

# Mathematical modeling of a biogenous filter cake and identification of oilseed material parameters

J. Očenášek<sup>a,\*</sup>, J. Voldřich<sup>a</sup>

<sup>a</sup>*New Technologies Research Centre, University of West Bohemia, Univerzitní 8, 306 14 Plzeň, Czech Republic*

Received 11 September 2009; received in revised form 18 December 2009

---

## Abstract

Mathematical modeling of the filtration and extrusion process inside a linear compression chamber has gained a lot of attention during several past decades. This subject was originally related to mechanical and hydraulic properties of soils (in particular work of Terzaghi) and later was this approach adopted for the modeling of various technological processes in the chemical industry (work of Shirato). Developed mathematical models of continuum mechanics of porous materials with interstitial fluid were then applied also to the problem of an oilseed expression. In this case, various simplifications and partial linearizations are introduced in models for the reason of an analytical or numerical solubility; or it is not possible to generalize the model formulation into the fully 3D problem of an oil expression extrusion with a complex geometry such as it has a screw press extruder.

We proposed a modified model for the oil seeds expression process in a linear compression chamber. The model accounts for the rheological properties of the deformable solid matrix of compressed seed, where the permeability of the porous solid is described by the Darcy's law. A methodology of the experimental work necessary for a material parameters identification is presented together with numerical simulation examples.

© 2009 University of West Bohemia. All rights reserved.

*Keywords:* filter cake, fractionation process, permeability, continuum mechanics

---

## 1. Introduction

Oilseed expression has become an important technology not only in the food industry, but also for biofuel manufacturing with rapeseed oil as the most common component. The oilseed expression process could be undertaken in linear compression chambers or in screw press extruders. Nowadays almost the whole rapeseed oil production and the production of other vegetable oils is processed by extruders. This led to an extensive development of experimental programs with the aim of the farther screw extruders optimization (see e.g. [5, 6]). On the other hand, no convenient mathematical models of the oilseed expression were proposed for the screw press geometry, or there are no computational software tools necessary for the numerical modeling applying existing models [11]. An exception is the work of [9] based on experiences of Japanese chemical engineers in comparable technologies of fractional processes. Unfortunately the proposed methodology is strongly based on the engineering intuition and a number of empirical relations that limit its generalization.

This situation is much different for the oilseed expression in a linear compression chamber. In this case, extensive experimental programs of the expression process were carried out [10, 6, 12, 5, 3], as well as their appropriate mathematical modeling were performed for the identification of relevant material parameters of various oilseeds.

---

\*Corresponding author. Tel.: +420 377 634 702, e-mail: ocenasek@ntc.zcu.cz.

The work of Shirato [7] could be considered as one of the most evolved analytical approach to model the compression of a filter cake. His model is a limiting case of the conservation laws model, as a number of assumptions need to be made in order to be able to derive an analytical solution. Nevertheless, the conservation laws 1D models based on mass and momentum balances were solved in the last decade (see e.g. [3]). Unfortunately, these models were formulated in such a manner that their three-dimensional generalization for a screw press geometry is not feasible.

### Nomenclature

$o$	suffix related to the initial state
$f$	suffix related to the fluid
$s$	suffix related to the solid matrix
$c$	empirical parameter of the equation suggested by Tiller and Yeh [–]
$K$	permeability [ $\text{m}^2$ ]
$p$	liquid pressure [Pa]
$p_s$	solid compressive pressure [Pa]
$p_f = \Phi p$	effective liquid pressure [Pa]
$P$	externally applied pressure of the volume $V$ [Pa]
$q$	superficial liquid flow [m/s]
$S$	piston area [ $\text{m}^2$ ]
$t$	time [s]
$v$	solid phase velocity [m/s]
$V$	local volume [ $\text{m}^3$ ]
$V_0$	the volume at $t = 0$ [ $\text{m}^3$ ]
$V_s$	local volume of the solid phase [ $\text{m}^3$ ]
$V_f$	local volume of the liquid phase [ $\text{m}^3$ ]
$w$	liquid velocity relative to the solid velocity [m/s]
$x$	spatial coordinate (distance from the filter) [m]
$\alpha$	specific cake resistance [m/kg]
$\gamma$	material parameter of the solid matrix analogous to the Biot coefficient [–]
$\epsilon$	(natural) volume strain of the solid matrix [–]
$\eta$	viscosity of the dash pot [Pa · s]
$\mu$	liquid viscosity [Pa · s]
$\Phi = V_f/V$	porosity [–]
$\Phi_0 = V_{f0}/V_0$	the porosity at $t = 0$ [–]
$\rho_s$	(intrinsic) solid phase density [ $\text{kg}/\text{m}^3$ ]
$\omega$	spatial convective (material) coordinate [m]

$$d\epsilon = dV/V \quad \epsilon = \ln\left(\frac{V}{V_0}\right) \quad V = V_s + V_f \quad P = p_s + p_f = p_s + \Phi p \quad q = \Phi w$$

During the oilseed compression process the bed of seeds becomes more compacted due to the expulsion of air. After a short time, when the pressure raises the so called oil point, the seeds are crushed, whereby the seed structure containing the oil is damage and the oil is freed. This allows the oil to drain through the bed of crushed nibs and allows it to fraction. During this stage the solid matrix and the freed oil carry the external load. Therefore, the biogenous filter cake can be, within the mathematical model, considered as a two-phase medium that consists of a porous material (the solid matrix) saturated with an interstitial fluid (the vegetable

oil) as the second phase. The theory of the filtration and consolidation of saturated porous media had received a lot of attention by geomechanics, ground engineering and engineering of building materials. The deformation behavior of the porous media are, within these fields, most frequently described by the Biot constitutive model (see e.g [1]). Unfortunately, the Biot model does not satisfactorily cover the case of large volume deformations, which are generally not of a big interest for the area of ground engineering or geomechanics, where the large volume deformations are undesirable or related to a failure state.

This paper is arranged as follows. In the following section of the paper a mathematical model of the filter cake compression process is derived from the basic conservation laws, where the primary unknown quantities are the (natural) volume strain  $\epsilon$  of the solid matrix, the effective oil pressure  $p_f$  and the solid velocity  $v$  as functions of the spatial coordinate  $x$  and the time  $t$ . The Darcy's law is used to describe the local cake permeability and the viscoelasticity of the solid matrix is considered. The velocity  $v$  (following the Euler continuum approach) is selected as a primary unknown quantity, since it is naturally possible to extend the model to the three-dimensional case of the screw press with a fixed boundary, as it was presented in [11]. In the case of a linear compression chamber there is a moving piston, meaning that the problem with a moving boundary is obtained, though this difficulty can be overcome by applying the spatial convective coordinate. In the section 3 setups of various experiments are proposed, which should be sufficient for the identification of material parameters applied in the mathematical model. The section 4 is devoted to identification of material parameters. Related numerical simulations based on the presented model were performed and the comparison of the simulation results with experimental observations published in the literature is given. Finally, the section 5 provides discussion and summary of achieved results.

## 2. Mathematical model for fractionation mechanism

The compression process of a filter cake, as shown in Fig. 1, is considered in this section. With regard to the large volume changes of the filter cake, the volume changes caused by the compressibility of the liquid and the intrinsic compressibility of the solid phase can be neglected. Thus, in following the solid phase and the liquid are assumed as incompressible with a constant density  $\rho_s$  and  $\rho_f$ , respectively.

Additionally it is assumed that the expression process is one-dimensional (therefore the friction between the compression cake and the walls of the compression chamber is negligible),

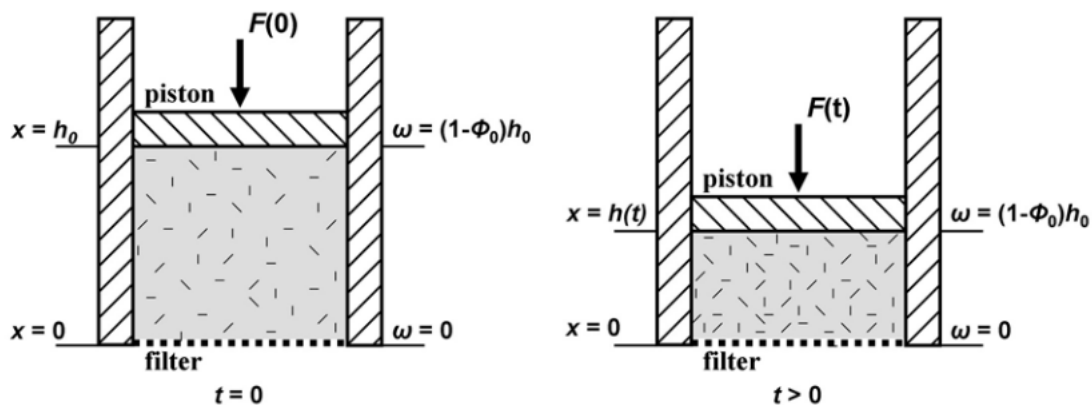


Fig. 1. The expression process at the start and for the time  $t > 0$  with the applied force  $F(t)$

that no solids pass through the filter medium, that the effect of inertia and gravity is negligible, that the resistance of the filter medium is negligible compared to that of the filter cake and that the local porosity  $\Phi$  is a smooth function of both spatial position and time.

### 2.1. Mass balance

The solid mass of a volume  $V_0$  of the solid matrix is  $m_s = \rho_s V_{s0} = \rho_s (V_0 - V_{f0}) = \rho_s V_0 (1 - \Phi_0)$ . At the time  $t$  the corresponding part of the solid matrix occupies a volume  $V$ , and by analogy we obtain  $m_s = \rho_s V (1 - \Phi)$ , thus  $1 - \Phi = (1 - \Phi_0) V_0 / V$ . Since  $-\epsilon = \ln(V_0 / V)$ , the following relation between the porosity  $\Phi$  and the volume strain  $\epsilon$  can be derived

$$\Phi = 1 - (1 - \Phi_0) \exp(-\epsilon). \tag{1}$$

Let  $\rho$  be the “distributed” density of the solid matrix in a volume  $V$ , i.e.  $\rho V = \rho_s V_s$ . From the solid mass balance it holds that  $\rho V = \text{const.}$  and so  $dV/V = -d\rho/\rho$ . As  $d\epsilon = dV/V$ , we obtain  $d\epsilon = -d\rho/\rho$  and furthermore  $\rho \frac{\partial \epsilon}{\partial t} = -\frac{\partial \rho}{\partial t}$ ,  $\rho \frac{\partial \epsilon}{\partial x} = -\frac{\partial \rho}{\partial x}$ . Applying the last two relations to the fundamental equation of the mass balance  $\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0$ , we find

$$\frac{\partial \epsilon}{\partial t} + v \frac{\partial \epsilon}{\partial x} - \frac{\partial v}{\partial x} = 0. \tag{2}$$

Since we assume that the densities  $\rho_s$  and  $\rho_f$  are constant, we can also write fundamental equations of the mass balance in the form

$$\frac{\partial \Phi}{\partial t} + \frac{\partial}{\partial x}(\Phi(w + v)) = 0, \quad \frac{\partial(1 - \Phi)}{\partial t} + \frac{\partial}{\partial x}((1 - \Phi)v) = 0.$$

Now, we obtain by their summation  $\frac{\partial v}{\partial x} + \frac{\partial}{\partial x}(\Phi w) = 0$ , i.e.

$$\frac{\partial v}{\partial x} + \frac{\partial q}{\partial x} = 0 \tag{3}$$

where  $q = \Phi w$  is the superficial flow of the oil.

### 2.2. Force balance

We can assume that shear stress is zero. The one-dimension equation of the force balance is then

$$-\frac{\partial P}{\partial x} = 0, \tag{4}$$

where  $P$  is the external pressure acting on a local volume  $V$ . Consequently,  $P$  is the function of time  $t$  only,  $P(t) = F(t)/S$ , where  $S$  is the area of the piston, and

$$P(t) = p_s(x, t) + p_f(x, t). \tag{5}$$

### 2.3. Constitutive equations

#### 2.3.1. Cake permeability

The relationship between the liquid velocity  $w$  and the liquid pressure gradient  $p_f$  is determined by the local cake permeability. The (intrinsic) liquid flow within pores of the solid matrix can be assumed to be laminar due to the Reynolds number (see e.g. [3]) and therefore the relationship can be described by the Darcy’s law

$$q = -\frac{K}{\mu} \frac{\partial p_f}{\partial x}, \tag{6}$$

where  $K$  is the permeability and  $\mu$  is the dynamic viscosity of the oil. In the case of fluid mechanics in porous mediums it is standard practice to use the Kozeny-Carman’s empirical law to evaluate the permeability of the medium in function of its porosity  $\Phi$ . However, the Tiller-Yeh’s approach (see [3, 10]) seems to be more promising when considering cake filtration. If

$$\alpha = \frac{1}{(1 - \Phi)K\rho_s} \tag{7}$$

is now the specific cake resistance, then we obtain by the Tiller-Yeh’s empirical law

$$\alpha = \alpha_0 \frac{1 - \Phi}{1 - \Phi_0}^c, \quad \text{i.e.} \quad K(\Phi) = \frac{(1 - \Phi_0)^c}{\alpha_0 \rho_s (1 - \Phi)^{c+1}}. \tag{8}$$

#### 2.3.2. Solid matrix compressibility

Experimental data published in the literature indicate that the solid matrix deformation behavior is not only elastic, but also exhibits a creep character. Therefore, modeling of the oil expression solely by recourse to the theory of poro-elasticity would be insufficient. There are several empirical relations published in the literature (see e.g. [3]). Within this paper, we chose to adopt the theory of Voigt in the generalized form

$$\mathcal{F}(\epsilon) + \eta \frac{d\epsilon}{dt} = -p_s + \gamma p, \tag{9}$$

where the  $\mathcal{F}(\epsilon)$  has the meaning of the pressure response of the solid matrix subjected to a volume deformation  $\epsilon$ , in the case of the zero fluid pressure (for the linear case  $\mathcal{F}(\epsilon) = E\epsilon$ ). The parameter  $\eta \neq 0$  is the viscosity of the dash pot and  $\gamma$  is a parameter analogous to the Biot coefficient taking into account the influence of the oil pressure.

### 2.4. Summary

When following functions  $\epsilon = \epsilon(x, t)$ ,  $p_f = p_f(x, t)$ ,  $v = v(x, t)$  are taken as primary unknowns, the mathematical model of the fractionation mechanism can be formulated as

$$\frac{d\epsilon}{dt} = \frac{\partial v}{\partial x}, \tag{10a}$$

$$\frac{\partial}{\partial x} \left( \eta \frac{\partial v}{\partial x} + \mathcal{F}(\epsilon) - \left(1 + \frac{\gamma}{\Phi(\epsilon)}\right) p_f \right) = 0, \tag{10b}$$

$$\frac{\partial}{\partial x} \left( \frac{K(\Phi(\epsilon))}{\mu} \frac{\partial p_f}{\partial x} - \frac{\partial v}{\partial x} \right) = 0 \tag{10c}$$

with the relation (1) for  $\Phi = \Phi(\epsilon)$  and the relation (8) for  $K = K(\Phi)$ . The presented approach is especially convenient for its generalization to the 3D problem of the oil expression in a screw press extruder. Here applied material parameters are  $\mu, \alpha_0, c, \eta, \gamma$ , the function  $\mathcal{F} = \mathcal{F}(\epsilon)$  and the initial porosity  $\Phi_0$ .

The first of the equations (10a) is parallel to the equation (2), because the derivative  $d/dt$  denotes the material derivative here. The second equation (10b) can be derived by substituting for the term  $d\epsilon/dt$  from the first equation (10a) into the equation (9) and also for the term  $p_s$  from the relation (5), after that by taking the derivative with respect to  $x$  and applying the equation (4). Finally, the term (10c) is obtained combining the Darcy’s law (6) and the balance equation (3).

In our one-dimensional case an even more simple form can be derived. The external pressure  $P$  acting on the (local) volume  $V$  does not depend on the spatial variable  $x$  and as we know,  $P(t) = F(t)/S$ , thus the pressure  $P$  can be kept in equations. This, at the same time, allows to eliminate the derivative  $\partial v/\partial x$  from the related terms as follows. The first equation of the new system can be derived from the equation (9) by substitution for the term  $p_s$  from (5) and applying the relation  $p = p_f/\Phi$  with respect to different liquid pressure definitions. The second equation is then obtained from the equation (10c) with the help of (10a) by replacing the derivative  $\partial v/\partial x$  by the derivative  $d\epsilon/dt$  which is finally expressed by (11a). For only two unknown quantities  $\epsilon = \epsilon(x, t)$  and  $p_f = p_f(x, t)$  we now obtain

$$\eta \frac{d\epsilon}{dt} + \mathcal{F}(\epsilon) - \frac{\gamma}{\Phi(\epsilon)} p_f = -P(t) + \bar{p}_f, \tag{11a}$$

$$-\frac{\partial}{\partial x} \frac{K(\Phi(\epsilon))}{\mu} \frac{\partial p_f}{\partial x} + \frac{1 + \gamma/\Phi(\epsilon)}{\eta} p_f = (P(t) + \mathcal{F}(\epsilon))/\eta. \tag{11b}$$

Initial and boundary conditions will be discussed later.

### 2.5. Formulation using the convective (material) coordinate

With respect to the piston displacement in the linear compression chamber, the part of the domain boundary, where equations (11) are being solved, is moving. To reformulate this problem into a problem with a fixed domain boundary, we will introduce the convective (material) coordinate  $\omega$  as

$$\omega(t, x) = \int_0^x (1 - \Phi(t, X)) dX. \tag{12}$$

It is important to realize that the coordinate  $\omega = \omega(t, x(t, A))$  is time independent for a selected material particle  $A$  of the solid matrix, i.e.  $\omega = \omega(A)$ . For the selection and  $x = x(t, A)$  it comes out that the integral on the right hand side of (12) is time independent, because by multiplying it with the (constant) density  $\rho_s$  and with the piston cross-section area  $S$  we obtain the mass of the solid matrix between the filter and a cross-section (perpendicular to the filter cake axis) passing through the material particle  $A$ . It is  $\omega(t, 0) = 0$  at the point of the filter and  $\omega(t, h(t)) = \omega(0, h_0) = \int_0^{h_0} (1 - \Phi_0) dX = (1 - \Phi_0)h_0$  just under the piston. By virtue of

$$\frac{\partial}{\partial x} = (1 - \Phi) \frac{\partial}{\partial \omega} \quad \text{and} \quad \frac{d\epsilon(t, x)}{dt} = \frac{\partial \epsilon(t, \omega)}{\partial t}$$

the problem (11) can be rewritten for the convective coordinate  $\omega$ . For the unknown functions  $\epsilon = \epsilon(t, \omega)$  and  $p_f = p_f(t, \omega)$ ,  $0 \leq \omega \leq (1 - \Phi_0)h_0$ , we then have the system of partial

differential equations

$$\eta \frac{\partial \epsilon}{\partial t} + \mathcal{F}(\epsilon) - \frac{\gamma}{\Phi(\epsilon)} p_f = -P(t) + p_f, \tag{13a}$$

$$-\frac{\partial}{\partial \omega} \frac{K(\Phi(\epsilon))}{\mu} (1 - \Phi(\epsilon)) \frac{\partial p_f}{\partial \omega} + \frac{1 + \gamma/\Phi(\epsilon)}{\eta(1 - \Phi(\epsilon))} p_f = \frac{1}{\eta} \frac{P(t) + \mathcal{F}(\epsilon)}{1 - \Phi(\epsilon)} \tag{13b}$$

with boundary conditions

$$p_f(t, 0) = 0, \quad \frac{\partial p_f}{\partial \omega}(t, (1 - \Phi_0)h_0) = 0 \tag{14}$$

for the situation at the Fig. 1 and with the initial condition

$$\epsilon(0, \omega) \equiv 0.$$

### 3. Experimental procedure

Material parameters like the density  $\rho_s$ , the viscosity  $\mu$  and the initial porosity  $\Phi_0$  can be estimated in a standard way and reliable values can be found in the literature [12, 2]. Therefore we focus our attention to other material parameters named in the section 2.4, that are  $c$ ,  $\alpha_0$ ,  $\eta$ ,  $\gamma$  and  $\mathcal{F}(\epsilon)$ .

#### 3.1. Drained experiment and “steady state” component of the volume deformation

The scheme of a drained experiment is illustrated at the Fig. 1, where the acting force  $F(t)$  rises and settles at the constant value of  $F_1$ . Farther we put our attention only to the initial unloaded configuration and the final steady state. The initial state is defined by the values  $h_0$ ,  $P_0 = F(0)/S = 0$ , i.e.  $p_{f0} = p_{s0} = 0$ . The final steady state is reached, when no more oil is being expeled from the filter cake and the height of the cake remains unchanged, so the gradients of quantities  $p_f$  and  $\epsilon$  are zero.

Then, at the time  $t_1$ , when the steady state is reached precisely enough, it is  $h(t_1) = h_1$ ,  $p(t_1, x) = 0$ ,  $d\epsilon/dt(t_1, x) = 0$ ,  $\epsilon(t_1, x) = \text{const.} = \epsilon_1$  and  $v(t_1, x) = 0$  for all  $x \in (0, h_1)$ . This yields  $p_s(t_1, x) = P(t_1, x) = P_1 = F(t_1)/S$  and it is  $\mathcal{F}(\epsilon_1) = -P_1$  for the corresponding deformation value  $\epsilon_1 = \ln(h_1/h_0)$ . The relation  $\epsilon \rightarrow \mathcal{F}(\epsilon)$  can be identified from various steady state configurations, characterized by different values of  $h_1$ .

#### 3.2. Undrained experiment and the material parameter $\gamma$

Within this subsection we will discuss two different steady state configurations of the undrained experiment as illustrated at the Fig. 2.

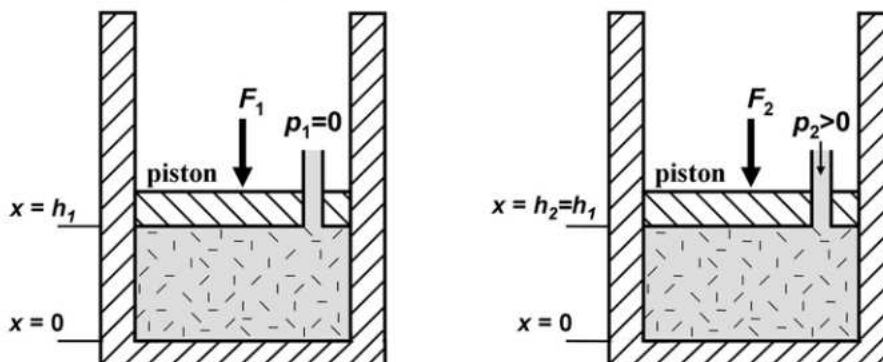


Fig. 2. Two steady states of an undrained compressive test for the identification of the parameter  $\gamma$

In both cases, the deformation  $\epsilon$  and the intrinsic fluid pressure  $p$  are constant in respect to the  $x$ . The experiment is performed so that the height of the filter cake is the same in both cases, i.e.  $h_2 = h_1$ , and so  $\epsilon_2 = \epsilon_1 = \ln(h_1/h_0)$ . However  $p_1 = 0$  in the first case, while the invariant pressure  $p_2 > 0$  of the oil is kept under the piston in the second case. Perceiving that it is  $d\epsilon/dt = 0$  and  $p_s = P - \Phi p = F/S - \Phi p$  for steady state, then by applying relations (9) and (1), it yields to the result

$$\gamma = \frac{1}{p_2}(F_1/S - F_2/S - \Phi p_2) \quad \text{with} \quad \Phi = 1 - (1 - \Phi_0)\frac{h_0}{h_1}.$$

These experiments for different values of  $\epsilon_1$ ,  $\Phi_0$  and  $p_2$  reveal, how effectively the parameter  $\gamma$  could be considered as a constant and how the dependence on these quantities is significant.

### 3.3. Drained experiment and permeability

The scheme of a drained experiment, supplied by an oil pressure line leading under the piston, is presented at the Fig. 3. Let us consider a steady state, when the piston stops to shift and when the pressure field as well as the total oil flow  $q_1$  are stabilized. Then  $d\epsilon/dt(t_1, x) = 0$  and  $v(t_1, x) = 0$  for all  $x \in (0, h_1)$ . With regard to the present oil flow and non-zero pressure gradient  $p$ , the deformation  $\epsilon$  and the porosity  $\Phi$  are not constant. By virtue of (11a) it can be written that

$$p_f(x) = \{\mathcal{F}(\epsilon(x)) + F_1/S\} / \{1 + \gamma/\Phi(\epsilon(x))\},$$

which by substitution into (11b) leads to a differential equation for the deformation  $\epsilon(x)$

$$\frac{\partial}{\partial x} \frac{K(\Phi(\epsilon(x)))}{\mu} \frac{\partial}{\partial x} \frac{\mathcal{F}(\epsilon(x)) + F_1/S}{1 + \gamma/\Phi(\epsilon(x))} = 0,$$

where functions  $\Phi = \Phi(\epsilon)$  and  $K = K(\Phi)$  have the form of (1) and (8), respectively.

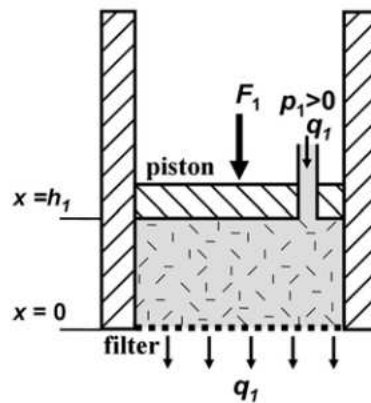


Fig. 3. Steady state of the drained compressive test with oil flow for the identification of the permeability

The corresponding boundary condition for  $\epsilon = \epsilon(0)$  at the side of the filter can be derived by resolving the following algebraic equation

$$0 = (\mathcal{F}(\epsilon) + F_1/S) / (1 + \gamma/\Phi(\epsilon)) ,$$

and by analogy, the boundary condition for  $\epsilon = \epsilon(h_1)$  at the side of the piston, by solving

$$\Phi(\epsilon) p_1 = (\mathcal{F}(\epsilon) + F_1/S) / (1 + \gamma/\Phi(\epsilon)) .$$



Referred material parameters  $\alpha_0$  and  $c$  of the Tiller-Yeh's empirical law (8), which characterize the permeability  $K$ , have to be fitted in such a way that the calculated oil flow rate matches well the real flow rate  $q_1$  from the experiment, i.e. that the following relation is fulfilled as precisely as possible for all  $x \in (0, h_1)$

$$\frac{K(\Phi(\epsilon(x)))}{\mu} \frac{\partial p_f}{\partial x}(x) = q_1.$$

### 3.4. Drained experiment and the viscosity of the solid matrix

In the case of identifying the solid matrix viscosity  $\eta$ , the analysis of the steady state configuration in the experiment from the Fig. 1 is insufficient and a complete record of the continuous process of the filter cake compression is necessary. The fitting of the material parameter  $\eta$  has to be then done so that the compression process calculated by means of equations (13a) and (13b) corresponds well to the experimental data.

## 4. Transient modes and identification of material parameters

The piecewise linear Galerkin method and the explicit integration scheme was applied for numerical simulation of the unsteady problem defined by equations (13a) and (13b).

### 4.1. Material parameters fitting

As mentioned, material parameters  $\mu$ ,  $\rho_s$  were taken from the literature [12] and [2] and the parameter  $\gamma$  was taken to be zero for simplicity, because results corresponding to the test from Fig. 2 were not published. Remaining parameters  $\eta$ ,  $c$ ,  $\alpha_0$  and the relation  $\mathcal{F}(\epsilon)$  were determined by fitting of our numerical results of the problem (13)–(14) to data from two different type of experiments, both adopted from the literature [12].

The first experiment is performed in the compression chamber with a setup corresponding to the scheme at Fig. 1 and continuously records the oil yield  $U_t$  as a function of time, when a constant pressure of 30 MPa is applied. The yield  $U_t$  is a mass fraction of the expelled oil at the time  $t$  to the initial oil content and thus it is defined as

$$U_t = \frac{h_0 - h(t)}{\Phi_0 h_0}, \tag{15}$$

The second type of experiment determines the total oil yield  $U_{10min}$  for various pressure values  $P$  applied by the piston in a linear compression chamber of the same configuration (see Fig. 4), where the period of 10 minutes was estimated in the work of Willems [12] as sufficient to characterize the expression process. However, there are no published experiments confirming that an equilibrium (i.e. a steady state) is reached after this period, especially for high values of the pressure  $P$ . Therefore the  $\mathcal{F}(\epsilon)$  can not be identified separately from parameters  $c$  and  $\alpha_0$  with enough accuracy as proposed in the section 3.1, because the assumption of the steady state is not sufficiently fulfilled.

We assume that the pressure  $\mathcal{F}(\epsilon)$  will increase with the growing deformation following the relation

$$\mathcal{F}(\epsilon) = A \frac{\epsilon}{\bar{\epsilon} - \epsilon}, \tag{16}$$

where  $\bar{\epsilon}$  is the volume deformation at the state with the zero oil content  $\Phi = 0$ , i.e.  $\bar{\epsilon} = \ln(1 - \Phi_0)$  and the parameter  $A$  is to be determined by fit to experimental data.

The discrepancy at low pressure values, especially for  $P = 10$  MPa is most probably caused by a threshold pressure called oil point, which indicates pressure at which the oil emerges from a seed kernel during mechanical oilseed expression. The typical value of the oil point for rapeseed is approximately 7–8 MPa. This threshold pressure is not yet incorporated in our model, so the oil yield at low pressure values is notably overestimated. Nevertheless, our research focuses to design extruders able to reach high yields so the filter cake behavior at high pressure states gains more interest.

Parameters  $A$ ,  $\eta$ ,  $c$  and  $\alpha_0$  were then fitted to correspond to the experimental data, from Figs. 4 and 5. Summary of the parameter values is given in Tab. 1.

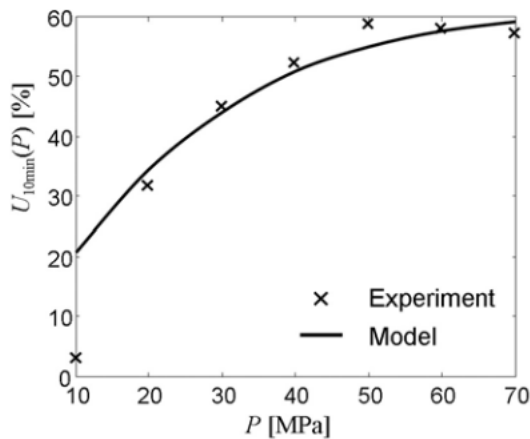


Fig. 4. The oil yield  $U_{10min}$  for various values of the external pressure  $P$

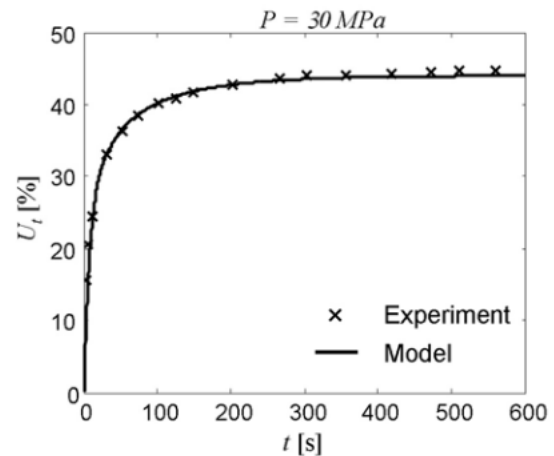


Fig. 5. Time dependence of the oil yield  $U_t$  for  $P = 30$  MPa

Table 1. Material parameters of rapeseed

$A$ [MPa]	$\rho_s$ [kg/m <sup>3</sup> ]	$\mu$ [Pa · s]	$\Phi_0$ [-]	$\eta$ [GPa]	$\alpha_0$ [m/kg]	$\gamma$ [-]	$c$ [-]
52	1 052	0.055	0.467	1.8	$1.1 \times 10^{11}$	0	12

It has to be stressed that additional experiments (conformable with Figs. 2, 3) added to published ones in [12] should be performed, because otherwise more than one parameter set can be found to approximate well the experimental data mentioned. For instance, after setting the form of  $\mathcal{F}(\epsilon)$  to be of (16), the viscosity  $\eta$  can be selected from quite wide range, while the parameter  $c$  and  $\alpha_0$  can be still adjusted to match a good fit of experiments.

#### 4.2. Modeling result analysis

By analyzing the simulation results we can observe that the permeability of the filter cake decreases for higher values of exponent  $c$  to the extent that the steady state is not reached within the period of 10 minutes and the time necessary to approach a steady state can be in order of magnitudes longer. For the parameter set from Tab. 1 this effect can be seen for higher loads  $P = 60$  and  $70$  MPa, how we can deduce for example from Figs. 6, 7 and 8.

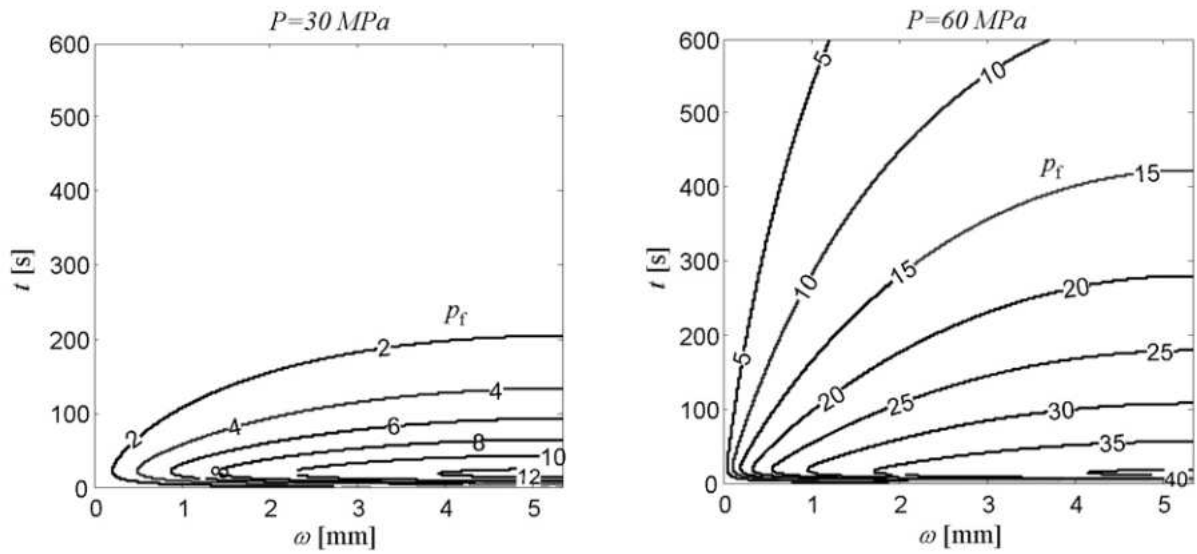


Fig. 6. Contour of the fluid pressure  $p_f$ , applied external pressure (a)  $P = 30$  MPa and (b)  $P = 60$  MPa

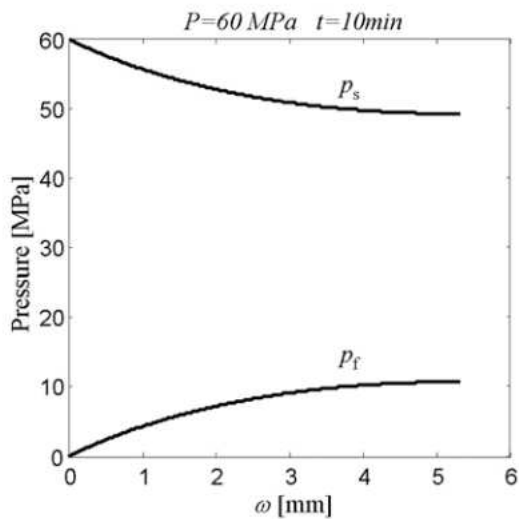


Fig. 7. Distribution of pressures  $p_f$  and  $p_s$  through the filter cake for  $t = 10$  min and  $P = 60$  MPa

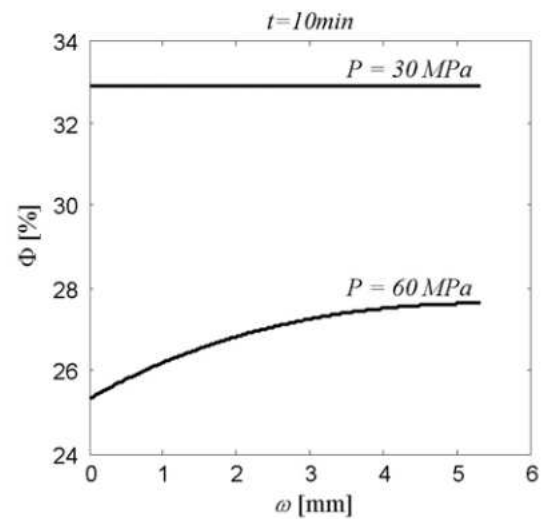


Fig. 8. Distribution of the porosity through the filter cake for  $t = 10$  min and for external pressures  $P = 30$  MPa and  $P = 60$  MPa

## 5. Conclusions and discussion

In the chapter 2 we have proposed the mathematical description of oil seed materials, which is covering their rheological properties as well as the permeability of these porous materials by means of the Darcy's law, where the appropriate description of the permeability turns out to be the Tiller-Yeh's empirical law. The proposed mathematical model also covers the possible effect of oil-pressure on the solid matrix behavior through a parameter analogous to the Biot coefficient.

The third chapter presents three different experiments in a linear compression chamber whose results are sufficient for the identification of suggested model parameters. Unfortunately, experimental results published up to now (e.g. for rapeseed see [12, 2]) are insufficient and more or less pertain only to the test from Fig. 1. Therefore, during the filter cake compression pro-

cess, it is difficult to distinguish the deformation part rising from the rheological behavior of the solid matrix from effects related just to the permeability.

Thus we can find markedly different material parameters in the literature (e.g. compare [12] and [3] for rapeseed parameters) for the commonly applied Shirato model [7, 5]. Hence, the results of the parameter identification that we present in chapter 4 are only illustrative.

Insufficient experimental results also pose fundamental questions of the filter cake behavior at higher external pressures. Since from published experiments it is not possible to reveal, how long is the relaxation time to reach the steady state or even, whether eventually the pores of the solid matrix get completely closed, i.e. achieving practically zero permeability at low porosity. In such case the Tiller-Yeh's empirical law would have to be modified.

Finally, it has to be remarked that the height of the filter cake is assumed to be sufficiently small in comparison to the diameter of the compression chamber, so the friction of the filter cake with the chamber wall can be neglected and that the temperature field of the filter cake is approximately uniform.

### **Acknowledgements**

The paper is based on work supported by the Ministry of Industry and Trade of the Czech Republic under the research project FR-TI 1/369. This project is solved in collaboration with FARMET, a. s., Česká skalice.

### **References**

- [1] Detournay, E., Cheng, A., *Fundamentals of poroelasticity*, Pergamon Press, 1993.
- [2] Faborode, M. O., Favier, J. F., Identification and significance of the oil-point in seed-oil expression, *J. Agric. Engng Res.*, 65 (1996), p. 335–345.
- [3] Kamst, G. F., *Filtration and expression of palm oil slurries as a part of the dry fractionation process*, Dissertation, Technische Universiteit Delft, 1995.
- [4] Olivier, J., Vaxelaire, J., Vorobiev, E., Modelling of cake filtration: An overview, *Separation Science and Technology* 42 (2007), p. 1 667–1 700.
- [5] Raß, M., *Zur Rheologie des Biogenen Feststoffs unter Kompression am Beispiel Geschaãlter Rapssaat*, Dissertation, Universität Gesamthochschule Essen, 2001.
- [6] Schein, Ch., *Zum Kontinuierlichen Trennpresen Biogener Feststoffe in Schneckenometrien am Beispiel Geschãlter Rapssaat*, Dissertation, Universität Duisburg-Essen, 2003.
- [7] Shirato, M., Murase, T., Iwata, M., Hayashi, N., Deliquoring by expression – Part 1 Constant-pressure expression, *Drying Technology* 4 (3) (1986), p. 363–386.
- [8] Shirato, M., Murase, T., Iwata, M., Pressure profile in a power-law fluid in constant-pitch, straight-taper and decreasing pitch screw extruders, *International Chemical Engineering*, 23 (2) (1983), p. 323–331.
- [9] Vadke, V. S., Sosulski, F. W., Shook, C. A., Mathematical simulation of an oilseed press, *JAOC* 65 (10) (1988), p. 1 610–1 616.
- [10] Venter, M. J., Kuipers, N. J. M., de Haan, A. B., Modelling and experimental evaluation of high-pressure expression of cocoa nibs, *Journal of Food Engineering* 80 (2007), p. 1 157–1 170.
- [11] Voldřich, J., *Mathematical modelling of the dry fractionation process of oilseed in presses with the linear or screw geometry (in Czech)*, Research Report, New Technology Research Center, UWB, 2009.
- [12] Willems, P., *Gas assisted mechanical expression of oilseeds*, Dissertation, Universiteit Twente, Nederland, 2007.