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NEW METHODS OF CIRCUIT ANALYSIS UNDER NONSINUSOIDAL CONDITIONS

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Abstract: The paper deals with a new method of an integral transformation of alternating nonharmonic periodic functions. The presented new methods serves to circuit analysis under nonsinusoidal conditions. This approach is based on the electromagnetic physical laws: Ohm´s law, induction law, equation of continuity, Kirchhoff´s laws and the mathematical definition of the scalar product of the time varying quantities.

Keywords: integral transformation, scalar product, time varying function, circuit parameter, circuit analysis.

1 Introduction

Although sinusoidal signals are undoubtedly the most common periodic signals, other types of periodic signals do arise in practical electrical engineering. There is a mathematical theorem which states that any periodic function can be expressed equivalently as the sum of a constant plus an infinite number of sine waves and cosine waves. Unfortunately, in this case, common power theory is not based on Kirchhoff´s laws.

The new methods are based on a scalar product of time varying quantities and physical properties real circuit elements. First we determine the circuit parameters of the electromagnetic phenomenon [1] and then we perform circuit analysis. The purpose of circuit analysis is to calculate the voltages and currents that appear in a circuit and then, on the basis of these, to determine the powers and energy.

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2 Voltage analysis

The elementary circuit model of the electromagnetic phenomenon is a series connection or a parallel connection of the circuit elements. Because a series model and parallel model are dual can be only analyzed either, for example the series model. This circuit contains three circuit elements: resistor, inductor and capacitor, as shown in Fig.1, with the linear parameters: resistance (R), inductance (L) and inverse capacitance (D).

Each circuit element is characterized by a relationship between the current through the element and the voltage between the terminals of the element. Adding the voltages, we have

$$u = u_R + u_L + u_D = Ri + L\frac{\mathrm{d}i}{\mathrm{d}t} + D\int_0^t i \cdot \mathrm{d}\xi = Ri + Li' + Di^x \quad (1)$$

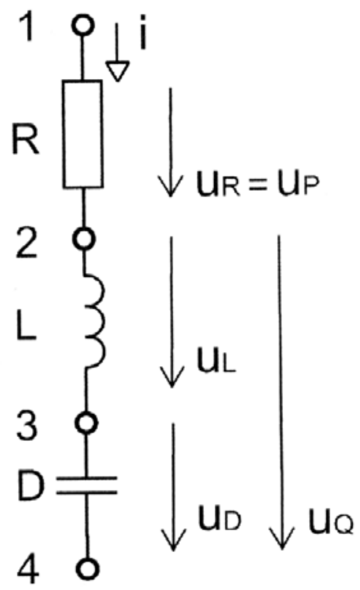

Fig. 1 The elementary series circuit model of the electromagnetic phenomenon

2.1 Root-mean-square values of the voltages

We may now proceed to standing the root-mean-square (rms) values of the voltages

$$U_R = RI, \quad U_L = LI', \quad U_D = DI^x,$$

where I, I', I' are rms values of current i, time derivation of current i' and time integral of current i^x. Rms value of the sum of the voltages from Eq. (1) is calculated according to

$$U^2 = \frac{1}{T}\int_0^T u^2 \cdot \mathrm{d}\,t = \frac{1}{T}\int_0^T (u_R + u_L + u_D)^2 \cdot \mathrm{d}\,t =$$
$$= \frac{1}{T}\int_0^T (u_R^2 + u_L^2 + u_D^2)\cdot \mathrm{d}\,t + \frac{2}{T}\int_0^T (u_R u_L + u_R u_D + u_L u_D)\cdot \mathrm{d}\,t =$$
$$= \frac{1}{T}\int_0^T [(Ri)^2 + (Li')^2 + (Di^x)^2]\cdot \mathrm{d}\,t + \frac{2}{T}\int_0^T (RLii' + RDii^x + LDi'i^x)\cdot \mathrm{d}\,t =$$
$$= (RI)^2 + (LI')^2 + (DI^x)^2 - 2LDI^2 = U_R^2 + U_L^2 + U_D^2 - 2LDI^2 =$$
$$= U_R^2 + U_Q^2 = U_P^2 + U_Q^2,$$

because of the identities

$$\int_0^T i \cdot i' \cdot \mathrm{d}\,t = 0, \quad \int_0^T i \cdot i^x \cdot \mathrm{d}\,t = 0, \quad \frac{1}{T}\int_0^T i' \cdot i^x \cdot \mathrm{d}\,t = -I^2.$$

The voltage across the terminals 1 and 2 are called the active component of the voltage (active voltage) - index P

$$u_R = u_P, \quad U_R = U_P,$$

and the voltage across the terminals 2 and 4 are called the reactive component of the voltage (reactive voltage) - index Q

$$u_Q = u_L + u_D$$

$$U_Q^2 = U_L^2 + U_D^2 - 2LDI^2. \tag{3}$$

Now we ixtend third term in Eq. (3) by

$$\frac{I'I^x}{I'I^x}.$$

By algebraic rearrangement

$$2LDI^2 \frac{I'I^x}{I'I^x} = 2LI'DI^x \frac{I^2}{I'I^x} = 2U_L U_D \cos\varepsilon$$

we have

$$U_Q^2 = U_L^2 + U_D^2 - 2U_L U_D \cos\varepsilon. \tag{3'}$$

Equation (3′) represents a general triangl.

2.1.1 Voltage vector diagram

The instantaneous voltage is the algebraic sum of the instantaneous active voltage and the instantaneous reactive voltage

$$u = u_P + u_Q, \tag{4}$$

whereas rms voltage is the geometric (vector) sum of the rms active voltage and the rms reactive voltage

$$U^2 = U_P^2 + U_Q^2 \tag{5}$$

as shown by a voltage vector diagram in Fig. 2.

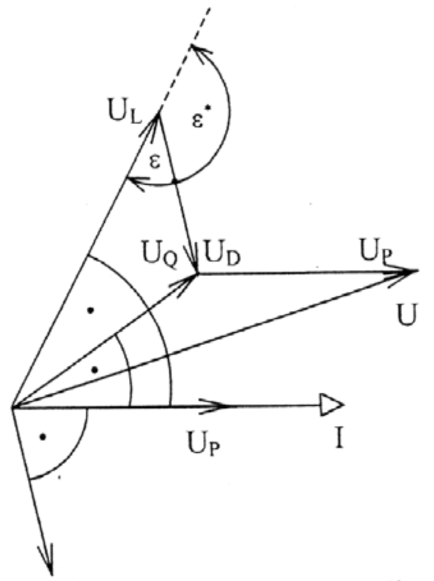

Fig. 2 The voltage vector diagram

3 Conclusion

The state of a circuit is known completely it the voltages across all terminals and the current is known. In this case we can calculate all powers. All instantaneous voltages and their rms values are unambiguously stated by physical laws.

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