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Fatigue crack shape prediction based on vertex singularity

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Abstract

Due to the existence of vertex singularity at the point where the crack intersects the free surface, stress distribution around the crack tip and the type of the singularity is changed. In the interior of the specimen the classical singular behaviour of the crack is dominant and can be described using analytic equations. Contrary to this, at the free surface or in the boundary layer close to free surface the vertex singularity is significant. The influence of vertex singularity on crack behaviour and a crack shape for a three-dimensional structure is described in this paper. The results presented make it possible to estimate fatigue crack growth rate and crack shape using the concept of the generalized stress intensity factor. The estimated fatigue crack shape can help to provide a more reliable estimation of the fatigue life of the structures considered.

Keywords: 3D vertex singularity, crack shape, fatigue crack propagation

1. Introduction

Due to the existence of vertex singularity fatigue crack in the point, where crack front intersects the free surface, singular stress distribution is changed and can influence crack behaviour. The change of the singular stress field leads to the change of the crack shape and fatigue crack growth rate close to the vertex point. The vertex singularity was investigated extensively last 30 years for e.g. [1, 2, 3, 4]. It was found that the singularity exponent induced by the free surface differs from 0.5 and depends on Poisson's ratio. Due to the change of the stress singular field close to the free surface crack front is uniquely shaped; see experimental works [3, 5]. The main objective of the article is to simulate real crack shapes using singularity exponent analysis and using generalized methodology for estimation of the fatigue crack growth rate. Assuming that plastic zone size is a value controlling FCGR and based on correlation of the plastic zones parameters for the standard fatigue crack and the V-notch fatigue crack growth rate (FCGR) was estimated.

2. Singular stress field close to the free surface

The usual modeling of the 3D cracks is related to the straight crack front, intersecting the free surface at $90°$. Under these conditions the stress singularity exponent is not constant along the crack front and differs from the value $1/2$. Generally it is possible to consider two different singular fields along the crack font of the 3D crack, see fig. 1. In the center of the specimen where

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plain strain conditions are prevailing the classical square-root singular field can be explained by the relation [7]:

$$\sigma_{ij} = K_I r^{-\frac{1}{2}} \cdot f_{ij}(\varphi) \tag{1}$$

where K_I is a stress intensity factor and $f_{ij}(\varphi)$ is a generally known shape function. In the area close to the free surface, more important is so called vertex singularity which can be expressed by the following relation [2]:

$$\sigma_{ij} \approx r^{-p} f_{ij}(p, \theta, \varphi) \tag{2}$$

In this case the stress singular field is spherical with the center in the vertex point (O) and shape functions depend in this case on local spherical coordinates r, φ, θ and vertex singularity exponent, see fig. 1.

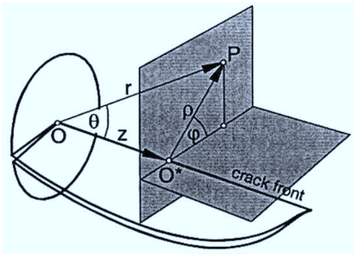

Fig. 1. Singular stress field close to the free surface [8]

Thus, for a description of the 3D crack front, semi-analytical solutions of two special cases exist. The order of the vertex singularity for semi-infinite space with vertex point can be found, for example, using the variation principle; see [1] for details. According to these results, the power of the vertex singularity is weaker than 0.5 and for particular Poisson's ratios varies between 0.5 (corresponding to $\nu = 0$) and 0.33 (corresponding to $\nu = 0.5$). This solution is valid only for thick structures and in addition is not consistent with classical fatigue crack description.

To avoid these problems, 2D stress distribution around the crack tip in each single plane perpendicular to crack front can be generally expressed as:

$$\sigma_{ij} = \frac{H_I}{r^p} \cdot f_{ij}(p, \theta) \tag{3}$$

where H_I is a generalized stress intensity factor, p is a singularity exponent and $f_{ij}(r, \theta)$ are corresponding shape functions. r, θ are local polar coordinates with the origin at the point on the crack front. In this case the stress and displacement distribution depends on the distance r from the crack tip as $\sigma_{ij} \approx r^{-p}$ and $u_i \approx r^{1-p}$.

Based on these assumptions, stress singularity exponent p along the crack front can be estimated numerically by direct method using log-log regression analysis [9, 10]. Estimation of the singularity exponent based on direct methods is highly sensitive to element size around the crack tip and distance for extrapolation. For this reason, the level of mesh refinement was referred to a convergence analysis carried out in 2D plane strain solutions of a crack in homogenous material and bi-material where the analytical solution for the stress singularity exponent is known (stress singularity exponent vary in these cases from 0 to 1). In these cases the value of the singularity exponent was estimated by direct method with deviation smaller than 2 %, see [11] for details. Final element size for 3D analysis corresponded to that with the highest refinement level in 2D.

3. Numerical model

The effect of the vertex singularity on the stress field around the crack front under small scale yielding conditions in the sense of linear elastic fracture mechanics was assessed using a middle tension (MT) specimen, see fig. 2.

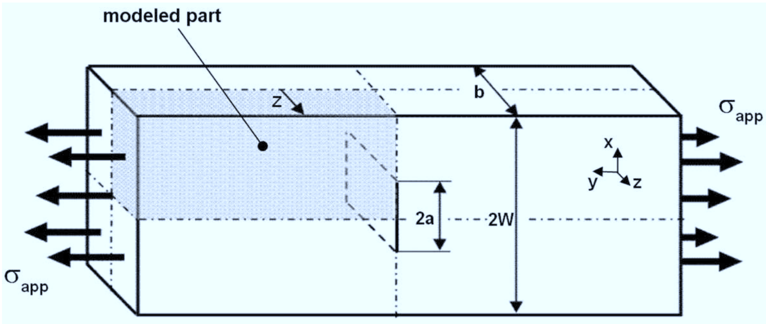

Fig. 2. Model of the MT specimen used for finite element calculations

The dimensions of the MT specimen were: crack length $2a = 25$ mm, $a/W = 0.5$ and thickness $b = 10$ mm. Material properties were considered homogenous and isotropic, defined by Young modulus $E = 210$ GPa and Poisson's ratio ν ranges from 0 to 0.5. Uniform applied tensile stress was applied, so only loading mode I was considered. Exploiting the symmetry in the specimen geometry and loading, only one-eighth of the MT-specimen was modeled by finite element analysis (using FEM software ANSYS) and the stress and displacement distribution were calculated. The mesh of finite elements was refined mainly close to the crack tip. A typical model of 3D structure contains approximately 250 000 isoparametric elements.

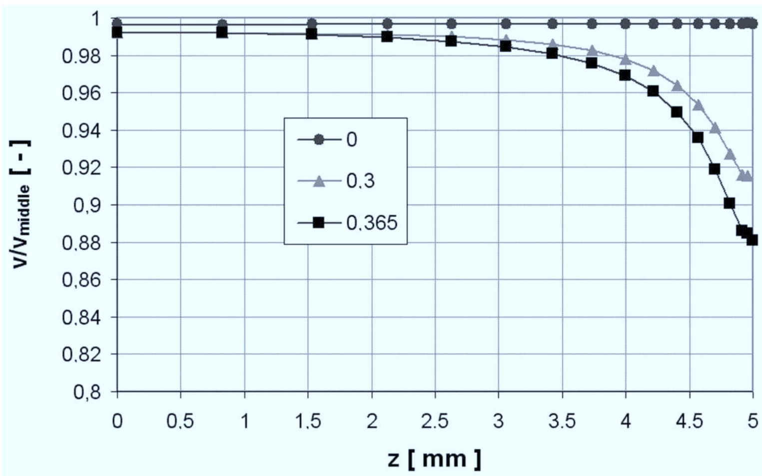

Fig. 3. Variation of ratio between real fatigue crack propagation rate v and crack propagation rate in the middle of the specimen v_{middle} along the crack front for three Poisson's ratios [12]

Displacements from a finite element analysis were used to estimate the power of the singularity using log-log regression. Based on previous extensive numerical simulations for different thicknesses of the specimen and also in comparison with the literature data, it was found that the stress singularity exponent along the crack front for relatively thick specimens ($b > 10\,\mathrm{mm}$) continuously decreases from 0.5 in the middle of the specimen to the particular vertex singularity.

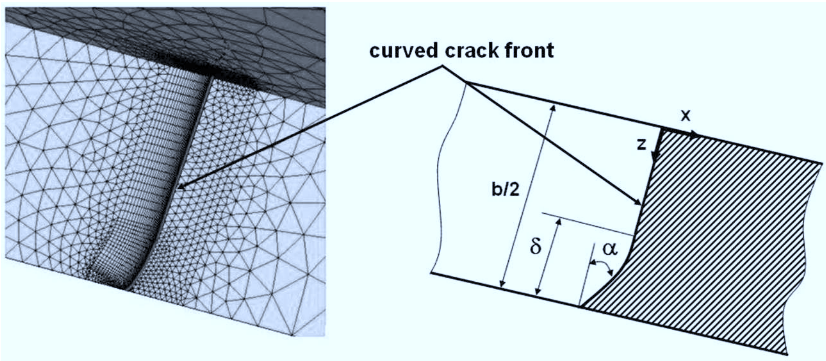

Fig. 4. Example of the finite element mesh close to curved crack front and schematic description of the MT specimen cross section area

Using methodology based on the generalized stress intensity factor, the distribution of the fatigue crack growth rate along the straight crack front was estimated [6, 12], see fig. 3. For a specimen with Poisson's ratio 0.365 the decrease of FCPR in region close to free surface is approximately 12 % in comparison with the centre of the specimen ($z = 0\,\mathrm{mm}$). The FCPR is not constant along the crack front and therefore a hypothetical originally straight crack starts to change its shape during its propagation. The FCPR in regions closer to the free surface is slower than in the centre of the specimen and typical curved crack front observed experimentally for through cracks is created. Therefore the aim of this article is to simulate the curvature of the crack front based on the methodology proposed for estimation of the fatigue crack growth and to compare the results with experimental data published in the literature.

Table 1. Thickness of the boundary region as a function of the Poisson's ratio

Poissn's ratio ν	boundary region δ
[–]	[mm]
0	0
0.2	0.4
0.3	0.9
0.36	1.3
0.4	1.8
0.5	3.7

To simulate this problem numerically a model of the MT specimen with curved crack front was developed. The dimensions of the MT specimen were similar to those in the previous model with straight crack (see fig. 2). Uniform applied tensile stress was applied. Exploiting the symmetry in the specimen geometry and loading, only one-eighth of the MT-specimen was modeled by finite element analysis (see Fig. 4).

The thickness of the boundary layer where the stress field is influenced by the vertex singularity was estimated in work [6]. According to previous numerical simulations [6, 11], it was found that the boundary layer thickness δ for thick specimens are dependent only on Poisson's ratio and the influence of the specimen geometry is insignificant. Therefore value δ corresponding to different Poison's ratio can be estimated, see tab. 1.

4. Estimation of the fatigue crack growth rate

Fatigue crack growth rate (FCGR) can be estimated following the concept published in [13]. Assuming that plastic zone size is a value controlling FCGR and based on correlation of the plastic zones parameters for the standard fatigue crack and the V-notch (see [6, 11] for a details), the relation between the generalized stress intensity factor H_I and the effective value of the stress intensity factor K_{eff} can be expressed in the form [6, 14]:

$$K_{eff} = \left(\frac{H_I \cdot s^{\frac{1}{2}} (2\pi)^{(p-1)}}{(1 + 4\nu^2 - 4\nu)^p \sigma_0^{(1-2p)}} \right)^{\frac{1}{2p}}, \tag{4}$$

where H_I is a generalized stress intensity factor corresponding to mode I loading, p is a singularity exponent and σ_0 is cyclic yield stress of the material. Function s depends on the singularity exponent and Poisson's ratio:

$$s = (1-p)^2(3p^2 + 3p^2q^2 + 6p^2q - 12q^2p - 12qp + 12q^2 + 4 + 16\nu^2 - 16\nu), \tag{5}$$

where q is function of the singularity exponent and V-notch angle (β) corresponding to particular singularity exponent [14]:

$$q = -\frac{\cos(p(\pi - \beta))}{\cos((2-p)(\pi - \beta))}. \tag{6}$$

The equation (6) makes it possible to recalculate a generalized stress intensity factor H_I with unit MPa m^p to an effective stress intensity factor K_{eff} with standard unit MPa $\mathrm{m}^{1/2}$. Then the effective stress intensity factor can be used as a parameter controlling fatigue crack propagation rate according the standard Paris-Erdogan law in the usual form:

$$\frac{\mathrm{d}a}{\mathrm{d}N} = C\,(K_{eff})^m \tag{7}$$

where C and m are material parameters characterizing the standard fatigue crack propagation rate in studied material, see [13] for more details.

5. Numerical results and discussion

According to experimental and numerical data from the literature [3, 4, 5], it seems reasonable to assume that real mode I fatigue cracks might choose to preserve the square-root singularity whole along the crack front. Based on this assumption, the different fatigue crack shapes, defined by different intersection angle between crack front and free surface α see fig. 4), were analyzed. A typical distribution of the singularity exponent along the crack front for two different Poisson's ratios 0.2 and 0.365 is shown in figs. 5, 6. The singularity exponent in the vertex point increases with an increase in the angle between crack front and free surface (intersection angle). Therefore, depending on Poisson's ratio there exists a unique intersection angle, where along the crack front a square-root singularity is present. Our results are also in the good comparison with the experimental data from the literature. Hayder et al. [3] on PMMA with $\nu = 0.365$ found intersection angle for fatigue crack $14.049°$.

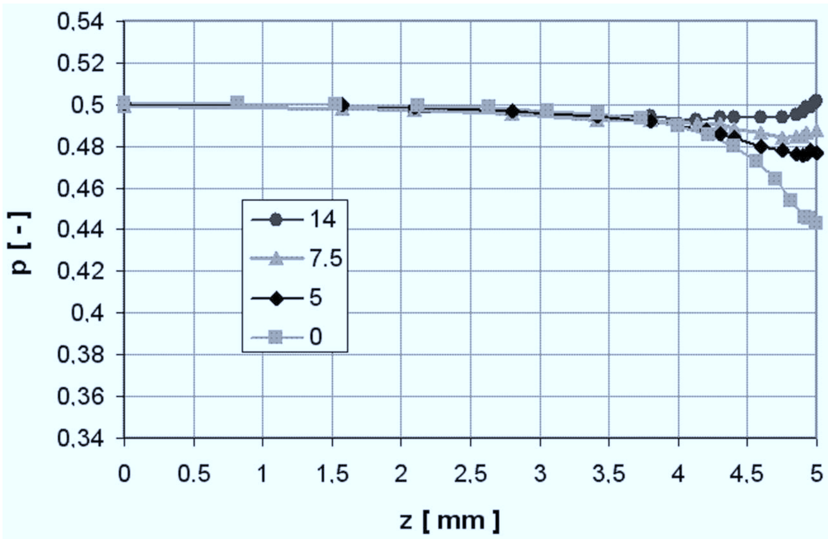

Fig. 5. Variation of the stress singularity exponent along crack front for different crack front angles α, poisson's ratio is 0.365

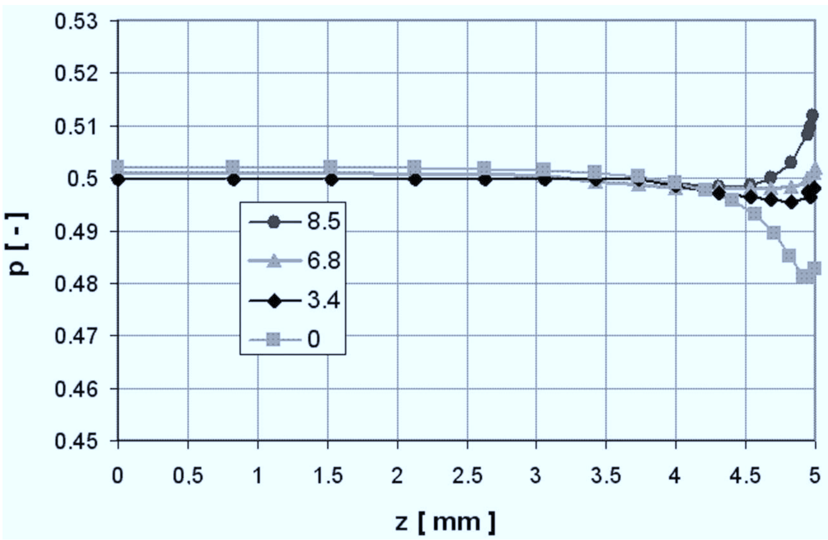

Fig. 6. Variation of the stress singularity exponent along crack front for different crack front angles α, poisson's ratio is 0.2

For the same material properties according to Pook [5] equation the intersection angle is estimated as $12.58°$. Based on numerical results presented on fig. 5, the stress singularity exponent is constant along crack front (with accuracy 1 %) for intersection angle $14°$.

Based on the methodology proposed in the previous chapter it is possible to estimate the fatigue crack growth rate in each single plane perpendicular to crack front. The ratio between real fatigue crack propagation rate v and FCPR in the center of the specimen v_{middle} along the crack front are presented in Fig. 7. For a specimen with intersection angle $0°$ (the crack front is perpendicular to the free surface) the decrease of FCPR in regions close to free surface is approximately 12 % in comparison with the centre of the specimen ($z = 0$ mm). Decrease of FCPR along the crack front leads to an increase in the intersection angle. The change of FCPR along the crack front decreases with an increase in the intersection angle. Thus, finally, the crack front is stabilized in a shape with constant FCPR along the crack front. This shape corresponds to the square root singularity along the crack front. Consequently, the proposed methodology for estimation of the fatigue crack growth rate is consistent with the numerical and experimental data and can be used for prediction of the residual fatigue life time of the considered structures.

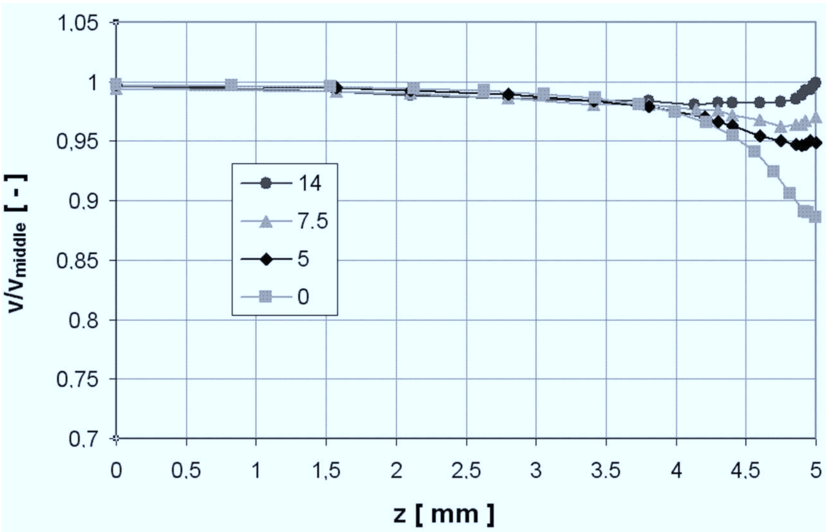

Fig. 7. Variation of ratio between real fatigue crack propagation rate v and crack propagation rate in the middle of the specimen v_{middle} along the crack front for four intersection angles. Poisson's ratio is 0.365

6. Conclusions

The influence of vertex singularity in the case of surface breaking cracks has been numerically investigated. For this reason, different middle tension specimens loaded under constant tension are considered. The vicinity of the intersection of the crack front and the free surface is analyzed due to three-dimensional singular behaviour. In this area the practically constant intersection angle (the angle between crack front and free surface) for a particular Poisson's ratio could be observed. The value of the intersection angle α was verified by singularity exponent analysis and using the methodology proposed for estimation of the fatigue crack growth rate. The numerically simulated intersection angle corresponds to classical square-root stress singularity. Hence, at least for mode I, the crack front is shaped to holds this type of the singularity along whole crack front. Thus it can be concluded, that vertex singularity is important for crack front formation and can sufficiently explain that phenomenon. Due to the final form of the crack which prioritizes square-root singularity, the classical stress intensity factor concept can be used for suitable fatigue crack propagation criteria even close to the free surface. The methodology proposed makes it possible to estimate fatigue crack behaviour in cases where the singular stress field around the crack tip has a singularity exponent different from $1/2$. The estimated fatigue crack growth rates can help to provide a more reliable estimation of the residual fatigue life time of the structures considered.

Acknowledgements

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References

[1] Z. Bažant, F. Estenssoro, Surface singularity and crack propagation, International J. Solids Structures (15) (1979) 405–426.

[2] J. P. Benthem, State of stress at the vertex of a quarter-infinite crack in a half space, International J. Solids Structures (13) (1977) 479–492.

[3] M. Heyder, K. Kolk, G. Kuhn, Numerical and experimental investigations of in the influence of corner singularities on 3D fatigue crack propagation, Engineering Fracture Mechanics (72) (2005) 2 095–2 105.

[4] P. F. P. Matos, D. Nowell, The influence of the Poisson's ratio and corner point singularities in three-dimensional plasticity-induced fatigue crack closure: A numerical study, International Journal of Fatigue, (30) (2008) 1 930–1 943.

[5] L. P. Pook, Some implications of corner point singularities, Engineering Fracture Mechanics (48) (1994) 367–378.

[6] P. Hutař, L. Náhlík, Z. Knésl, Numerical investigations of corner singularities in cracked bodies, Key Engineering Materials (348–349) (2007) 377–380.

[7] T. L. Anderson, Fracture Mechanics Fundamentals and Applications, Second Edition, CRC Press, 1995.

[8] A. Dimitrov, F. G. Buchholz, E. Schnack, Effects in cracks propagation, 5$^{\text{th}}$ World Congress on Computational Mechanics, Wiena, 2002.

[9] H. Awaji, Y. Toshimitsu, The variation of the stress singularity at the notch tip as notch angle and radius of curvature, Computers & Structures (22) (1986) 25–30.

[10] P. Hutař, L. Náhlík, Z. Knésl, The effect of the singularity induced by the free surface on fatigue crack growth in thin structures, Key Engineering Materials (385–387) (2008) 317–320.

[11] P. Hutař, L. Náhlík, Influence of corner point singularity on crack behaviour, Engineering Mechanics, Svratka, 2007.

[12] P. Hutař, L. Náhlík, Z. Knésl, S. Seitl, The influence of vertex singularities on fatigue crack shape, 17th European Congress on Fracture, Brno, 2008.

[13] Z. Knésl, L. Náhlík, J. C. Radon, Influence of interface on fatigue threshold values in elastic bimaterials, Computational Material Science (28) (2003) 620–627.

[14] J. Klusák, Lineární elastická lomová mechanika V-vrubu, Ph.D.-thesis, Brno, 2002.