

# Identification of material properties of sandwich structure with piezoelectric patches

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## Abstract

The work focuses on light-weight sandwich structures made of carbon-epoxy skins and foam core which have unique bending stiffness compared to conventional materials. The skins are manufactured by vacuum autoclave technology from unidirectional prepregs and the sandwich is then glued together. The resulting material properties of the structure usually differ from those provided by manufacturer or even those obtained from experimental tests on separate materials, which makes computational models unreliable. Therefore, the properties are identified using the combination of experimental analysis of the sandwich with attached piezoelectric transducer and corresponding static and modal finite element analyses. Simple mathematical optimization with repetitive finite element solution is used. The model is then verified by transient analysis when the piezoelectric patch is excited by harmonic signals covering the first two eigen-frequencies and the induced oscillations are measured by laser sensor.

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## 1. Introduction

Light-weight structures are nowadays necessary components in modern state-of-the-art products in all sorts of industries. The increasing requirements on structural performance call for the usage of embedded sensors and actuators, resulting in the construction of the so-called adaptive, smart or even intelligent structures that can thus respond to loading conditions in real time [10]. This enables for instance to monitor the condition of the structure [5], suppress vibrations or to adapt the desired shape [1, 8], provided that proper electronic control circuits are applied.

Sandwich structures are one of the most practical forms of composite light-weight structures. Their use has spread in many industrial branches, e.g. in building and transport industries, but especially in aircraft industry and in shipbuilding. Their main advantage over the conventional materials are the high ratio of the flexural stiffness to weight, reasonable price and service life, or damping properties. Sandwich structures consist of a core of soft flexible material enclosed in two outer skins. The skins carry the bending load, while the core transfers the shear forces and thus increases the flexural stiffness by holding the outer coating layers together [4].

The finite element modeling of the piezoelectric materials began with the first implementation in 1970 [2]. Many models have then been developed to simulate the piezoelectric effect, ranging from the simple ones using the similarity to the theory of thermo-elasticity to models [3], multi-purpose elements programmed for commercial software [11, 12], up to complex models with full piezoelectric coupling incorporating layerwise approach for electric potential across layers [7] or quadratic variation of electric potential across the layer thickness [6].

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The purpose of this work is to set up numerical model of hybrid sandwich and to use the piezoelectric actuator to identify the structural properties using experiment and corresponding simulation. This should be further extended for the application of monitoring the structural health in the future.

## 2. Analogy between piezoelectricity and thermal expansion

Let us consider the theory of piezoelectricity which assumes symmetrical hexagonal piezoelectric structure and only the laminar piezoelectric effect (also called  $d_{31}$  effect, both direct and converse), i.e., the material is polarized in the thickness direction and the electric potential varies linearly across the thickness [9].

The classical stress-strain law (Hooke's law)

$$\boldsymbol{\sigma} = \mathbf{C} \boldsymbol{\varepsilon}, \quad (1)$$

$\boldsymbol{\sigma}$  being the stress vector,  $\mathbf{C}$  stress-strain matrix and  $\boldsymbol{\varepsilon}$  the strain vector, is extended in this case by the piezoelectric coupling, hence

$$\begin{aligned} \boldsymbol{\sigma} &= \mathbf{C} \boldsymbol{\varepsilon} - \mathbf{e} \mathbf{E} \\ \mathbf{D} &= \mathbf{e}^T \boldsymbol{\varepsilon} + \boldsymbol{\varepsilon} \mathbf{E}, \end{aligned} \quad (2)$$

where  $\mathbf{e}$  is the piezoelectric coupling matrix,  $\mathbf{E}$  is the electric field vector,  $\mathbf{D}$  is the vector of electric flux density (electric displacement), and  $\boldsymbol{\varepsilon}$  is the dielectric permittivity matrix.

In many applications, the electric potential can be considered known (the piezoelectric material is in the actuator mode) and, therefore, the second equation in (2) does not need to be solved for the electro-mechanical behavior. This allows to model the problem of piezoelectricity using the analogy with thermal expansion [3]. This can prove very helpful if the used software does not contain the piezoelectric features.

The stress-strain law with thermal expansion for one-dimensional problem can be written as

$$\sigma = E(\varepsilon - \alpha \Delta T), \quad (3)$$

where  $E$  is the Young's modulus,  $\alpha$  the coefficient of thermal expansion and  $\Delta T$  the change in temperature. The corresponding piezoelectric equation is

$$\sigma = E\varepsilon - e \frac{U}{d} \quad (4)$$

with  $U$  being the voltage across electrodes and  $d$  the distance between the electrodes. The analogy is obvious and it is possible to write directly the resemblance between

$$\alpha \sim \frac{e}{E} \quad \text{and} \quad \Delta T \sim \frac{U}{d}. \quad (5)$$

## 3. Experiment

Experimental investigation of oscillations caused by harmonic excitations of hybrid sandwich cantilever beam was carried out. The beam consists of foam core and composite skins and there is a piezoelectric patch glued to its upper surface (see fig. 1). The foam is Rohacell 71, the composite is made of Panex unidirectional carbon-epoxy prepregs with fibers running along the

axial direction, the patch is DuraAct P876.A12 attached with HBM Z70 glue. The thickness of the core is 5.6 mm, each skin is 0.7 mm thick and the beam's length and width are 450 mm and 50 mm, respectively. The dimensions of the patch are  $61 \times 35 \times 0.5$  mm but the size of the active material (PIC155), enclosed in a protective foil, is only  $50 \times 30 \times 0.2$  mm (see fig. 2).

The beam was clamped at one end so that the gap between the fixture and the active piezoelectric area was 10 mm, i.e., the free length of the beam was 400 mm. The piezoelectric patch was loaded by sine signal from the generator connected to voltage multiplier with final amplitude of 100 V. Laser sensor was used to measure the deflections of the free end. The scheme of the experimental setup is shown in fig. 3.

Firstly, the two lowest eigen-frequencies of the structure were found by sweeping the generator frequency and searching for the largest steady oscillations. The values are  $f_1 = 52$  Hz and  $f_2 = 318$  Hz. Secondly, using constant voltage 100 V the static deflection was measured to be  $u = 0.37 \mu\text{m}$ . The last analysis investigated the response to frequencies around the two eigen-frequencies, namely the intervals  $\langle 1, 70 \rangle$  Hz and  $\langle 290, 340 \rangle$  Hz. The amplitudes of the steady oscillations  $A$  were measured for each frequency using the laser sensor.



Fig. 1. Detail of sandwich beam with applied piezoelectric patch



Fig. 2. Top and bottom view of the DuraAct P876.A12 piezoelectric transducers (patches)

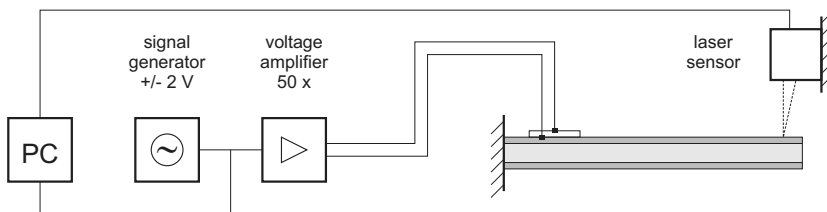


Fig. 3. Scheme of experimental setup

#### 4. Numerical analysis

Finite element model of the investigated structure was designed in MSC.Marc/Mentat software utilizing the analogy between piezoelectricity and thermal expansion. The beam consisted of eight-node solid elements (with assumed strain option) as shown in fig. 4. The detail of how the materials were modeled within the structure is obvious from the cross-section in fig. 5. The prescribed boundary conditions for the simulation of the clamped part are shown in fig. 6.

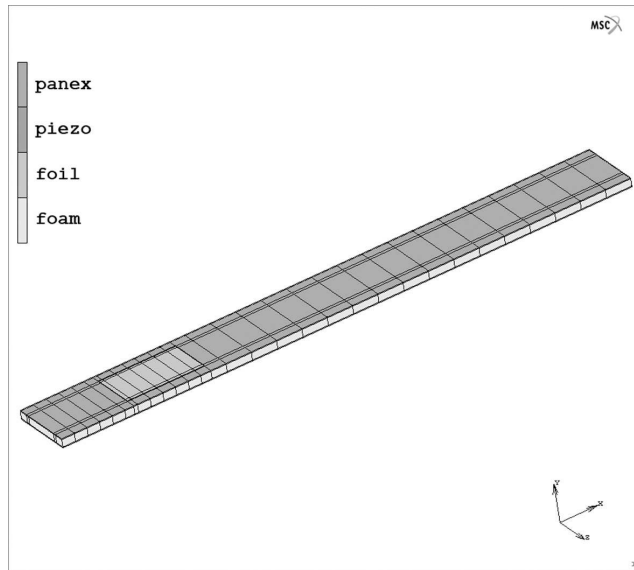


Fig. 4. Finite element model of sandwich and attached piezoelectric patch

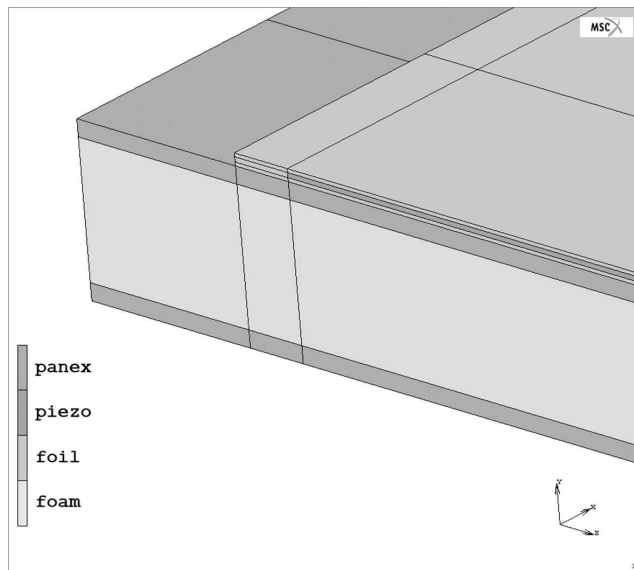


Fig. 5. Detail of sandwich and patch cross-section

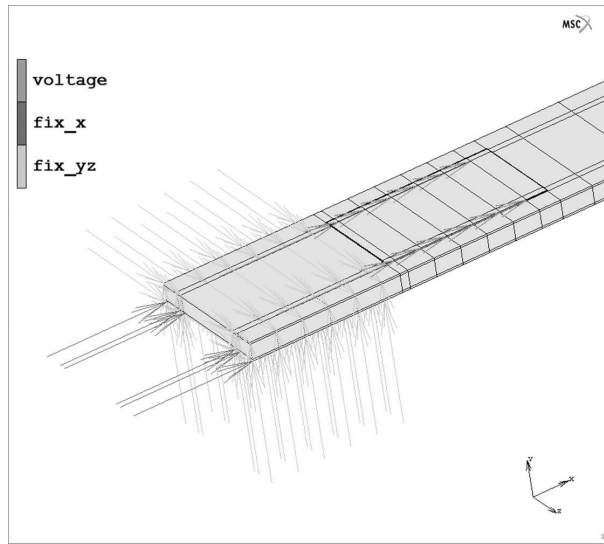


Fig. 6. Applied boundary conditions

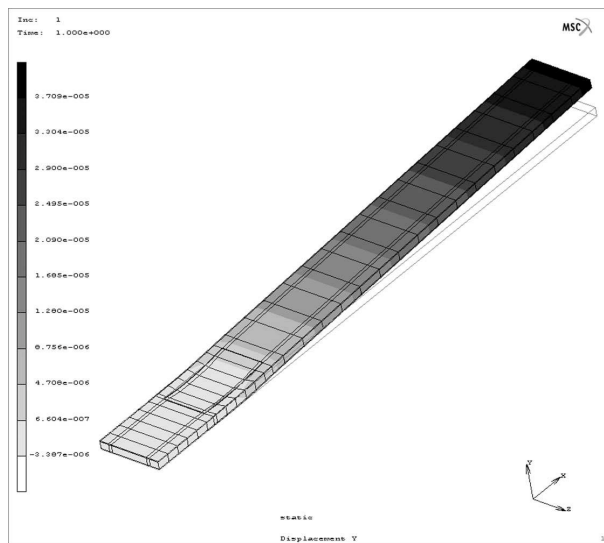


Fig. 7. Deflection induced by static load (voltage)

In the beginning, the material properties obtained from data sheets or previous static tests were used. The modal analysis, however, did not results in good agreement with experimental eigen-frequencies even after mesh refinement. As the properties of the composite were trustworthy and the influence of the piezoelectric patch could be considered negligible in this case, the Young's modulus of the foam was sought. Simple optimization loop with interval partitioning was used to minimize the error

$$\Delta = \sum_{i=1}^2 \left\{ (f_i^{EXP} - f_i^{FEA})^2 \right\} \quad (6)$$

where  $f_i^{EXP}$  are the measured eigen-frequencies and  $f_i^{FEA}$  are the calculated values. The resulting frequencies differed by less than 0.1 %.

Another unknown was the piezoelectric constant  $e$  of patch. As the bending mechanical behavior of the model could be considered reliable now, the piezoelectric constant was obtained by comparing the static deflection of the beam which was found in the experiment. The resulting static deflection of the numerical model is shown in fig. 7. The optimal values of all material properties used are displayed in tab. 1. Since only the bending behavior is of interest it is possible to assume the composite and piezoelectric materials to be isotropic (i.e. the elasticity constants correspond to axial components).

Table 1. Material properties

	Composite	Foam	Foil	PIC155
$E$ [MPa]	107 950	69	8 241	61 800
$\nu$	0.322 5	0.3	0.3	0.3
$\rho$ [kg/m <sup>3</sup> ]	1 468	76.89	1 528	7 760
$e$ [C/m <sup>2</sup> ]				-15.6

The main part of the investigation was to compare the measured frequency characteristics of the hybrid sandwich beam with the results of the numerical model. The transducer was excited by harmonic signals covering similar spectra as those in the experiment. The calculated and measured amplitudes of steady oscillations  $A$  are shown in fig. 8 and 9. As it showed to be difficult to reach fully steady oscillations in both cases, two types of results are used for each set of data. The symbol FFT denotes the amplitude (of a frequency being the same as the excitation frequency) obtained from the signal by Fast Fourier Transform and the symbol P2P denotes the maximum amplitude in the signal (corresponding to peak-to-peak values).

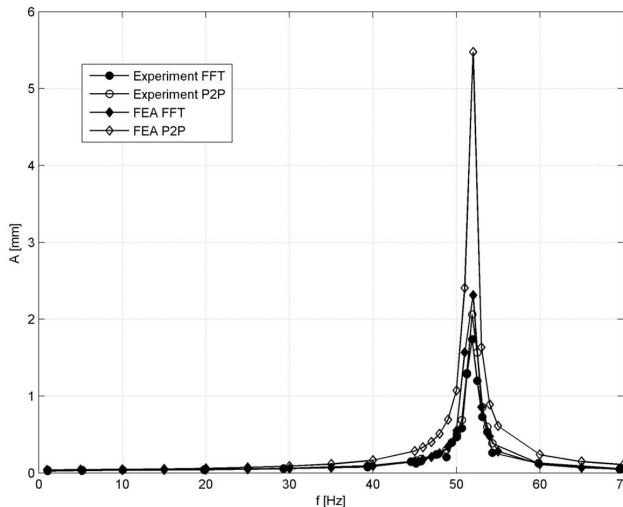


Fig. 8. Amplitude characteristics around first eigen-frequency

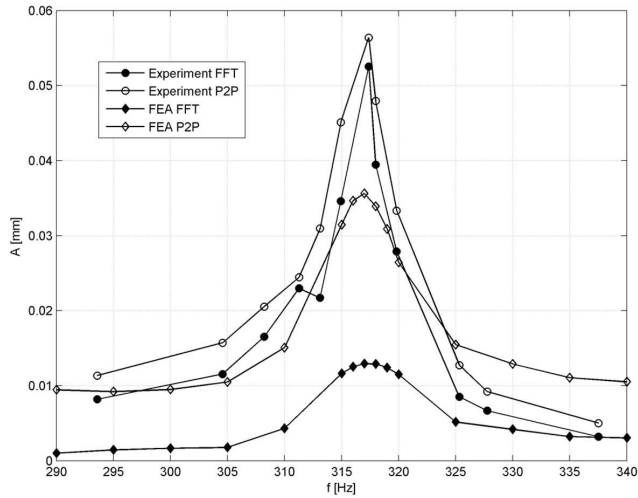


Fig. 9. Amplitude characteristics around first eigen-frequency

## 5. Conclusions

Experimental analysis of frequency characteristics of hybrid sandwich cantilever beam was carried out. The beam consists of foam core, composite skins and a piezoelectric patch glued to its surface. Corresponding finite element model was designed in MSC.Marc/Mentat software using the analogy between piezoelectricity and thermal expansion. The material properties of the foam were sought by modal analysis using simple optimization loop with interval partitioning. Also, the proper value of the piezoelectric constant of the patch active material had to be identified by comparing the calculated static deflection with experiment.

The main part of the investigation was to compare the calculated and measured frequency characteristics when the transducer material was excited by harmonic signals around the first two eigen-frequencies and the resulting steady oscillations were measured by laser sensor placed at the free end.

The following study will focus on simplified macro model of the piezoelectric patch, and the influence of material and numerical damping.

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