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MODELLING OF ELECTRIC ACTIVITY OF CARDIAC CELL

DR. ING. JIŘÍ BÜLLOW

Abstract: *Electric and magnetic phenomena of living organisms and their modelling belong to great research area of the Department of theory of electrical engineering in University of West Bohemia. This paper brings a new approach focused on cardiac cell's membrane ion transport and its modelling in MATLAB environment.*

Key words: *Bioelectronics, cell, membrane, ion transport, mathematical modelling*

INTRODUCTION

In several last decades great effort was devoted to mathematical modelling of cardiac cell's dynamics of their ion channels. See, e.g., [1], [2], and [3] that contain examples of mathematical models in MATLAB and in the C language. These models are only approximate and not much suitable for exact description of the ion channel dynamics. Therefore, they are not suitable for medical prediction. It is because they avoid such parts of measured ion channel developments that cannot be described analytically.

The methodology presented herein is based on an approach using the Hausdorff space, transformation of time axis that becomes one of the spatial coordinates, and a minimal set of deterministic analytical functions. All of these features build blocks for discrete time models suitable for the simulations. A comparison of the computer result with biomedical data obtained from the real cardiac cell is included.

1 MATHEMATICAL APPROACH - SPACE

In the above problem one deals with experimentally measured biomedical data. A level of their accuracy is determined by the time sampling frequency and signal accuracy of the data acquisition technology – usually D/A measuring PC card. It causes that the resultant curve of biological object behaviour looks like a non-analytical function. The Hausdorff space is suitable for description this situation because its main feature is that any two different points can be distinguished.

2 MATHEMATICAL APPROACH – FUNCTIONS SET

One needs to define a set of functions which are analytical, and have parameters that can be identified with substantial factors of biological object. Moreover, the set of these functions should be as small as possible.

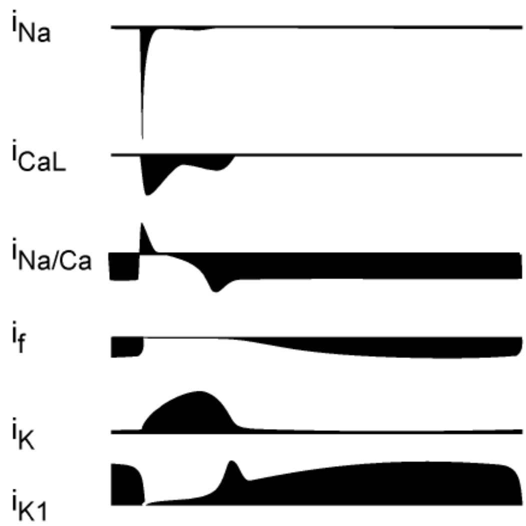

Fig.1: Ion currents [2]

Fig. 1 in [2] illustrates the shapes of cell's membrane ion transport current developments. They can consist of four functions: rounded step, slope hill, slide and wave.

2.1 Rounded step

This function is defined in the following way:

$$y = am_0 \ \text{for} \ t < t_0 , \tag{1}$$

$$y = am_1 \ \text{for} \ t > t_1 , \tag{2}$$

and for $t \in \langle t_0, t_1 \rangle$:

$$x = a \cdot \left(\frac{t - t_0}{t_1 - t_0} - 1 \right), \tag{3}$$

$$y_0 = \sqrt{\frac{r^2 - b^2 x^2}{a^2}} , \tag{4}$$

$$y = am_0 + y_0 \cdot \left(\frac{am_1 - am_0}{b} \right), \tag{5}$$

where t is a time point of time vector $\boldsymbol{t}$, t_0 is the starting time of the step, t_1 is the ending time of the step, am_0 is the amplitude of the step in time t_0, am_1 is the amplitude of the step in time $t \geq t_1$, and, finally, $a = b = r = 1$ are parameters of an ellipse segment used to realize this function.

2.2 Slope hill

This function is defined as

$$y = am \cdot \exp\left[-\left(\ln\left(t + t_a \right) \right)^2 \right] \tag{6}$$

for time point t from time vector $\boldsymbol{t}$ from the symmetric interval $t \in \langle -t_a, +t_a \rangle$, where am is the amplitude of the slope hill.

2.3 Slide

This function is defined by

$$y = 0 \quad \text{for} \ t < t_0 , \tag{7}$$

$$y = am \ \text{for} \ t = t_0 , \tag{8}$$

$$y = am \cdot \exp\left(-\left(t - t_0 \right) \cdot s \right) \ \text{for} \ t > t_0 , \tag{9}$$

where t_0 is the starting time of the function, am is the amplitude, and s is the slope factor.

2.4 Wave

This function is defined as follows:

$$n = \tanh\left(s_0 \left(t - t_0 \right) \right), \ \text{for} \ n < 0 : n = 0 , \tag{10}$$

$$d = 0.5\left(1 + \tanh\left(s_1 \left(t - t_1 \right) \right) \right), \ \text{for} \ d < 0 : d = 0 , \tag{11}$$

$$y = am \cdot \left(n - d \right). \tag{12}$$

The significance of the above parameters is analogous to other functions described here.

3 RESULT - ILLUSTRATIVE EXAMPLE

For example, the ion current denoted i_{CaL} in Fig. 1 (the second one from the top) can be modelled using definition formula (6)

$$y = y_1 + y_2 , \tag{13}$$

where

$$y_1 = am_1 \cdot \exp\left[-\left(\ln\left(t + t_{a1} \right) \right)^2 \right], \tag{14}$$

$$y_2 = am_2 \cdot \exp\left[-\left(\ln\left(t + t_{a2} \right) \right)^2 \right], \tag{15}$$

where y_1, y_2 are slope hill functions with parameters: $-t_{a1} = 0$, $+t_{a1} = 500$, $am_1 = -0.5$, $-t_{a2} = 100$, $+t_{a2} = 500$, $am_2 = -0.2$. Symbol t is a time from the time vector $\boldsymbol{t}$. The result of (15) is depicted in Fig.2.

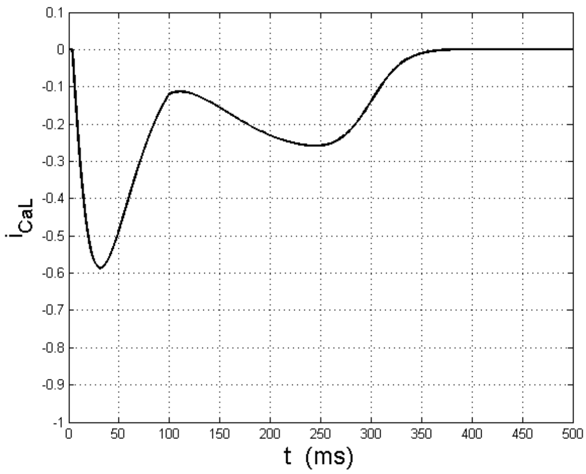

Fig.2: Approximation of i_{CaL} current

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Jiří Büllow, Dr. Ing., Department of Theory of Electrical Engineering, Faculty of Electrical Engineering, University of West Bohemia, Univerzitní 26, 306 14 Plzeň, tel. 420/37763 4601, e-mail: bullow@kte.zcu.cz