

# Some problems of asymmetric load of a three-phase transformer

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**Summary – The paper presents dependence of asymmetric receivers of RLC type on the output parameters of a transformer. The analysis has been carried out for the case of the transformer supplied from a symmetrical three-phase network. Results of the analysis are presented in the form of characteristic patterns illustrating the influence of the changes of the receiver parameters on distribution of the currents in transformer windings, the changes in output voltage, and the factors of the degree of voltage and current asymmetry.**

## 1. INTRODUCTION

All the electric three-phase devices are designed for operating at symmetrical system of voltages and currents. Consequences of asymmetric conditions particularly negatively affect electrical rotating machines the parameters of which, namely impedances of particular symmetrical components, are of different values. As it was shown in the paper any asymmetrical load of the transformer causes definite results, affecting both the transformer and the whole power network. The paper presents selected characteristic results of the asymmetric loads in the form of plots. Particular attention has been paid to the effect of asymmetric load on such characteristic values like the factor of the degree of voltage and current asymmetry, that are limited by the regulations.

## 2. CHARACTERISTICS OF THE PROBLEM

The paper analyzes the problem of influence of transformer load asymmetry on its basic output parameters. The calculation is carried out with the assumption that impedances of the receiver may vary for each of the phases and, in consequence, may be considered as independent variables. In order to make easier the analysis of the effect of particular variables, the phase impedances have been expressed as the functions of the  $k_1$ ,  $k_2$ , and  $k_3$  factors and presented in the equations (1):

$$\begin{pmatrix} Z_{zu}(k_1) \\ Z_{zv}(k_2) \\ Z_{zw}(k_3) \end{pmatrix} = \begin{pmatrix} k_1 Z_{zu} e^{j\phi_u 2\frac{\pi}{3}} \\ Z_{zv} e^{jk_2\phi_v 2\frac{\pi}{3}} \\ k_3 Z_{zw} e^{j\phi_w 2\frac{\pi}{3}} \end{pmatrix} \quad (1)$$

The number of the coefficients may be higher, according to the assumed number of the independent variables. In order to analyze the problem the method of symmetrical components has been used. This was achieved by transformation of the receiver impedances to the form of symmetrical components, in which  $Z_1$ ,  $Z_2$ , and  $Z_0$  are impedances of symmetrical components of positive, negative, and neutral sequences, respectively. This is illustrated by the equations (2):

$$\begin{pmatrix} Z_1(k_1) \\ Z_2(k_2) \\ Z_0(k_3) \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} Z_{zu}(k_1) \\ Z_{zv}(k_2) \\ Z_{zw}(k_3) \end{pmatrix} \quad (2)$$

The assumption that primary winding voltage of the transformer is symmetrical, allows to derive a system of voltage – current equations, that in the matrix notation take the form (3) – see Appendix 1.

The currents of symmetrical components are described by the relationships (4) – see Appendix 2.

Phase currents are calculated from the formulae (5):

$$\begin{pmatrix} I_a(k_1, k_2, k_3) \\ I_b(k_1, k_2, k_3) \\ I_c(k_1, k_2, k_3) \end{pmatrix} = \begin{pmatrix} 1 & 1 & 1 \\ a^2 & a & 1 \\ a & a^2 & 1 \end{pmatrix} \begin{pmatrix} I_1(k_1, k_2, k_3) \\ I_2(k_1, k_2, k_3) \\ I_0(k_1, k_2, k_3) \end{pmatrix} \quad (5)$$

The phase voltages are presented as the product of currents and impedances of particular phases, that is illustrated by the equations (6):

$$\begin{aligned} U_a(k_1, k_2, k_3) &= I_a(k_1, k_2, k_3) Z_{zu}(k_1) \\ U_b(k_1, k_2, k_3) &= I_b(k_1, k_2, k_3) Z_{zv}(k_2) \\ U_c(k_1, k_2, k_3) &= I_c(k_1, k_2, k_3) Z_{zw}(k_3) \end{aligned} \quad (6)$$

Once the phase voltages are determined, the voltages of symmetrical components may be defined, making use of the relationships (7):

$$\begin{pmatrix} U_1(k_1, k_2, k_3) \\ U_2(k_1, k_2, k_3) \\ U_0(k_1, k_2, k_3) \end{pmatrix} = \begin{pmatrix} 1 & a & a^2 \\ 1 & a^2 & a \\ 1 & 1 & 1 \end{pmatrix} \frac{1}{2} \begin{pmatrix} U_a(k_1, k_2, k_3) \\ U_b(k_1, k_2, k_3) \\ U_c(k_1, k_2, k_3) \end{pmatrix} \quad (7)$$

The voltages of symmetrical components serve as a basis for calculation of characteristic factors necessary for evaluating the degree of voltage asymmetry.

$K_u$  denotes the ratio of symmetrical component of the negative to positive sequence voltage, presented by the relationships (8):

$$K_u(k_1, k_2, k_3) = \frac{|U_2(k_1, k_2, k_3)|}{|U_1(k_1, k_2, k_3)|} \quad (8)$$

while  $K_{u0}$  is for the ratio of symmetrical component of zero to positive sequence voltage (9):

$$K_{u0}(k_1, k_2, k_3) = \frac{|U_0(k_1, k_2, k_3)|}{|U_1(k_1, k_2, k_3)|} \quad (9)$$

Values of the factors should not exceed the levels of 0.01, 0.02, 0.03, according to the regulations applied and the measurement goal. The lowest value of the factor is related to three-phase AC motors. These motors should not be used in the networks where the ratio of symmetrical component of negative to positive sequence voltage exceeds 0.01. It should be noticed that the ratio of these sequence voltages should not exceed 0.03 in the power networks.

In some cases the ratio of symmetrical component of the negative to positive sequence current is also calculated. It is denoted by  $K_1$  and expressed in the form (10):

$$K_1(k_1, k_2, k_3) = \frac{I_2(k_1, k_2, k_3)}{I_1(k_1, k_2, k_3)} \quad (10)$$

### 3. EXAMPLE CALCULATION

The calculation has been made on the example of a power transformer of 800kVA power and 15000/400-231V voltages, with the short-circuit voltage  $U_z=5.6\%$ . It was assumed that the transformer operates with the star/zigzag or delta/star connection. Figure 1 shows current intensity images at the transformer secondary circuit as functions of the  $k_1$  factor

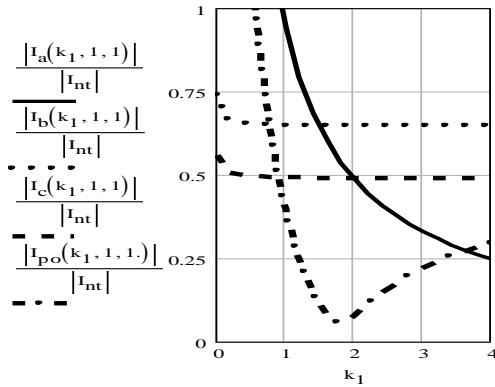


Fig. 1. Current intensity images of transformer secondary winding vs. the  $k_1$  factor

Figures 2 depict the images of the voltage asymmetry factors as functions of the  $k_1, k_2,$  and  $k_3$  factor

### Appendix 1

$$\begin{pmatrix} I_1(k_1, k_2, k_3) \\ I_1(k_1, k_2, k_3) \\ I_1(k_1, k_2, k_3) \end{pmatrix} = \begin{bmatrix} (Z_z + Z_0)(Z_0 + Z_{\mu 0}) - Z_1 Z_2 & Z_1^2 - Z_2(Z_{\mu 0} + Z_0) & Z_2^2 - Z_1(Z_0 + Z_z) \\ Z_2^2 - Z_1(Z_0 + Z_{\mu 0}) & (Z_0 + Z_z)(Z_0 + Z_{\mu 0}) - Z_1 Z_2 & Z_1^2 - Z_2(Z_0 + Z_z) \\ Z_1^2 - Z_2(Z_0 + Z_z) & Z_2^2 - Z_1(Z_0 + Z_z) & (Z_0 + Z_z)^2 - Z_1 Z_2 \end{bmatrix} \frac{1}{D} \begin{pmatrix} U_{ntp} \\ 0 \\ 0 \end{pmatrix} \quad (3)$$

### Appendix 2

$$\begin{pmatrix} I_1(k_1, k_2, k_3) \\ I_2(k_1, k_2, k_3) \\ I_0(k_1, k_2, k_3) \end{pmatrix} = \begin{pmatrix} M_{11}(k_1, k_2, k_3) \\ M_{21}(k_1, k_2, k_3) \\ M_{01}(k_1, k_2, k_3) \end{pmatrix} \frac{U_{nt}}{D(k_1, k_2, k_3)} \quad (4)$$

where

$$M_{11}(k_1, k_2, k_3) = (Z_z + Z_0(k_1, k_2, k_3))(Z_0(k_1, k_2, k_3) + Z_{\mu 0}) - Z_1(k_1, k_2, k_3)Z_2(k_1, k_2, k_3)$$

$$M_{21}(k_1, k_2, k_3) = (Z_2(k_1, k_2, k_3))^2 - (Z_1(k_1, k_2, k_3))(Z_0(k_1, k_2, k_3) + Z_{\mu 0})$$

$$M_{01}(k_1, k_2, k_3) = Z_1(k_1, k_2, k_3)^2 - Z_2(k_1, k_2, k_3)(Z_0(k_1, k_2, k_3) + Z_z)$$

$$D(k_1, k_2, k_3) = D_1(k_1, k_2, k_3) + D_2(k_1, k_2, k_3) + D_3(k_1, k_2, k_3)$$

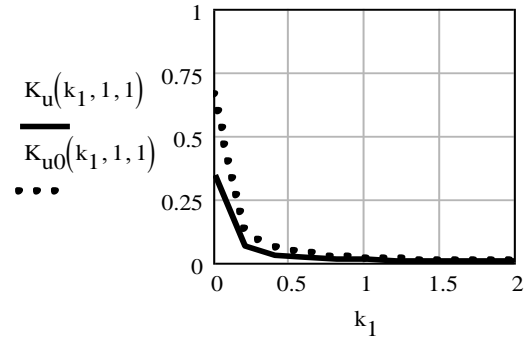


Fig. 2. Images of the voltage asymmetry factors vs. the  $k_1$  factor

### 4. SUMMARY AND CONCLUSIONS

The current and voltage images presented above illustrate the effect of the degree of asymmetry and the load character on the transformer output parameters, inclusive, among others, the asymmetry factors the limits of which are defined by the regulations. The use of the  $k_1, k_2,$  and  $k_3$  factors enables observation of the effect of the degree and type of asymmetry on the transformer output parameters. Possible analysis of the influence of the symmetry degree on the asymmetry factor defined as the ratio of negative to positive voltage sequence is of significant practical meaning. It may be noticed that even slight load asymmetry significantly affects the degree of the voltages asymmetry.

### LITERATURE

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