

Using Intrinsic Surface Geometry Invariant for 3D Ear Alignment

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ABSTRACT

In this study we derive novel surface fiducial point's detection that is computed from the differential surface geometry. The fiducial points are intrinsic, local, and relative invariants, i.e., they are preserved under similarity, affine, and nonlinear transformations that are piecewise affine. In our experiment, the fiducial points, computed from high order surface shape derivatives, are used in a non-iterative geometric-based method for 3D ear registration and alignment. The matching is achieved by establishing correspondences between fiducial points after a sorting based on a set of absolute local affine invariants derived from them. Experimental results showed that our purposed surface feature is suitable for further application to 3D ear identification because its robustness to geometric transformation.

Keywords— 3D ear registration, Surface geometric invariant, Zero torsion

1. INTRODUCTION

Biometrics is an emerging technique that involves the use of physiological and behavioral characteristics to determine the identity of an individual. At the present time, the physical biometrics, for example fingerprints, facial patterns and eye retinas, are developed and enhanced. In this work, we are interested to identify humans by ear structure. The ear anatomy has a lot of unique structures which do not change with changing event or age. In addition, compared with the other biometrics techniques, the ear data can be registered in a non-invasive way [Che07a, Bur00a].

Geometric Invariance is a central problem in visual information system, computer vision, pattern recognition image registration, and robotics. The term invariance is referred to the geometrical properties of the relative distance among a collection of static spatial features of an object [Gov99a-Pin13a].

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Our study is focused on registration techniques for biomedical 3D images, Surface registration. Surface registration is a vital step in medical imaging literature. Various techniques have been proposed for surface registration from which the following general methodology can be defined. Firstly, the landmarks or specific structures are extracted from each image to be registered. Secondly, correspondence between the extracted landmarks is established. Thirdly, choice of geometric transformation, such as rigid transformation affine transformation or polynomial transformation, is entertained. Fourthly, the geometric transformation parameters are estimated. Lastly, the two surfaces are aligned. Surface registration methods can be categorized into polynomial transformation [Sin09a], similarity-based [Woo98a] surface-based [Bes92a], energy-based [Por94a] and fiducial point - based registration [Kan81a-Ibr98a].

In this paper, we consider the problem of 3D ear alignment in the different orientations. The data is 3D and obtained using a laser scanner. Our approach is based on the differential geometry of the surface, consists of two processes, start from computing intrinsic local fiducial points on the surface and on curves that reside on the surface. A fast non-iterative alignment is purposed in our study that establishes reliable correspondences between fiducial points

without any prior knowledge of the overall nonlinear global transformation that took place after the changes. This is achieved through the construction of a set of ordered novel absolute local affine invariants. With enough fiducial points collection as correspondents, the overall nonlinear transformation is computed and the ear before and after the transformation are aligned. Finally, this paper deals with the computation of the fiducial points that are based on high order surface shape and curve derivatives, which are honorable for their sensitivity to measurement errors, round off error and distortion.

This paper is organized as the following: - Section 2 introduces the intrinsic local geometric fiducial points on surfaces. Section 3 shows how to construct a set of absolute invariants derived from these fiducial points and how to manage the correspondences between two partial sets of fiducial points residing on two surfaces that are transformations of one another. The experimental results on robustness of surface geometric invariance applied compared with the 3D ear are described in section 4. The finally, discussions and conclusions are given in section 5.

2. GEOMETRIC INVARIANT SHAPE MEASURE

2.1 Parabolic Contour Points

Local and invariant intrinsic properties are presented by the Frenet frames [8], which states that for a curve $r(s)$ parameterized by arc length s , the tangent $t(s) = r^{(1)}(s)$, the curvature $k(s) = r^{(2)}(s)$, the vector $b(s) = t(s) \times k(s)$, and the torsion $\tau(s) = -\langle r^{(2)}(s), b^{(1)}(s) \rangle$ determines as a set of local coordinates on the curve at each point that completely characterizes the curve at that point, where $r^{(k)}(s)$ stands the k^{th} order derivative of r with respect to s , and \times is the cross product operation.

As we are interested in finding the relative and absolute invariant to the affine transformation, we observe that since arc length is not preserved under the affine transformation, neither $t(s)$ nor $b(s)$ cannot be used because they are not relative invariants. We seek to find geometric invariance on the surface which is intrinsic, local and affine invariant. When a surface undergoes an affine transformation, the parabolic contours are the affine transformed parabolic contours of the original curve, i.e., they are preserved. Similarly, the fiducial points residing on these contours are also preserved under the affine transformation. In this section, we briefly introduce theory related to the geometric invariance.

Parabolic contours are space curves that reside on a surface when either one of the two principal curvatures, k_1 or k_2 , is zero [Do76a, Mil97a]. In that case, the Gaussian curvature ($K_G = k_1 k_2$), which is

intrinsic, and vanishes at these points. For a surface, represented by the parameterization $r(u, v): U \subset R^2 \rightarrow S$, the Gaussian curvature is given by the determinant of the second fundamental form parameters [Do76a]

$$K_G = k_1 k_2 \propto \begin{vmatrix} e & f \\ f & g \end{vmatrix} \quad (1)$$

Where

$$e = \langle N, r^{(2,0)} \rangle, f = \langle N, r^{(1,1)} \rangle, g = \langle N, r^{(0,2)} \rangle$$

$$N = \langle r^{(1,0)}, r^{(0,1)} \rangle, r^{(1,0)} = \frac{\partial r}{\partial u}, r^{(0,1)} = \frac{\partial r}{\partial v}$$

$$r^{(2,0)} = \frac{\partial^2 r}{\partial u^2}, r^{(0,2)} = \frac{\partial^2 r}{\partial v^2}, r^{(1,1)} = \frac{\partial^2 r}{\partial u \partial v}$$

The parabolic contours are given by solving

$$eg - f^2 = \langle N, r^{(2,0)} \rangle \langle N, r^{(0,2)} \rangle - \langle N, r^{(1,1)} \rangle^2 = 0 \quad (2)$$

The parabolic contours, which are based on the Gaussian curvature, are intrinsic [Do76a] and preserved under the affine transformation.

The mean curvature was computed from half of Gaussian curvature.

$$K_m = 1/2 (k_1 k_2) \quad (3)$$

2.2 Zero Volume and Zero torsion Points on Parabolic Contour Curves

Given a parabolic contour curve $r(t)$ we can also obtain intrinsic curve points by creating volume relative invariants. One such relative affine invariant can be had by considering the volume of the parallelepiped spanned by the zero, first, and second curve derivatives given by the scalar triple product

$$v_1(t) = \langle r^{(0)}(t) \times r^{(1)}(t), r^{(2)}(t) \rangle \quad (4)$$

Where $r^{(k)}(t)$ is the k^{th} derivative of the curve with respect to parameter t . Equation (5) is a relative affine invariant when the affine transformation has zero translation, i.e., when it is a purely linear transformation. Another relative invariant that carries in the case of a nonzero translation, is the volume of the parallelepiped that is spanned by the first, second, and third curve derivatives given by the scalar triple product

$$v_2(t) = \langle r^{(1)}(t) \times r^{(2)}(t), r^{(3)}(t) \rangle \quad (5)$$

3. ALIGNING THE SURFACES

To align two surfaces that we need to establish the corresponding fiducial points on the two set of

parabolic contours from which we can estimate the overall transformation using least squares. To establish these correspondences and in the absence of the knowledge of the transformation parameters a priori, we construct absolute invariants derived from the fiducial points residing on any contours. These are blind to the transformation and remain unchanged before and after the affine transformation.

3.1 Constructing Absolute Invariant from the Fiducial Points

The set of absolute invariants are constructed from the sequence of relative invariants formed by the volumes of the parallelepipeds spanned by set of four intrinsic surface fiducial points on observed contour. The volume spanned by a set of four points m, n, k and l , is given by $V(m,n,k,l) = |<r_m-r_l>x(r_n-r_l), (r_k-r_l)>|$. When the surface is affine mapped, the volume $V_a(m,n,k,l)$ spanned by the mapped points m, n, k and l , relates to

$$V_a(m,n,k,l) = \det\{[L]\} V(m,n,k,l) \quad (6)$$

To facilitate the process of finding the correspondences by reducing to string matching, we rearrange the fiducial points in accordance with the order described below.

For a collection of n intrinsic fiducial points, we pick an intrinsic surface fiducial point, say point i . By using this point as well as the other three points selected from the combination $\binom{n-1}{3}$, we compute the volume spanned by these four vectors. The smallest volume out of the $\binom{n-1}{3}$ computed volumes is assigned as the point i relative invariant. We restart the process with the next point and the remaining $(n-2)$ points excluding the i^{th} point. The process repeats until the list is depleted. The order of the intrinsic surface fiducial point is then sorted according to increasing volume. If both surfaces have n fiducial points, the two sequences of volume patches on the two surfaces would be.

$$(V(1) < V(2) < \dots < V(n-3) \& V_a(1) < V_a(2) < \dots < V_a(n-3))$$

where there volume patches are related by

$$V_a(i) = \det\{[L]\} V(k), k = 1, 2, \dots, n-3 \quad (7)$$

The absolute invariants on the original and transformed surface are the defined as the ratio of the consecutive volume element in the ordered sequence, i.e.,

$$I(k) = \left\{ \frac{V(k)}{V((k+1) \bmod (n-3))} \right\}, k = 1, 2, \dots, n-3$$

$$I_a(k) = \left\{ \frac{V_a(k)}{V_a((k+1) \bmod (n-3))} \right\}, k = 1, 2, \dots, n-3 \quad (8)$$

3.2 Establishing Correspondence of the Fiducial Points

In the absence of noise and occlusion, each of $I_a(k)$ will have a counterpart $I(k)$ with $I_a(k)=I(k)$, with that counterpart easily determined through a circular shift involving n comparison where n is the number of invariants. To allow for noise and distortion, a smaller error percentage between counterpart invariants is tolerated. The lower the error percentage, the stricter the matching is. In this our experiment, an error percent of 5% is used. A run length method is applied to decide on the correspondences between the two ordered set of zero-torsion points. For every starting point on the transformed set, this run length method computes a sequence of consecutive invariants that satisfies $|I(k)-I_a(k)| < 0.05 |I(k)|$ and declare a match based on the longest string. Once this correspondence is found, these matched fiducial points are used to estimate the polynomial transformation.

4. EXPERIMENT AND RESULTS

The experiments are divided into two parts. In part one; we would like to test the robustness of the proposed method to ear feature extraction. In this test, 3D ear cloud point data are subjected. In the second part, we would like to test the aligning curvature feature of our method.

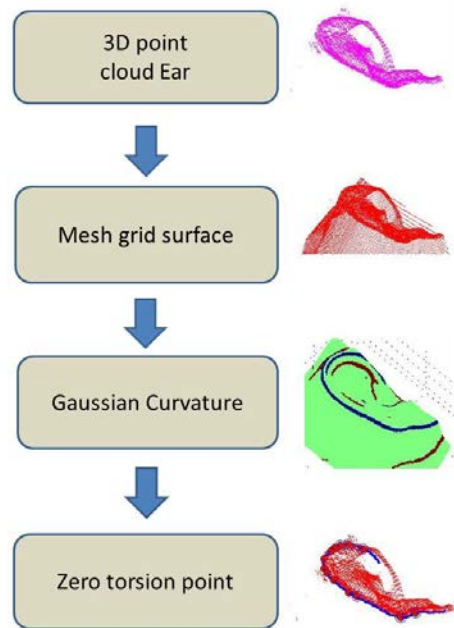


Figure 1. Process of extracting color-mapped images of zero torsion points of surfaces computed from Gaussian curvature.

A. Feature Extraction

The experiment of the robustness of the surface geometric invariant feature on the 3D ear surface extracted from cross-sectional contours to certain class of geometric transformation is described in this section. The process which shows in Fig.1, is as follows:

- (i) Extract 3D coordinate from a set of dense 3D data.
- (ii) Numerically compute to create the grid on the surface
- (iii) Compute Gaussian curvature. The parabolic contours are then derived by approximately solving by Equation (2).
- (iv) Compute Zero torsion points. We obtain intrinsic curve points by creating volume relative invariants which following by Equation (4, 5).

B. Aligning the Surfaces

In this process, the experiment is based on real 3D scans of the same person taken under different orientations. The ear point cloud data is collected in 5 positions. We elect to use the distance map that displays the distance between any point of one surface and the closest point on the other surface after undoing the transformation to the second surface. The two ears taken of the same person at different orientations and with two different positions. Corresponding fiducial points are found, the affine transformation estimated using LSE fitting and the two surfaces are then aligned. The alignment is shown in Fig. 4 whereas we show the two surfaces before and after the alignment in Fig. 2 and 3 respectively. The average distance map error after alignment is shown in table 1.

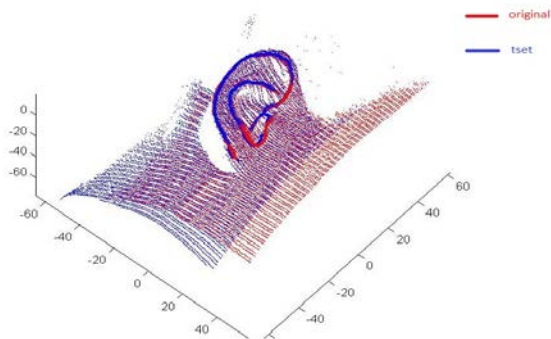


Figure 2. The pre-process of ear alignment

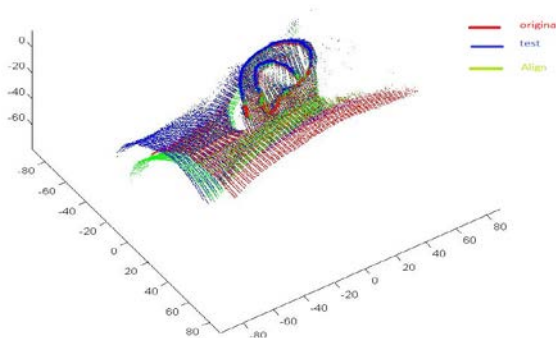


Figure 3. The result of ear alignment



Figure 4. The ear alignment in the presence of occlusion

Error	
Mean	STD
1.908	0.723

Table 1. Distance map error after alignment in the case of occlusion (Unit in mm)

5. DISCUSSIONS AND CONCLUSIONS

In this study, we introduced geometric-based methods to perform shape matching by aligning 3D surfaces. For the 3D-to-3D alignment, a novel collection of surface fiducial points, which are the points on the affine-invariant contours, e.g. parabolic contours, where the volume of parallelepiped spanned by two derivative vectors is zero, are computed. In addition, the fiducial points are preserved under affine transformations. To establish correspondences between the fiducial points on the two shapes, a set of absolute invariants were derived based on the volumes confined between parallelepipeds spanned by sets of the fiducial point quadruplets. Once the correspondences were established, the parameters of a relevant transformation were estimated and the two

surfaces were aligned. The performance of our method is demonstrated by the ability to register the 3D ear data scanned under a host of shape transformations, including ones that arise from change in ear position. Alignment errors, which were found to be within the 3D scanner resolution of 0.8 mm. This will be particularly relevant to applications where there is intra-class variability in the 3D, or due to the use of different modalities.

6. REFERENCES

- [Che07a] Chen, H., and Bhanu, B. Contour Matching for 3D Ear Recognition. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, vol 29, no.4, pp.718-737, 2007.
- [Bur00a] Burge, M., and Burger, W. Ear Biometrics in Computer Vision. *Proc. 15th International Conf. of Pattern Recognition*, vol. 2, pp. 822–826, 2000.
- [Gov99a] Govindu, V., and Shekhar, C. Alignment using Distributions of Local Geometric Properties. *IEEE Trans. Patt. Anal. Machine Intell.*, vol. 21, no. 3, pp.1031-1043, 1999.
- [Bes92a] Besl, P., and McKay, N. D. A method of registration of 3-D shapes. *IEEE Trans. Pattern Analysis and Machine Intelligence*, vol 14, no.2, pp.239–256, 1992.
- [Pin13a] Pintavirooj, C., Cohen, F. S., and Tosranon, P. 3D face alignment and registration in the presence of facial expression differences. *IEEJ Trans. Electrical and Electronic Engineering.*, vol 8, no.4, pp.395-407, 2013.
- [Sin09a] Singh, M., R. Brechner, R. R., and Henderson, W. V. Neuromagnetic Localization using Magnetic Resonance Imaging. *IEEE Trans. Med. Imag.*, vol 11, no. 1, pp: 129-134, 1992.
- [Woo98a] Woods, R. P., Grafton, S. T., Holmes, C. J., Cherry, S. R. and Mazziotta, J. C. Automated Image Registration: I, General Methods and Intrasubject, Intramodality Validation. *Journal of Computer Assisted Tomography*, Vol. 22, No. 1, pp. 139-152, 1998.
- [Bes92a] Besl, P. J., and McKay, N. D. A Method for Registration of 3D Shapes. *IEEE Trans. Pattern Analysis Mach. Intell.*, vol 14, no.2, pp:239-256, 1992.
- [Por94a] Porrill, J., and Ivins, J. A Semiautomatic tool for 3-D Medical Image Analysis using Active Contour Models. *Med. Inform.*, vol. 19,no.1, pp:81-90,1994.
- [Kan81a] Kanal, L. N., Lambird, B. A., Lavine, D. and Stockman, G. C. Digital Registration of Images from Similar and Dissimilar sensors. *Proceedings of the International Conference on Cybernetics and Society*, pp: 347-351, 1981.
- [Gos86a] Goshtasby, A. Piecewise Linear Mapping Functions for Image Registration. *Pattern recog.*, vol 19, no.6, pp: 459-466, 1986.
- [Ibr98a] Ibrahim, W. S., and Cohen, F.S. Registration Coronal Histological 2D Sections of a Rat Brain with Coronal Sections of a 3D Brain Atlas Using Geometric Curve Invariants. *IEEE. Trans. on Medical Imaging, TMI*, vol 17, no. 6, 1998.
- [Do76a] Do Carmo, M. P. *Differential geometry of curves and surfaces*, Prentice hall, Englewood Cliffs, NJ. 1976.
- [Mil97a] Millman, R. S., and Parker, G. D. *Elements of differential geometry*. Prentice Hall, Englewood Cliffs, NJ. 1977.