCLUSIONS

This paper proposed a shutdown point estimation method with higher accuracy by converting comprehensive error of 3 directions to 2directions. The simulation results show that our method is feasible and valid.

6. REFERENCES

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