THE CALCULATION OF PARAMETERS OF WEIBULL DISTRIBUTION

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ABSTRACT

This contribution deals with the problems of failure intensity of big power plant's blocks basely on bathtub curve of blackouts, command of operation and blackouts caused by wastage. The failure intensity is in most cases expresed by the Weibull two-parameters distribution.

1. INTRODUCTION

The time between failure and failure-free time of each power plant block are known from the reliability information system, which is extracted every month from information given by power plants. The first task is to bring order to data - the mathematical statistic processing. This can be done by a histogram or a graph of cumulative frequency. The position of distribution is characterized by a mean value on an axis where we put data. A dispersal of data around the mean value is described by one of the degree of distribution. For projection of the mean value is in most cases used an arithmetic mean and for expression of size of the distribution is used a standard deviation.

The arithmetic mean $\overline{x}$

$$\overline{x} = \frac{\sum_{j=1}^{j=k} n_j \cdot x_j}{n} \tag{1.1}$$

The standard deviation

$$s = \sqrt{\frac{\sum_{j=1}^{j=k} \left(n_j x_j^2\right)}{n} - \left(\frac{\sum_{j=1}^{j=k} n_j x_j}{n}\right)^2} \tag{1.2}$$

where

x_j ... value

n_j ... frequency

n ... number of values

k ... number of classes

From these calculations we obtain the distribution of time which express a dependency of reliability of the power block R_i on time t_i. Through these points we interpose a curve which has this type of function

$$R_{(t)} = \exp\left(-\frac{k}{m+1} \cdot t^{m+1}\right) \tag{1.3}$$

The regression function is nonlinear in parameters *k,m* therefor it isn't possible to aplicate the least sguare method. The calculaton of regression function is based on Taylor series aroud the points k_0, m_0

$$R_{(t)} = R_0 + \left(\frac{dR}{dk}\right)_0 (k-k_0) + \left(\frac{dR}{dm}\right)_0 (m-m_0) + ...$$
$$+\left[\frac{1}{2!}\left(\frac{\partial^2 R}{\partial k^2}\right)_0 (k-k_0)^2 + 2\left(\frac{\partial^2 R}{\partial k \cdot \partial m}\right)_0 (k-k_0)(m-m_0) + \left(\frac{\partial^2 R}{\partial m^2}\right)(m-m_0)^2\right] + ...$$

If we skip the term of second and higher order we get

$$R(t) = R_0 + (\frac{\partial R}{\partial k})_0 (k-k_0) + (\frac{\partial R}{\partial m})_0 (m-m_0) \tag{1.4}$$

This new regression function is linear in parameters *k, m* and it is possible to aplicate the least square method. It is also necessary to implement an iteration process so we can compensate the mistake which is a result of skipping the rest of Taylor series.

The calculation of parameters of Weibull distribution by the least square method

We have data which was gained through empirical observation

$R_{(t)}$	R_1	R_2	$\cdots$	R_i	$\cdots$	R_n
t	t_1	t_2	$\cdots$	t_i	$\cdots$	t_n

(1.5)

Data gives the statistical dependency of the reliability of power plant block R_i on time t_i. Our goal is to interpose the curve through these points.

$$R_{(t)} = e^{-\frac{k}{m+1} t^{m+1}} \tag{1.8}$$

Now we get the parameters of Weibull distribution *k, m* so the curve is as similar as possible to the data gained through empirical observation.

In this case the regression function is nonlinear in parameters *k, m*, it is not possible to aplicate the least square method directly. The procedure will be:

1. first of all we will guess approximate figure for *k* and *m* from data obtained through empirical observation $t_i, R_i, i = 1-n$; we name these figures k_0, m_0. It is possible to choose for example $k = -\frac{1}{m_s}; m = 0$ this is origined from the exponencial distribution.

2. in the next step we calculate the regression function with Taylor series around the points k_0, m_0

$$R_{(t)} = R_0 + \left(\frac{\partial R}{\partial k}\right)_0 (k-k_0) + \left(\frac{\partial R}{\partial m}\right)_0 (m-m_0) +$$
$$+\frac{1}{2!}\left[\left(\frac{\partial^2 R}{\partial k^2}\right)_0 (k-k_0)^2 + 2\left(\frac{\partial^2 R}{\partial k \partial m}\right)_0 (k-k_0)(m-m_0) + \left(\frac{\partial^2 R}{\partial m^2}\right)_0 (m-m_0)^2\right] + \ldots$$

we skip the term of second and higher order we get

$$R_{(t)} = R_0 + \left(\frac{\partial R}{\partial k}\right)_0 (k-k_0) + \left(\frac{\partial R}{\partial m}\right)(m-m_0) \tag{1.4}$$

This new regression function is linear in parameters *k*, *m* and it is possible to aplicate the least squaer method. It is also necessary to implement an iteration process so we can compensate the mistake which is a result of skipping the rest of Taylor series.

In (1.4) means

$$R_0 = e^{-\frac{k_0}{m_0+1}t^{m_0+1}} = f_{0(t)}$$
$$\frac{\partial R}{\partial k} = e^{-\frac{k}{m+1}t^{m+1}}\left(-\frac{t^{m+1}}{m+1}\right)$$
$$\left(\frac{\partial R}{\partial k}\right) = -\frac{t^{m_0+1}}{m_0+1} e^{-\frac{k_0}{m_0+1}t^{m_0+1}} = f_{1(t)}$$
$$\frac{\partial R}{\partial k} = e^{-\frac{k}{m+1}t^{m+1}}\left(-k\frac{(m+1)t^{m+1}\lg t - t^{m+1}}{(m+1)^2}\right) = -kt^{m+1}\frac{(m+1)\lg t - 1}{(m+1)^2} e^{-\frac{k}{m+1}t^{m+1}}$$
$$\left(\frac{\partial R}{\partial k}\right)^2 = -k_0 t^{m_0+1}\frac{(m_0+1)\lg t - 1}{(m_0+1)^2} e^{-\frac{k_0}{m_0+1}t^{m_0+1}} = f_{2(t)}$$

It is

$$R_{(t)} = f_{0(t)} + f_{1(t)}(k-k_0) + f_{2(t)}(m-m_0) \tag{1.9}$$

3. now we use the least square method

$$F_{(m,k)} = \sum_{i=1}^{n}\left[R_{(t_i)} - R_i\right]^2 = \min \tag{1.10}$$

Figures t_i a R_i are number from table 1.

4. to (1.10) we insert (1.9)

$$F_{(m,k)} = \sum_{i=1}^{n}\left[f_{0(t_i)} + f_{1(t_i)}(k-k_0) + f_{2(t_i)}(m-m_0) - R_i\right]^2 = \min$$

Function $F_{(m,k)}$ is minimal if

$$\frac{\partial F_{(m,k)}}{\partial k}=0 \qquad \frac{\partial F_{(m,k)}}{\partial m}=0$$

It will be

$$\sum_{i=1}^{n} 2\left[f_{0(t_i)} + f_{1(t_i)}\left(k-k_0\right)+ f_{2(t_i)}\left(m-m_0\right)-R_i \right]f_{1(t_i)} = 0$$

$$\sum_{i=1}^{n} 2\left[f_{0(t_i)} + f_{1(t_i)}\left(k-k_0\right)+ f_{2(t_i)}\left(m-m_0\right)-R_i \right]f_{2(t_i)} = 0$$

From these two equation we express $\left(k-k_0\right)$ a $\left(m-m_0\right)$

$$\left(k-k_0\right)\sum_{i=1}^{n} f_{1(t_i)}^2 + \left(m-m_0\right)\sum_{i=1}^{n} f_{1(t_i)} f_{2(t_i)} = \sum_{i=1}^{n} f_{1(t_i)}\left[R_i - f_{0(t_i)}\right]$$

$$\left(k-k_0\right)\sum_{i=1}^{n} f_{1(t_i)} f_{2(t_i)} + \left(m-m_0\right)\sum_{i=1}^{n} f_{1(t_i)}^2 = \sum_{i=1}^{n} f_{1(t_i)}\left[R_i - f_{0(t_i)}\right]$$

We insert this terminology

$$A_{11} = \sum_{i=1}^{n} f_{1(t_i)}^2 \qquad B_1 = \sum_{i=1}^{n} f_{1(t_i)}\left[R_i - f_{0(t_i)}\right]$$

$$A_{12} = \sum_{i=1}^{n} f_{1(t_i)} f_{2(t_i)} \qquad B_2 = \sum_{i=1}^{n} f_{2(t_i)}\left[R_i - f_{0(t_i)}\right]$$

$$A_{22} = \sum_{i=1}^{n} f_{2(t_i)}^2$$

We obtaine

$$\left(k-k_0\right)A_{11} + \left(m-m_0\right)A_{12} = B_1$$

$$\left(k-k_0\right)A_{12} + \left(m-m_0\right)A_{22} = B_2$$

By solving we get

$$k-k_0 = \frac{B_1A_{22} - B_2A_{12}}{A_{11}A_{22} - A_{12}^2}$$

$$m-m_0 = \frac{B_1A_{12} - B_2A_{11}}{A_{11}A_{22} - A_{12}^2}$$

We finaly obtaine

$$k = k_0 + \frac{B_1A_{22} - B_2A_{12}}{A_{11}A_{22} - A_{12}^2} \qquad (1.11)$$

$$m = m_0 + \frac{B_2 A_{11} - B_1 A_{12}}{A_{11} A_{22} - A_{12}^2} \tag{1.12}$$

The calculation procedure will be:

1. pick k_0, m_0
2. determine function

$$f_{0(t)} = e^{-\frac{k_0}{m_0+1} t^{m_0+1}}$$

$$f_{1(t)} = -\frac{t^{m_0+1}}{m_0+1} e^{-\frac{k_0}{m_0+1} t^{m_0+1}} = -\frac{t^{m_0+1}}{m_0+1} f_{0(t)}$$

$$f_{2(t)} = -k_0 t^{m_0+1} \frac{(m+1)\lg t - 1}{(m+1)^2} e^{-\frac{k_0}{m_0+1} t^{m_0+1}} = k_0 \frac{(m_0+1)\lg t - 1}{m_0+1} f_{1(t)}$$

3. with these function we express following constants:

$$A_{11} = \sum_{i=1}^{n} f_{1(t_i)}^2 \qquad B_1 = \sum_{i=1}^{n} f_{1(t_i)} \left[R_i - f_{0(t_i)} \right]$$

$$A_{12} = \sum_{i=1}^{n} f_{1(t_i)} f_{2(t_i)} \qquad B_2 = \sum_{i=1}^{n} f_{2(t_i)} \left[R_i - f_{0(t_i)} \right]$$

$$A_{22} = \sum_{i=1}^{n} f_{2(t_i)}^2$$

4. with these figures we calculate another $\underline{k}, \underline{m}$ figures:

$$k = k_0 + \frac{B_1 A_{22} - B_2 A_{12}}{A_{11} A_{22} - A_{12}^2}$$

$$m = m_0 + \frac{B_2 A_{11} - B_1 A_{12}}{A_{11} A_{22} - A_{12}^2}$$

5. with calculated figures $\underline{k}, \underline{m}$ we pull back to the point 2. and redo the procedure as many times as we need to get the absolute figure of difference of two following $\underline{m}$ is smaller than the accuracy limit ε

The equations below are adjusted for computer.
We name functions:

$$f_{1(t)} = \exp(-\frac{xt^{y+1}}{y+1})$$

$$f_{2(t)} = -\frac{t^{y+1}}{y+1}$$

$$f_{3(t)} = x(\log t - \frac{1}{y+1})$$

1. We pick the initial figures x, y (for example $x = 0,01$ and $y = 0$).
2. Then we calculate these equations.

$$A_{11} = \sum_{i=1}^{N} f_1^2(t_i) f_2^2(t_i)$$

$$A_{12} = \sum_{i=1}^{N} f_1^2(t_i) f_2^2(t_i) f_3(t_i)$$

$$A_{13} = \sum_{i=1}^{N} f_1^2(t_i) f_2^2(t_i) f_3^2(t_i)$$

$$B_1 = \sum_{i=1}^{N} [R_i - f_1(t_i)] f_1(t_i) f_2(t_i)$$

$$B_2 = \sum_{i=1}^{N} [R_i - f_1(t_i)] f_1(t_i) f_2(t_i) f_3(t_i)$$

3. We calculate new figures x, y.

$$x_{k+1} = x_k + \frac{B_1 A_{22} - B_2 A_{21}}{A_{11} A_{22} - A_{12}^2}$$

$$y_{k+1} = y_k + \frac{B_2 A_{11} - B_1 A_{12}}{A_{11} A_{22} - A_{12}^2}$$

4. If $|x_{k+1} - x_k| + |y_{k+1} - y_k| \langle \varepsilon$ are final figures x_{k+1}, y_{k+1}, and there is not $|x_{k+1} - x_k| + |y_{k+1} - y_k| \langle \varepsilon$ we repeat the procedure with new figures x_{k+1}, y_{k+1} from the 2. point.

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