September 7. - 9.9.2009
Cheb, Czech Republic

THE USE OF A MULTIDIMENSIONAL APPROXIMATION IN DETERMINING A MODEL OF A PHOTOVOLTAIC CELL

M.SC., ENG. MAREK STANISŁAWSKI, CSC.
M.SC., ENG. GRZEGORZ TRZMIEL, CSC.

Abstract: *The paper includes a proposal of formulating a model of a photovoltaic cell based on measurement datafiles obtained during faultless operation of the considered cell. The authors determine the coefficients describing faultless operation of the tested cell with the use of an application. In order to achieve possibly the best results the multidimensional mean-square point approximation has been chosen. The paper presents a short characterization of the approximation, its practical use in the considered problem, and the obtained results.*

Key words: *multidimensional approximation, mean-square error, modeling, photocell*

INTRODUCTION

The diagnosis of aging in preliminary stage enables prolongation of the effective operation of a photovoltaic system by introduction of proper maintenance and overhaul procedures [1,2]. In order to detect improper operation of the cell some of its selected output parameters are compared to the model values, i.e. the ones that occur during its faultless operation. Such a procedure is possible provided that an appropriate mathematical model is formulated that describes the considered cell.

Having this in view, an application has been designed aimed at optimal determining the model coefficients based on the file including the data recorded during faultless operation of the photovoltaic cell.

1 THE MEASURING STAND

The laboratory measurement stand includes (Fig. 1): the photovoltaic module Shell ST20 [5], the decade resistor D14, the TH-03 converter (temperature measurement: the EL015 sensor; the lighting quality measurement: the EL031 sensor), connecting wires with plugs, a PC computer, the PIC18F8722 microcontroller, digital meters (verification of correctness of the AC converter reading), an intermediate system (including a voltage divider with the Zener diode preventing the overvoltage and the current measurement shunt) serving for connecting the photovoltaic module, the resistor, the meters, and the microcontroller.

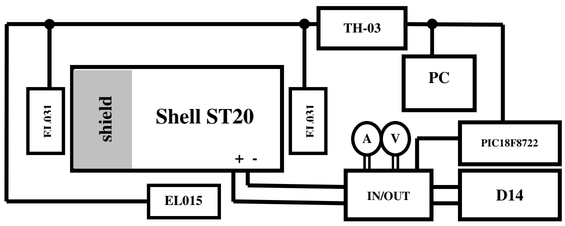

Fig.1: Block diagram of the measuring stand

The laboratory stand is used for measuring and database archiving of the following parameters, with the use of previously designed applications: D – the lighting quality expressed in the percent relative scale; T - the temperature of module in Celsius scale, I – the output current [A]; U – the voltage at the load [V]. For the purposes of formulation of the equivalent model a constant load value R has been assumed.

Taking into account that lighting D was measured with the sensors operating in percent scale and, additionally, representation of the lighting level in radiation power density units D_r was required, the lighting value has been measured with the use of the sensors operating in both above mentioned modes. The parameter D_r [W/m^2] was calculated from the formula (1) determined with the help of the mean-square point approximation.

The approximation has been carried out on the file of previous measurements made with EL031 sensors (Fig.1) measuring the lighting D in percent scale and similar measurements performed with the irradiation meter [W/m²].

$$D_r = d_0 + \sum_{h=1}^{6} d_h \cdot \exp(hB_D D) \tag{1}$$

where: d_0, d_1 – the coefficients determined in the approximation process [W/m²], B_D - an empirical constant [1/%], h – degree of the base function [-].

2 MATHEMATICAL MODEL OF THE PHOTOCELL

For the consideration purposes was used a well known relationship [6,7] expressed by the formula (2).

$$I = I_{ph0} D_r + J_0(T - T_0) - I_0\left\{\exp\left[\frac{q(U + IR_S)}{\alpha k_B T}\right] - 1\right\} - \frac{U + IR_S}{R_W} \tag{2}$$

where: I_{ph0} – a parameter determining the short-circuit current of the irradiated cell at 1000W/m² [mA·(W/m²)$^{-1}$], I_0 – the "dark" (saturation) diode current [mA], q – the elementary charge (1,6·10^{-19}C), α – the diode quality factor (for an ideal photocell α=1, under real conditions 1<α<2), k_B – the Boltzmann constant (1.381·10^{-23}J/K), J_0 – the temperature factor [mA/K], T – actual temperature of cell operation [K], T_0 – the reference temperature – in Standard Test Conditions (STC) i.e. for the power density of the light radiation equal to 1000W/m², T_0=298,15 K, R_S – the series resistance summarizing the contact and base resistance as well as the resistances of other cell layers [Ω], R_W – the shunt resistance representing the current leakage along the cell edge [Ω], U – output voltage of the photovoltaic panel [V].

The equivalent diagram representing the relationship (2) is shown in Fig. 2.

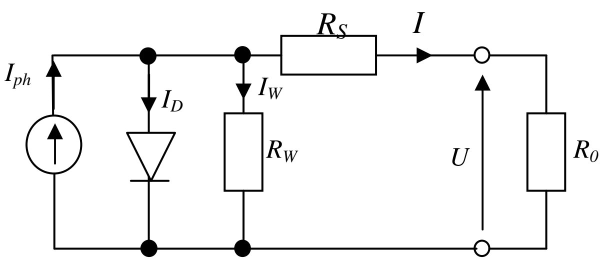

Fig.2: Equivalent diagram of the photocell for a five-parameters model [1,2]: R_0 – the load resistance [Ω], I_{ph} – the current flowing through the irradiated cell [mA], I_D – the current of a large surface diode [mA], I – the load current (at the diode output) [mA], U – voltage [V] at the receiver R_0 (at the cell output), I_W – the current flowing through the shunt resistance [mA]

For simplicity the parameters R_S and R_W are hereinafter omitted assuming R_W=∞ and R_S=0. Therefore, a model described by the equation (3) is obtained, the electric diagram of which is presented in Fig. 3.

$$I = I_{ph0} D_r + J_0(T - T_0) - I_0\left\{\exp\left[\frac{qU}{\alpha k_B T}\right] - 1\right\} \tag{3}$$

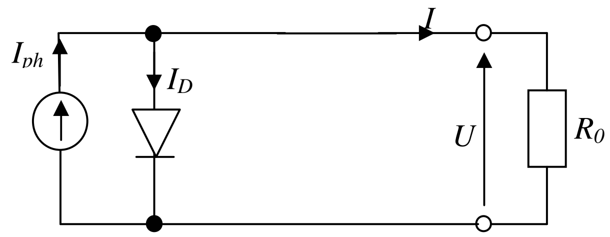

Fig.3: Ideal equivalent diagram of the photocell for a three-parameters model [1]

The currents I_{ph} and I_D occurring in the diagrams are described by the formulas (4) and (5).

$$I_{ph} = I_{ph0} D_r + J_0(T - T_0) \tag{4}$$

$$I_D = I_0\left\{\exp\left[\frac{qU}{\alpha k_B T}\right] - 1\right\} \tag{5}$$

The I_0 current is expressed by the formula (6).

$$I_0 = I_{d0}\left(\frac{T}{T_0}\right)^3 \exp\left[\frac{qE_q m}{\alpha k_B}\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \tag{6}$$

where: I_{d0} – the "dark" diode current [mA], E_q – the energetic potential barrier [V], m – the number of the cells connected in series.

Substitution of the equation (6) to the model (3) provides the final formula of the model (7).

$$I = I_{ph0} D_r + J_0(T - T_0) - I_{d0}\left(\frac{T}{T_0}\right)^3 \cdot \exp\left[B_1\left(\frac{1}{T_0} - \frac{1}{T}\right)\right] \cdot \left[\exp\left(\frac{B_2 U}{T}\right) - 1\right] \tag{7}$$

The model (7) is correct for various values of the R_0 load, while the parameters B_1 and B_2 are determined by the formulas (8) and (9).

$$B_1 = \frac{qE_q m}{\alpha k_B} \tag{8}$$

$$B_2 = \frac{q}{\alpha k_B} \tag{9}$$

Further substitutions (10-16) provide a symbolic form of the model (7) in the shape of the equation (17).

$$T_1 = (T - T_0) \tag{10}$$

$$T_2 = \left(\frac{T}{T_0}\right)^3 \tag{11}$$

$$T_3 = \left(\frac{1}{T_0} - \frac{1}{T}\right) \tag{12}$$

$$z = \frac{U}{T} \tag{13}$$

$$a_1 = I_{ph0} \tag{14}$$

$$a_2 = J_0 \tag{15}$$

$$a_3 = I_0 \tag{16}$$

$$I = a_0 + a_1 D_r + a_2 T_1 + a_3 T_2 \cdot \exp(B_1 T_3) \cdot [\exp(B_2 z) - 1] \tag{17}$$

In the above formula the current value I of the photocell output depends on five parameters D, T_1, T_2, T_3, z, that, in turn, depend on the three measured values D, T, U. While formulating the model the B_1 and B_2 constants are determined empirically, whereas the a_0-a_3 coefficients are computed with the use of the multivariate approximation method.

3 APPROXIMATION OF A FUNCTION OF SEVERAL VARIABLES

In the present paper the function of three variables was approximated by expression of the formula (17) in the form (18).

$$I = a_0 + a_1 f_1(D_r) + a_2 f_2(T) + a_3 f_3(T, U) \tag{18}$$

The base functions are defined by the relationships (19-21).

$$f_1(D_r) = D_r \tag{19}$$

$$f_2(T) = T_1 \tag{20}$$

$$f_3(T, U) = T_2 \cdot \exp(B_1 T_3) \cdot [\exp(B_2 z) - 1] \tag{21}$$

According to the approximation principles [3,4] such a set of the a_j coefficients is sought as to obtain minimal value of the mean-square point error S (22).

$$S = \sum_{i=0}^{n} \left(f_i - I(D_{ri}, T_i, U_i)\right)^2 \tag{22}$$

where: $n+1$ – the number of measurements samples subject to the approximation.

The necessary condition for S error minimization includes computation of the derivatives with regard to all the coefficients of the equation S according to the relationship (23), that allows to obtain a system of $(m+1)$ linear equations, where m=3, m - being maximal number of the a coefficient of the equation (18).

$$\sum_{j=0}^{m} \frac{\partial S}{\partial a_j} = 0 \tag{23}$$

Taking into account that the system of equations obtained this way is usually ill-conditioned, it is solved with the Cramer method.
The a_0-a_3 coefficients determined for the assumed B_1 and B_2 values and collected measurements I, D_r, T, U enable calculating the I current of a given photovoltaic panel with only small error, provided the measured parameters D_r, T and U are known.

4 RESULTS OF THE CALCULATION

The optimal model is considered to be the one that for any investigated lighting level D (in the entire considered D range) and any load level (investigated range from 2 to 400Ω) responds with a result charged with the least absolute error determined by the formula (24).

$$\Delta I_{sr} = \frac{1}{(n+1)} \sum_{i=0}^{n} \left| I_i^{'} - I_i \right| \tag{24}$$

where I'_i - value of the i-th current sample determined by the model, I_i - value of the i-th actual current taken while measuring during faultless operation, $n+1$ - the number of measurement samples.

The above model enables detecting abnormal operation of the monitored cell by comparison between the measurements and the model.

The range of the lighting quality D included a large spectrum of the radiation power D_r (from 150.39 to 593.37 W/m^2). The range of the load R variability (from 2 to 400Ω) results from practical wish to operate the considered panel in the closest possible vicinity to the maximal power point.

Many measurement series and approximations carried out in order to determine an optimal mathematical model of the considered photovoltaic panel provided the following values of the sought parameters:
a_0=64.366 mA, a_1=1.364 mA(W/m^2)$^{-1}$,
a_2=-15.052 mA/K, a_3=-3.313 mA, B_1=150 K,
B_2=306 K/V.

In the discussed case the average absolute error amounted to ΔI_{sr}=45.4mA, that may be considered as satisfactory, taking into account the neglect of R_S and R_W.
Figure 4 presents comparison of the I=f(U) characteristics for given parameters D and T (the black curve depicting the measurements, the red one – the model response).

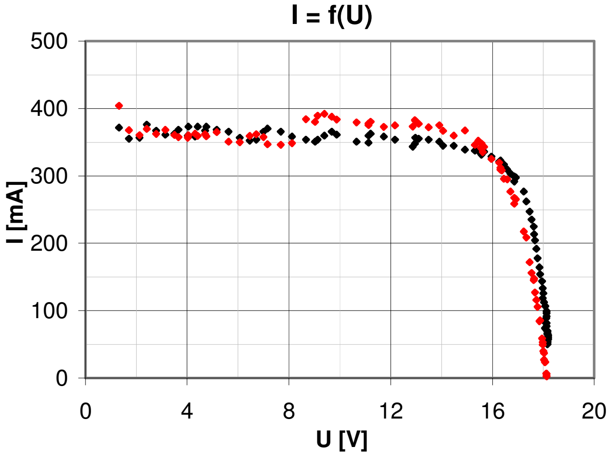

Fig.4: The I=f(U) characteristics for the irradiation level D_r=238 W/m^2 and temperature T=29 ºC (colour codes explained in the text)

Discrepancies between the real and model samples occurring in Fig.4 result mainly from inaccuracy of the devices and measuring sensors. In spite of them, the mathematical model enables detecting more severe defects of the monitored photovoltaic module.

5 Conclusions

The application designed here enabled obtaining satisfactory results in the process of photovoltaic cell modeling. Further works shall consist mainly in enlarging the measurement database during faultless operation and in improving its accuracy for various load values that should enable to obtain a more accurate equivalent model. It should be noticed that the mathematical model may be used in the conditions contained within the variability range occurring under the modeling measurements. Therefore, the measuring base used for purposes of model definition should be possible large. The model may be used for assessment of the tested object state in the future.

6 References

[1] Ciok Z.: Ochrona środowiska w elektroenergetyce – tom XXIX z serii 'Podstawowe problemy współczesnej techniki', PWN, Warszawa 2001.

[2] Kandyda A., Rodacki T.: Przetwarzanie energii w elektrowniach słonecznych, Wydawnictwo Politechniki Śląskiej, Gliwice 2000.

[3] Korn G.A., Korn T.M.: Matematyka dla pracowników naukowych i inżynierów cz.2, PWN, Warszawa 1983.

[4] Płoski Z.: Słownik Encyklopedyczny – Informatyka, Wyd. Europa, 1999.

[5] Shell Solar: Product Information Sheet – Shell ST20 Photovoltaic Solar Module, 2004.

[6] Jastrzębska G.: Odnawialne źródła energii i pojazdy proekologiczne, *Wyd. Naukowo-Techniczne, Warszawa 2007*.

[7] Ikegami T., Maezono T., Nakanishi F., Yamagata Y., Ebipara K.: Estimation of equivalent circuit parameters of PV module and its application to optimal operation of PV system, *Solar Energy, 2001*.

Marek Stanisławski, Grzegorz Trzmiel
Poznan University of Technology
Institute of Electrical Engineering and Electronics
Piotrowo 3a, 60-965 Poznan, Poland, e-mail:
marek.stanislawski@put.poznan.pl
grzegorz.trzmiel@put.poznan.pl

The paper financed from resources on science in the year 2009 as the investigative project.