WAVELET ANALYSIS OF STOCK RETURN ENERGY DECOMPOSITION AND RETURN COMOVEMENT – A CASE OF SOME CENTRAL EUROPEAN AND DEVELOPED EUROPEAN STOCK MARKETS

Silvo Dajčman, Alenka Kavkler

Introduction

Stock market integration, stock market comovement and return spillovers between developed and developing stock markets, particularly CEE markets, are of great importance for international investors making financial decisions. Increased comovement of stock markets returns may diminish the advantage of internationally diversified investment portfolios [30].

The most common method of measuring stock market comovement is linear correlation, expressed by Pearson's correlation coefficient, a symmetric, linear dependence metric [30] suitable for measuring dependence on multivariate normal distributions [11]. However correlations may be nonlinear or time-varying ([10[, [52]), and dependence between two stock markets as the market rises may be different than the dependence as the market falls [34]. A more accurate understanding of stock market interdependencies may be achieved by applying econometric methods. In the literature the following methods are often used to measure the level of stock market comovement: correlation coefficients (e.g. [28], [31]), Vector Autoregressive (VAR) models ([20], [33]), cointegration analysis ([19], [36]), GARCH models ([1], [5], [10], [50]) and regime switching models ([13], [45]). A novel but promising approach is a wavelet analysis of stock market comovement.

Candelon et al [2] argue that comovement analysis should consider the distinction between short- and long-term investors. From a portfolio diversification point of view, short-term investors are more interested in the comovement of stock returns at higher frequencies (short term movements), and long-term investors focus more on the lower frequency comovements. As such, one must resort to frequency domain analysis to obtain insights into comovement at the frequency level ([29], [35], [43], [48]). In such a context, with both the time horizon of economic decisions and the strength and direction of economic relationships between variables that may differ according to the time scale of the analysis, wavelet analysis may prove to be a useful analytical tool [41].

Economic and financial phenomena may exhibit different characteristics on different time scales, and thus wavelet analysis tools enable us to investigate the multiscale features of these phenomena. As wavelets are localized in both time and scale, unlike Fourier analyses and spectral analyses, they thus provide a convenient and efficient way of representing complex variables or signals [42]. Moreover, because of its translation and scale properties, nonstationarity in the data is not a problem when using wavelets and prefiltering is not needed [41]. Wavelet analysis is suitable for detecting seasonal and cyclical patterns, structural breaks, trend analyses, fractal structures and multiresolution analyses [8]. Wavelets in finance are primarily used as a signal decomposition tool (e.g. [32], [16], [14], [17], [18], [51]), or a tool to detect

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interdependence between variables ([23], [24], [25], [26], [27]).

Lee [29] developed a new testing technique based on the discrete wavelet transform, in order to study the relationships between U.S. and Korean stock market returns in the period 1995 to 2000, examining two stock indices in each market, namely the Dow Jones Industrial Average (DJIA) and the NASDAQ for the United States and the Korean Composite Stock Price Index (KOSPI) and the Korean Security Dealers Automated Quotations (KOSDAQ) for South Korea. By examining the relationships between high-frequency fluctuations in stock returns, obtained from the reconstruction of the data by wavelet details, [29] finds evidence of return spillover effects from the U.S. stock markets to Korean counterparts, but not vice versa.

In a similar way, [12] focuses on return spillovers in stock markets at different time scales using wavelet analysis. She considers eight stock indices that comprise the G7 countries, Emerging Asia, Western Europe, Eastern Europe and the Middle East, the Emerging Far East, Latin America, North America, and the Pacific region for the period 1990 to 2002. The author's estimation results show evidence of stock market return spillovers from the G7 countries to Western Europe, Eastern Europe and the Middle East, Emerging Asia, Europe, Latin America, and North America. However, return spillovers of these regions to the G7 countries are weaker at different time scales. Similarly, return spillovers from North America to Latin America, Emerging Asia, the Emerging Far East, and the Pacific region, and from both Western Europe and Latin America to North America are found. [48] investigate seven international stock markets – Ireland, the United Kingdom, Portugal, the United States, Brazil, Japan and Hong Kong – and their comovement and spillover effects, using a testing method suggested by [29]. They find evidence of intra-European comovement, namely between the stock markets of Ireland, the UK and Portugal. Further, they find comovement between the U.S. and Brazilian markets and similar intra-Asian comovement, namely between the stock markets of Japan and Hong Kong.

We will use maximal overlap discrete wavelet transform (MODWT) to analyse multiscale stock market return volatility dynamics and return comovement between CEE and developed European stock markets. To our knowledge, this is the first study to apply this methodology to CEE stock markets. The more recent empirical literature on the interdependence of CEE stock markets to developed stock markets predominantly apply correlation analysis ([21], [47]), Granger causality tests ([22], [36]), cointegration analysis ([36], [49]) and GARCH modelling ([3], [44].

The structure of the paper is as follows. Econometric methodology is described in the first chapter. Maximal overlap discrete wavelet transform (MODWT) is explained and some practical issues for MODWT analysis are addressed. In the second chapter, we present the data, describe in detail our empirical study of return comovement and energy decomposition, and interpret the results. Main implications of the empirical analysis are revisited in the conclusion.

1. Econometric Methodology

To study the comovement of the CEE stock markets (Slovenia, the Czech Republic and Hungary) with developed European stock markets (Austria, France, Germany and United Kingdom), we apply methodology of [29]. Interdependence between stock markets exists in two forms [53] – comovement, which measures the contemporaneous relationship between volatilities, and spillover, which indicates the lead-lag relationship. Stock market return spillover analysis is based on the idea that if news (a shock) in one stock market (reflected in its return) in time *t – 1* affects the returns of another stock market in time *t*, there are return spillovers, and the returns of the first market explain the returns of the second market. However, if there is a high degree of stock market comovement, then in the observed time period (e.g. one day, one week, etc.), stock returns synchronously move in the same direction.

We focus on return comovement analysis by estimating the following model (by ordinary least squares):

$$r_{D_A,t}(\tau_j) = a + br_{D_A,t}(\tau_j) + \varepsilon_A \qquad (1)$$

where:

$r_{D_A,t}(\tau_j)$ = from MODWT wavelet details reconstructed returns of the stock index A at scale τ_j,

$r_{D_A,t}$ (τ_j) = from MODWT wavelet details reconstructed returns of the stock index B at scale τ_j,
α = a regression constant,
b = a regression parameter,
ε_A = error of the regression model.

The proposed model is estimated in the following steps. First we transform the indices' return series by MODWT. We obtain wavelet and scaling coefficients, which we use to study the energy decomposition of the return series at different time scales (j). This inspection allows us to determine which time scales capture the most volatility of the indices' return series. In the next step we use only those scales that capture the greatest share of energy to reconstruct the original return time series from the wavelet details. In this way we obtain reconstructed returns of the indices on a scale-by-scale basis. For pairs of reconstructed return time series we then estimate the ordinary least squares model proposed by equation (1).

1.1 Description of the Maximal Overlap Discrete Wavelet Transform (MODWT)

The MODWT is a linear filtering operation that transforms a series into coefficients related to variations over a set of scales. It is similar to the discrete wavelet transform (DWT), but it gives up the orthogonality property of the DWT to gain other features that render MODWT more suitable for our analysis of stock market return comovement [39], as: i) the ability to handle any sample size, regardless of whether the series is dyadic (i.e. of size 2^{J_0}, where J_0 is any positive integer); ii) increased resolution at coarser scales as the MODWT oversamples the data; iii) translation-invariance, which ensures that MODWT crystal coefficients do not change if the time series is shifted in a "circular" fashion; iv) the MODWT produces a more asymptotically efficient wavelet variance estimator than the DWT.

1.1.1 MODWT Wavelet Coefficients and Scaling Coefficients

Wavelets are small waves, whereas by contrast, sinus and cosinus are large waves. A wavelet, by definition, is any function that is square-integrable and integrates to zero. The wavelet transform is a mechanism that allows us to quantify how averages of a time series over particular scales change from one interval of time to the next [40]. These changes are quantified in wavelet coefficients, which form the bulk of any discrete wavelet transform [38].

Let $\boldsymbol{X}$ be an N-dimensional vector whose elements represent the real-valued time series $\{X_t : t = 0,..., N-1\}$ (we use the same notation as [40]). For any positive integer, J_0, the level J_0 MODWT of X is a transform consisting of the $J_0 + 1$ vectors $\tilde{W}_1,..., \tilde{W}_{J_0}$ and $\tilde{\boldsymbol{V}}_{J_0}$, all of which have dimension N. The vector $\tilde{\boldsymbol{W}}_j$ contains the MODWT wavelet coefficients associated with changes on scale $\tau_j = 2^{j-1}$ for $(j = 1,..., J_0)$, while $\tilde{\boldsymbol{V}}_{J_0}$ contains MODWT scaling coefficients associated with averages on scale $\lambda_{J_0} = 2^{J_0}$. Based on definition of MODWT coefficients, we can write [40]:

$$\tilde{\boldsymbol{W}}_J = \tilde{\Lambda}_j \boldsymbol{X} \tag{2a}$$

and

$$\tilde{\boldsymbol{V}}_{J_0} = \tilde{\Gamma}_{j_0} \boldsymbol{X} \tag{2b}$$

where $\tilde{\Lambda}_j$ and $\tilde{\Gamma}_{j_0}$ are $N \times N$ matrices of containing the values of the wavelet and scaling filters. Vectors are denoted by bold fonts.

By definition, the elements of $\tilde{\boldsymbol{W}}_J$ and $\tilde{\boldsymbol{V}}_{J_0}$ are outputs obtained by filtering $\boldsymbol{X}$, namely:

$$\tilde{W}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{h}_{j,l} X_{t-l \bmod N} \tag{3a}$$

and

$$\tilde{V}_{j,t} = \sum_{l=0}^{L_j-1} \tilde{g}_{j,l} X_{t-l \bmod N} \tag{3b}$$

for $t = 0,..., N-1$, where $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ are jth level MODWT wavelet and scaling filters, defined in terms of the jth-level equivalent wavelet and scaling filters for a discrete wavelet transform (DWT) (for details see [40]).

The MODWT treats the series as if it were periodic, whereby the unobserved samples of the real-valued time series $X_{-1}, X_{-2},..., N_{-N}$ are assigned the observed values at $X_{N-1}, X_{N-2},..., X_0$. The MODWT coefficients are thus given by circularly filetering:

$$\widetilde{W}_{j,t} = \sum_{l=0}^{N-1} \widetilde{h}^{\circ}_{j,l} X_{t-l \bmod N} \quad (4a)$$

and

$$\widetilde{V}_{j,t} = \sum_{l=0}^{N-1} \widetilde{g}^{\circ}_{j,l} X_{t-l \bmod N} \quad (4b)$$

for $t = 0,..., N-1$; $\tilde{h}^{\circ}_{j,l}$ and $\tilde{g}^{\circ}_{j,l}$ are periodization of $\tilde{h}_{j,l}$ and $\tilde{g}_{j,l}$ to circular filters of length N.

This periodic extension of the time series is known as analyzing $\{X_t\}$ using "circular boundary conditions" ([6], [40]). There are $L_j - 1$ wavelet and scaling coefficients that are influenced by the extension ("the boundary coefficients"). Since L_j increases with j, the number of boundary coefficients increases with scale. Excluding boundary coefficients in the wavelet variance, wavelet correlation and covariance provides unbiased estimates [6].

1.1.2 MODWT Energy and Additivity Decomposition

One of the important uses of the MODWT is to decompose the sample variance of a time series on a scale-by-scale basis. Since the MODWT is energy conserving, the following equation holds [39]:

$$\|X\|^2 = \sum_{j=1}^{J_0} \|\widetilde{\boldsymbol{W}}_j\|^2 + \|\widetilde{\boldsymbol{V}}_{J_0}\|^2 \quad (5)$$

and a scale-dependent analysis of variance from the wavelet and scaling coefficients can be derived as [6]

$$\hat{\upsilon}_X^2 = \|X\|^2 - \overline{X}^2 = \frac{1}{N}\sum_{j=1}^{J_0} \|\widetilde{\boldsymbol{W}}_j\|^2 + \frac{1}{N}\|\widetilde{\boldsymbol{V}}_{J_0}\|^2 - \overline{X}^2 \quad (6)$$

Wavelet variance is defined for stationary and nonstationary processes with stationary backward differences. Considering only the non-boundary wavelet coefficient, obtained by filtering stationary series with MODWT, the wavelet variance $\upsilon^2_X(\tau_j)$ is defined as the expected value of $\widetilde{W}^2_{j,t}$. In this case, $\upsilon^2_X(\tau_j)$ represents the contribution to the (possibly infinite) variance of $\{X_t\}$ at the scale $\tau_j = 2^{j-1}$ and can be estimated by the unbiased estimator [40]:

$$\hat{\upsilon}_X^2(\tau_j) = \frac{1}{M_j}\sum_{t=L_j-1}^{N-1} \widetilde{W}^2_{j,t} \quad (7)$$

Where $M_j \equiv N - L_j - 1 > 0$ is the number of non-boundary coefficients at the jth level.

It is possible to prove that the asymptotic distribution of $\hat{\upsilon}^2_X(\tau_j)$ is Gaussian, a result that allows the formulation of confidence intervals for the estimate ([37], [46]).

Another useful characteristic of MODWT is additive decomposition. The time series X can be recovered from its MODWT via [40]:

$$\boldsymbol{X} = \sum_{j=1}^{J_0} \widetilde{\Lambda}_j^T \widetilde{\boldsymbol{W}}_j \mp \Gamma_{J_0}^T \widetilde{\boldsymbol{V}}_{J_0} = \sum_{j=1}^{J_0} \widetilde{D}_j + \widetilde{S}_{J_0} \quad (8)$$

which defines a MODWT-based multiresolution analysis (MRA) of X in terms of the jth level MODWT details $\tilde{D}_j = \tilde{\Lambda}^T_j \tilde{W}_j$, which capture local fluctuations over the whole period of a time series at each scale, and the J_0-th level of MODWT smooth $\tilde{S}_{j_0} = \Gamma^T_{j_0} \tilde{V}_{j_0}$, which provides a "smooth" or overall "trend" of the original signal. Adding $\tilde{D}_j$ to $\tilde{S}_{j_0}$, for $J = 1,2,..., J_0$, gives an increasingly accurate approximation of the original signal.

1.2 MODWT Parameters

Some practical issues, besides the handling of appropriate boundary conditions, should be addressed before the start of MODWT analysis:

- Choice of wavelet filter. MODWT is less dependent on the wavelet filter choice than is discrete wavelet transform [40], but different wavelet filter properties may still result in different wavelet analysis results. A reasonable choice of the filter must consider the specific analysis goal we want to achieve (such as isolation of transient events in a time series, analysis of variance, multiresolution analysis, etc.) and the properties we need in a filter to achieve that goal [40]. Choosing a wavelet filter of the shortest width $(L = 2,4,6)$ can sometimes introduce undesirable artefacts into the resulting analyses. Alternatively, while wavelet filters with a large L can be a better match to the characteristic features in a time series, their use can result in more coefficients being influenced by boundary conditions and an increase in computational burden. [40] suggest a strategy of using the smallest L that gives reasonable results. The Daubechies class of wavelets possesses appealing regularity characteristics and produces transforms that are effectively localized differences of adjacent weighted averages. The least asymmetric (LA) subclass, known as symmlets, has approximate linear phase and exhibits near symmetry about the filter midpoint. This linear phase property means that events and sinusoidal components in the wavelet and scaling coefficients at all levels can be aligned with the original time series. For the MODWT, this alignment is achieved by circularly shifting the coefficients by an amount dictated by the phase delay properties of the basic filter [6]. LA filters are available in even widths L. A wider filter is smoother in appearance and reduces the possible appearance of artefacts in a multiresolution analysis due to the filter shape. It also results in stronger uncorrelation between wavelet coefficients across scales for certain time series, which is useful for deriving confidence bounds from certain wavelet-based estimates [7]. Taking all these considerations into account, LA(8) filter is an appropriate choice [40], as it yields coefficients that are approximately uncorrelated between scales while having a filter width short enough such that the impact of boundary conditions is tolerable [6].

- Choice of level J_0. The appropriate choice depends primarily on the time series at hand [40]. For complete decomposition of a series of length $N = 2^J$ (J is any positive integer number) using the DWT, the maximum number of levels in the decomposition is J. The MODWT can accommodate any sample size N and, in theory, any J_0. In practice, the largest level is commonly selected such that $J_0 \leq log_2(N)$ in order to preclude decomposition at scales longer than the total length of the time series. The selection of J_0 determines the number of octave bands and thus the number of scales of resolution in the decomposition [6].

2. Empirical Results

2.1 Description of the Data

Data on stock indices return are calculated as differences of logarithmic daily closing value of indices (i.e. $ln(P_t) - ln(P_{t-1})$, where P_t is the index value in time t). The following indices are considered: LJSEX (Slovenia), PX (Czech Republic), BUX (Hungary), ATX (Austria), CAC40 (France), DAX (Germany) and FTSE100 (Great Britain). The first day of observation is 1 April 1997, the last day is 12 May 2010. Days of no trading on any of the observed stock markets were left out. Total number of observations amounts to 3,060 days. Data sources from the LJSEX, PX and BUX indices are their respective stock exchanges; data sources for the ATX, CAC40, DAX and FTSE100 indices is Yahoo Finance.

Table 1 presents some descriptive statistics of the data. We observe a higher spread between maximum and minimum daily returns

Tab. 1: Descriptive statistics for stock index return time series

	Min	Max	Mean	Std. deviation	Skewness	Kurtosis	Jarque-Bera statistics
LJSEX	-0.1285	0.0768	0.0003521	0.01062	-0.87	20.19	38,073.93***
PX	-0.1990	0.2114	0.0002595	0.01667	-0.29	24.62	59,654.93***
BUX	-0.1803	0.2202	0.0004859	0.02021	-0.30	15.90	21,260.91***
ATX	-0.1637	0.1304	0.0002515	0.01558	-0.40	14.91	18,153.48***
CAC40	-0.0947	0.1059	0.0001206	0.01628	0.09	7.83	2,982.52***
DAX	-0.0850	0.1080	0.0002071	0.01756	-0.06	6.58	1,635.47***
FTSE100	-0.0927	0.1079	0.0000774	0.01361	0.09	9.30	5,069.61***

Source: Own calculations

Note: Jarque-Bera statistics:*** indicates that the null hypothesis (of normal distribution) is rejected at the 1% significance, ** indicates that the null hypothesis is rejected at the 5% significance and * indicates that the null hypothesis is rejected at 10% significance in the PX and BUX indices than in other indices. Standard deviations of daily returns are smallest for the LJSEX index. The Jarque-Bera test rejects the hypothesis of normally distributed observed time series, all indices are asymmetrically (left) distributed around the sample mean, and kurtosis is greater than with normally distributed time series.

2.2 Tests of Time Series Stationarity

To test stationarity of stock index return time series, Augmented Dickey-Fuller (ADF), Phillips-Perron (PP) and Kwiatkowski-Phillips-Schmidt-Shin (KPSS) tests are applied. Test results are presented in Table 2.

Tab. 2: Results of time series tests of stationarity

	KPPS test (a constant + trend)	KPSS test (a constant)	PP test (a constant + trend)	PP test (a constant)	ADF test (a constant + trend)	ADF test (a constant)
LJSEX	0.249*** (11)	0.591** (12)	-44.099*** (0)	-43.795*** (3)	-37.229*** (L=1)	-37.128*** (L=1)
PX	0.158* (10)	0.170 (10)	-55.022*** (10)	-55.029*** (10)	-16.676*** (L=8)	- 16.676*** (L=8)
BUX	0.065 (6)	0.065 (6)	-54.295*** (6)	-54.304*** (6)	-54.301*** (L=0)	- 54.310*** (L=0)
ATX	0.186** (12)	0.191 (13)	-53.586*** (15)	-53.594*** (15)	- 40.604** (L=1)	- 40.608*** (L=1)
CAC40	0.110 (15)	0.250 (15)	-57.840*** (14)	-57.787*** (14)	- 36.142*** (L=2)	- 36.108*** (L=2)
DAX	0.099 (1)	0.105 (1)	-57.805*** (3)	-57.812*** (3)	- 57.692*** (L=0)	- 57.698*** (L=0)
FTSE100	0.089 (9)	0.101 (9)	-58.284*** (7)	-58.287*** (7)	-29.112*** (L=3)	- 29.111*** (L=3)

Source: Own calculations

Notes: KPSS and PP tests are performed for two models: for the model with a constant, and for the model with a constant plus trend. The Bartlet Kernel estimation method is used with Newey-West automatic bandwidth selection. Optimal bandwidth is indicated in parenthesis under statistics. For the ADF test, two models are applied: autoregression (AR), and trend stationary model; number of lags to be included (L) for the ADF test was selected by SIC criteria (30 was a maximum lag). Exceeded critical values for rejection of the null hypothesis are marked by *** (1% significance level), ** (5% significance level) and * (10% significance level).

The null hypothesis of the KPSS test, indicating that the time series is stationary, for a model with a constant plus trend, can be rejected at the 5% significance level for the return series of LJSEX and ATX. Since trend is not significantly different from zero, we give advantage to KPSS model results with no trend. For that model we cannot reject the null hypothesis of stationary process for any stock index return series, expect for LJSEX, at the 1% significance level. The null hypothesis of PP and ADF tests is rejected for all stock indices. On the basis of the stationarity tests, we conclude that all index return time series are stationary.

2.3 Empirical Results of the Energy Decomposition and Return Comovement between Stock Markets

2.3.1 Energy Decomposition Results

MODWT transformations of the indices return series are performed by using a Daubechies least asymmetric filter with a wavelet filter length of 8 (LA8). This is a common wavelet filter used in other empirical studies on financial market interdependencies ([15], Ranta 2010). The maximum number of levels in the decomposition is 6 ($J_0 = 6$) to achieve an optimal level balance between sample size and the length of the filter.

Scale 1 measures the dynamics of returns over 2 to 4 days, scale 2 over 4 to 8 days, scale 3 over 8 to 16 days, scale 4 over 16 to 32 days, scale 5 over 32 to 64 days and scale 6 over 64 to 128 days. To obtain unbiased estimates, only non-boundary wavelet coefficients must be considered. There are 2,619 MODWT wavelet coefficients not affected by boundary condition.

Table 3 shows that 38 percent of LJSEX return variability is captured by scale 1 (2 to 4 day dynamics). Scale 2 captures 29.6 percent of all LJSEX return variability. It is evident that most energy, for all indices, is captured by scales 1 and 2. This finding is in accordance with other studies. [12] finds that the first and second scale of all indices investigated explain at least 60 percent of return variability; in the U.S. equity market the scales explain 75 percent. [29] finds that these two scales capture around 70 percent of energy for Korean stock market indices and around 80 percent for U.S. stock market indices. It follows that, from the point of view of stock markets return comovement analysis, the most interesting are the first two scales.

Tab. 3: A scale-based energy decomposition of stock indices returns (in % of the index return energy)

	W_1	W_2	W_3	W_4	W_5	W_6	V_6	Total
LJSEX	38.0	29.6	18.2	6.1	3.2	2.0	2.9	100
PX	50.8	26.2	12.2	5.2	2.6	1.5	1.5	100
BUX	47.5	26.7	13.5	5.9	2.8	2.1	1.5	100
ATX	49.5	28.2	12.3	5.2	2.3	1.0	1.5	100
CAC40	51.4	28.0	11.9	4.7	2.0	0.9	1.1	100
DAX	52.4	26.1	11.4	5.3	2.3	1.1	1.4	100
FTSE100	52.2	28.6	11.1	4.6	2.0	0.8	0.7	100

Source: Own calculations

Note: $W_j (j = 1,...,6)$ are MODWT wavelet coefficients at scale j, and V_6 are MODWT scaling coefficients.

2.3.2 Stock Market Comovement Analysis Results

To estimate regression model (1), we reconstruct the returns series using the first and second high-frequency wavelet details, D_1 and D_2, and then apply OLS to obtain parameter estimates of regression (1).

As wavelet energy decomposition indicates that most of the energy is captured by the first two scales, we estimate model (1) by using reconstructed indices returns for these two scales in the following manner:

- we estimate model (1) on the returns series reconstructed from D_1 $(r_D(\tau_1))$;
- we estimate model (1) on the summed returns series of reconstructed returns at scales 1 and 2 $(r_D(\tau_1) + r_D(\tau_2))$;
- for comparison purposes, we also estimate model (1) on raw (non-MODWT transformed) indices return series.

The strength of comovement is measured by R^2 and the significance of the regression parameter b ([29], [48]).

Estimation results for the Slovenian stock market (LJSEX) are presented in Table 4. Parameter estimates of the regression models are highly significant, as indicated by t-statistics, which shows that there exists comovement between LJSEX returns and returns of other investigated indices at the daily returns level, at the scale 1 level, and for the case of the summated first two scales. The adjusted R^2 reveals that the volatility of LJSEX raw returns is best explained by the volatility of PX and ATX returns. It follows that Slovenian stock market comoves more with stock markets of Austria and the Czech Republic and less with other observed stock markets.

Tab. 4: Results of the estimation of the regression model (1), LJSEX is dependent variable (part 1)

PX→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0003* (1.74)	0.2002*** (17.46)	0.1040
$r_D(\tau_1)$	-0.0000 (-0.0000)	0.1596*** (16.29)	0.0917
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.31)	0.1396*** (27.22)	0.2203
BUX→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0004* (1.79)	0.1301*** (12.60)	0.0568
$r_D(\tau_1)$	0.0000 (0.00)	0.0909*** (9.98)	0.0363
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.37)	0.0903*** (13.32)	0.0631
ATX→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0004 (1.82)	0.2195*** (17.34)	0.1027
$r_D(\tau_1)$	0.0000 (0.0000)	0.132*** (11.79)	0.0500
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (2.81)	0.2268*** (32.28)	0.2845
CAC40→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0004* (1.95)	0.1417*** (11.58)	0.0484
$r_D(\tau_1)$	0.0000 (0.0000)	0.0734*** (7.10)	0.0185
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.06)	0.1423*** (13.86)	0.0680
DAX→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0004* (1.92)	0.1257*** (11.12)	0.0448
$r_D(\tau_1)$	0.0000 (0.00)	0.0701*** (7.45)	0.0204
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.48)	0.0911*** (9.06)	0.0301

Tab. 4: Results of the estimation of the regression model (1), LJSEX is dependent variable (part 2)

FTSE100→LJSEX			
	A constant	Parameter b	R^2adj
Raw returns	0.0004** (2.02)	0.1655*** (11.45)	0.0474
$r_D(\tau_1)$	-0.0000 (-0.00)	0.0929*** (7.65)	0.0215
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.25)	0.1651*** (12.12)	0.0528

Source: Own calculations

Notes: PX→LJSEX indicates that LJSEX is a response variable and PX is the explanatory variable. Other arrows are explained by analogy. In the parenthesis, under estimated regression parameters, values of t-statistics are given, with critical values: 1.645 at the 10% significance level (rejection of the null hypothesis at this level is indicated by *), 1.961 at the 5% significance level (indicated by **), and 2.578 at the significance 1% (indicated by ***).

On scale 1, the LJSEX return's comovement with other stock markets is reduced. As with the raw return data, LJSEX return volatility is best explained by PX return volatility. Volatility in CAC40 and FTSE100 returns explains only about 2 percent of LJSEX return volatility. Taking summated returns of scales 1 and 2, which correspond to a time span of 2 to 8 days, more LJSEX return volatility can be explained by volatility in foreign stock indices, especially PX and ATX indices. Two to eight day PX return dynamics can explain about 22 percent of LJSEX return dynamics over this time horizon, whereas ATX returns explain approximately 28 percent.

The finding that raw returns and summed scales 1 and 2 returns are more connected than scale 1 returns was also obtained by [29] and [48]. [29], who studies return spillovers between U.S. and Korean stock markets using lagged returns of the explanatory variable finds significant return spillovers from the U.S. to the Korean stock market. Significance of parameter estimates and R^2 for raw return and summed scales 1 and 2 return series were slightly higher than for scale 1 series. [48] find strong co-movements only between pairs of Irish, UK and Portuguese stock market returns. The UK and Irish stock markets were most connected, as R^2 for the raw return series reached 0.32, for scale 1 returns 0.22, and for summed scale 1 and scale 2 returns 0.25.

Parameter estimates of the regression models for the Czech stock market are also highly significant (Table 5). Higher adjusted R^2 reveals that the Czech stock market returns comove more synchronously with the Hungarian and developed stock market returns, as is the case for Slovenia. PX return volatility is best explained by ATX and BUX return volatility. Interestingly, both Czech and Slovenian stock markets seem to comove with the Austrian stock market to a greater extent than with other developed stock markets. This finding could be explained by historical ties, strong economic ties, investments of Austrian enterprises in these two countries, and equity connection between the observed stock markets. Namely, the stock exchanges in Ljubljana, Prague, Vienna and Budapest are owned by a common holding company.

Tab. 5: Results of the estimation of the regression model (1), PX is dependent variable (part 1)

LJSEX→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0000 (0.13)	0.5214*** (17.46)	0.1040
$r_D(\tau_1)$	0.0000 (0.00)	0.5768*** (16.29)	0.0917
$r_D(\tau_1) + r_D(\tau_2)$	0.0000** (2.09)	1.5806*** (27.22)	0.2203
BUX→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0001 (0.49)	0.4991*** (35.31)	0.3225
$r_D(\tau_1)$	0.0000 (0.00)	0.4945*** (33.49)	0.2997
$r_D(\tau_1) + r_D(\tau_2)$	0.0000 (1.04)	0.6191*** (30.58)	0.2630
ATX→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0001 (0.53)	0.6675*** (38.87)	0.3658
$r_D(\tau_1)$	0.0000 (0.00)	0.6522*** (36.76)	0.3402
$r_D(\tau_1) + r_D(\tau_2)$	0.0000 (1.20)	1.0046*** (50.49)	0.4933
CAC40→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0003 (0.91)	0.5456*** (31.72)	0.2774
$r_D(\tau_1)$	0.0000 (0.00)	0.5219*** (30.68)	0.2643
$r_D(\tau_1) + r_D(\tau_2)$	0.0000 (1.21)	0.766*** (23.56)	0.1747
DAX→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0002 (0.81)	0.4564*** (27.84)	0.2282
$r_D(\tau_1)$	0.0000 (0.00)	0.4159*** (25.71)	0.2014
$r_D(\tau_1) + r_D(\tau_2)$	0.0000 (1.57)	0.5704*** (17.56)	0.1051

Tab. 5: Results of the estimation of the regression model (1), PX is dependent variable (part 2)

FTSE100→PX			
	A constant	Parameter b	R^2adj
Raw returns	0.0003 (1.12)	0.6541*** (32.39)	0.2859
$r_D(\tau_1)$	-0.0000 (-0.00)	0.6222*** (31.22)	0.271
$r_D(\tau_1) + r_D(\tau_2)$	0.0000 (0.74)	1.1472*** (27.69)	0.2263

Source: Own calculations.

Notes: LJSEX→PX indicates that PX is a response variable and LJSEX is the explanatory variable. Other arrows are explained by analogy. Critical values of the two-sided t-statistics for rejection of the null hypothesis (i.e. regression parameter is equal 0) at 2,617 degrees of freedom are: 1.645 at the 10% significance level (indicated by *), 1.961 at the 5% significance level (indicated by **), and 2.578 at the significance 1% (indicated by ***).

As in the case of LJSEX, reconstructed scale 1 returns of PX exhibit smaller interdependence with foreign stock markets than do raw return series. PX 2 to 8 day return dynamics (i.e. scale 1 plus scale 2) also exhibit less comovement with foreign stock market returns, with exception of the Slovenian and Austrian stock market, than raw return series.

All parameter estimates of regression model (1) for BUX are significant (Table 6). At the aggregated (raw) return series, BUX volatility is best explained by PX volatility, followed by FTSE100 and DAX return volatility. Similar to PX, diversification benefits at the 2 to 8 day investment horizon seem to be greater than at the scale 1 horizon or at the daily horizon.

Tab. 6: Results of the estimation of the regression model (1), BUX is the dependent variable (part 1)

LJSEX→BUX			
	A constant	Parameter b	R^2adj
Raw returns	0.0001 (0.14)	0.4391*** (12.60)	0.0568
$r_D(\tau_1)$	0.0000 (0.00)	0.4032*** (9.98)	0.0363
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (5.25)	0.7026*** (13.32)	0.0631
PX→BUX			
	A constant	Parameter b	R^2adj
Raw returns	0.0000 (0.20)	0.6466*** (35.31)	0.3225
$r_D(\tau_1)$	0.0000 (0.00)	0.6066*** (33.49)	0.2997
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (4.88)	0.4253*** (30.58)	0.263

Tab. 6: Results of the estimation of the regression model (1), BUX is the dependent variable (part 2)

	ATX→BUX		
	A constant	**Parameter b**	**R^2adj**
Raw returns	0.0001 (0.38)	0.6294*** (29.63)	0.2509
$r_D(\tau_1)$	0.0000 (0.00)	0.5565*** (25.75)	0.2018
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (4.83)	0.4811*** (22.72)	0.1644
	CAC40→BUX		
	A constant	**Parameter b**	**R^2adj**
Raw returns	0.0002 (0.72)	0.5802*** (28.93)	0.2421
$r_D(\tau_1)$	0.0000 (0.00)	0.5173*** (26.53)	0.2116
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.78)	0.5863*** (21.43)	0.1489
	DAX→BUX		
	A constant	**Parameter b**	**R^2adj**
Raw returns	0.0002 (0.65)	0.5507*** (30.07)	0.2565
$r_D(\tau_1)$	-0.0000 (-0.00)	0.4956*** (28.24)	0.2333
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.70)	0.5034*** (18.85)	0.1192
	FTSE100→BUX		
	A constant	**Parameter b**	**R^2adj**
Raw returns	0.0003 (0.91)	0.704*** (30.00)	0.2556
$r_D(\tau_1)$	-0.0000 (-0.00)	0.623*** (27.31)	0.2214
$r_D(\tau_1) + r_D(\tau_2)$	0.0000*** (3.40)	0.8861*** (25.32)	0.1965

Source: Own calculations

Notes: LJSEX→BUX indicates that BUX is a response variable and LJSEX is the explanatory variable. Other arrows are explained by analogy.

As in the Czech case, the Hungarian stock market return volatility is more synchronized with developed European stock market volatility than the Slovenian stock market. Similar conclusions were also reported by studies of [21] and [22]. This finding can be attributed to the fact that the Czech and Hungarian stock markets have attracted many foreign investors [3], while the Slovenian stock market has struggled to do so. Further, the liquidity of shares listed on the Ljubljana stock exchange is significantly smaller than on the Prague and Budapest stock exchanges. According to [4], Ljubljana stock exchange equity turnover in 2010 was €0.7 billion, that of the Prague stock exchange €30.5 billion and that of the Budapest stock exchange €39.9 billion. As argued by [9], stock market liquidity can significantly explain stock market comovement. However, the authors investigate the comovement of daily return series and did not investigate return comovement at particular time scales. As noted by [53], the financial market consists of a variety of agents with different time horizons, and therefore it is postulated that market linkage could differ across time scales. Our findings confirm this – the strength of comovement between stock markets is scale dependent.

Comovement analysis should consider the distinction between short- and long-term investors [2]. The findings of the survey then have important implications for foreign financial investors who already hold international portfolios that exactly replicate those of non-Czech or non-Hungarian stock markets; international investing in the Czech or Hungarian stock markets with investment horizons corresponding to scale 2 (4 to 8 days) brings greater international diversification benefits than shorter (2 to 4 day horizon) international trading diversification strategies. When moving from raw (daily) to scale 1 (2 to 4 day) and scale 1 plus scale 2 (2 to 8 day) return dynamics, the comovement between stock markets reduces, but the advantages of international diversification grow. The Slovenian stock market differs from the Czech and Hungarian markets also in this respect, as when the scale is increased the benefits of diversification are reduced.

Conclusion

The studies of the interdependence of CEE stock markets with more developed European stock markets has so far predominantly applied correlation analysis, Granger causality tests, cointegration analysis and GARCH modelling. In this study we applied a novel approach of maximal overlap discrete wavelet transform to analyse multiscale stock market return volatility dynamics and return comovement between CEE (Slovenia, the Czech Republic and Hungary) and developed European stock markets (Austria, France, Germany and the UK). Two MODWT features are used for this purpose: MODWT energy decomposition, and additivity decomposition. The results of MODWT energy decomposition show that the first two scales of indices return series capture from 68 percent to 81 percent of the return series variability. We then applied methodology of [29] to study stock market comovement and found that the Czech and Hungarian stock markets comove more closely between themselves and the developed European markets than does the Slovenian stock market. The degree of comovement between the Austrian stock market and the Czech and Slovenian stock markets is higher than for other observed developed stock markets, probably due to historical reasons and strong economic ties. The unique finding of the study is that when moving from raw (daily) to scale 1 (2 to 4 day) and scale 1 plus scale 2 (2 to 8 day) return dynamics, the comovement between stock markets reduces, but the advantages of international diversification increase.

References

[1] BAE, K.H., KAROLYI, A.G. and STULZ, R.M. A new approach to measuring financial contagion. *The Review of Financial Studies*. 2003, Vol. 16, Iss. 13, pp. 717-763. ISSN 0893-9454.

[2] CANDELON, B., PIPLACK, J. and STRAETMANS, S. On measuring synchronization of bulls and bears: The case of East Asia. *Journal of Banking and Finance*. 2008, Vol. 32, Iss. 6, pp. 1022-1035. ISSN 0378-4266.

[3] CAPORALE, M.G. and SPAGNOLO, N. *Stock market integration between three CEEC's* [online]. Brunel University Working Paper No. 10-9, 2010. [cit. 2011-02-03]. Available from: http://www.brunel.ac.uk/9379/efwps/1009.pdf.

[4] CEEG – CEE Stock Exchange Group. *Fact sheet January 2011* [online]. Wien, 2011 [cit. 2011-20-05]. Available from: http://www.ceeseg.com.

[5] CHO, J.H. and PARHIZGARI, A.M. East Asian financial contagion under DCC-GARCH. *International Journal of Banking and Finance*. 2008, Vol. 6, Iss. 1, pp. 16-30. ISSN 1675-7227.

[6] CORNISH, R.C., BRETHERTON, C.S. and PERCIVAL, D.B. Maximal Overlap Discrete Wavelet Statistical Analysis with Application to Atmospheric Turbulence. *Boundary-Layer Meteorology*. 2006, Vol. 119, Iss. 2, pp. 339-374. ISSN 0006-8314.

[7] CRAIGMILE, P.F. and PERCIVAL, D.B. Asymptotic Decorrelation of Between-Scale Wavelet Coefficients. *IEEE Transactions on Information Theory*. 2005, Vol. 51, Iss. 3, pp. 1039-1048. ISSN 0018-9448.

[8] CROWLEY, M.P. *An intuitive guide to wavelets for economists* [online]. Bank of Finland Research Discussion Paper No. 1/2005, 2005. [cit. 2011-10-05]. 71 p. (PDF). Available from: http://www.suomenpankki.fi/en/julkaisut/tutkimukset/keskustelualoitteet/Documents/0501netti.pdf. ISBN 952-462-189-4.

[9] DIDIER T., LOVE, I. and MARTÍNEZ PERÍA, M.S. What explains comovement in stock market returns during the 2007-2008 crisis? *International Journal of Finance and Economic* [online]. 2011, Vol. 17, Iss. 2 [cit. 2011-05-05], pp. 182-202. ISSN 1099-1158.

[10] ÉGERT, B., KOČENDA, E. Time-varying synchronization of European stock markets. *Empirical Economics*. 2010, Vol. 40, Iss. 2, pp. 393-407. ISSN 0377-7332.

[11] EMBRECHTS, P., MCNEIL, A.J. and STRAUMANN, D. Correlation and Dependence in Risk Management: Properties and Pitfalls. In M.A.H. DEMPSTER (ed.). *Risk Management: Value at Risk and Beyond*. Cambridge: Cambridge University Press, 1999. pp. 176-223. ISBN 0-521-78180-9.

[12] FERNANDEZ, V. Time scale decomposition of price transmission in international markets. *Emerging Markets Finance and Trade*. 2005, Vol. 41, Iss. 4, pp. 57-90. ISSN 1540-496X.

[13] GARCIA, R. and TSAFACK, G. *Dependence structure and extreme comovements in international equity and bond markets* [online]. CIRANO Scientific Series. 2009. [cit. 2011-05-05]. 54 p. (PDF). Available from: http://www.cirano.qc.ca/pdf/publication/2009s-21.pdf. ISSN 1198-8177.

[14] GENÇAY, R., SELÇUK, F. and WHITCHER, B. Scaling properties of foreign exchange volatility. *Physica A: Statistical Mechanics and its Applications*. 2001, Vol. 289, Iss. 1-2, pp. 249-266. ISSN 0378-4371.

[15] GENÇAY, R., SELÇUK, F. and WHITCHER, B. Differentiating intraday seasonalities through wavelet multi-scaling. *Physica A*. 2001, Vol. 289, Iss. 3-4, pp. 543-556. ISSN 0378-4371.

[16] GENÇAY, R., SELÇUK, F. and WHITCHER, B. *An Introduction to Wavelets and Other Filtering Methods in Finance and Economics*. San Diego (CA): Academic Press, 2002. ISBN 0-12-27-9670-5.

[17] GENÇAY, R., SELÇUK, F. and WHITCHER, B. Systematic risk and timescales. *Quantitative Finance*. 2003, Vol. 3, Iss. 2, pp. 108-116. ISSN 1469-7688.

[18] GENÇAY, R., SELÇUK, F. and WHITCHER, B. Multiscale systematic risk. *Journal of International Money and Finance*. 2005, Vol. 24, Iss. 1, pp. 55-70. ISSN 0261-5606.

[19] GERRITS, R.J. and YUCE, A. Short- and long-term links among European and US stock markets. *Applied Financial Economics*. 1999, Vol. 9, Iss. 1, pp. 1-9. ISSN 0960-3107.

[20] GILMORE, G.C. and MCMANUS, G.M. International portfolio diversification: US and Central European equity markets. *Emerging Markets Review*. 2002, Vol. 3, Iss. 1, pp. 69-83. ISSN 1566-0141.

[21] HARRISON, B. and MOORE, W. Stock market comovement in the European Union and transition countries. *Financial Studies* [online]. 2009, Vol. 13, Iss. 3 [cit. 2011-05-05], pp.124-151. ISSN 2582-8654.

[22] HOROBET, A. and LUPU, R. Are Capital Markets Integrated? A Test of Information Transmission within the European Union. *Romanian Journal of Economic Forecasting*. 2009, Vol. 10, Iss. 2, pp. 64-80. ISSN 1582-6163.

[23] IN, F. and KIM, S. The hedge ratio and the empirical relationship between the stock and futures markets: A new approach using wavelet analysis. *Journal of Business*. 2006, Vol. 79, Iss. 2, pp. 799-820. ISSN 002-9398.

[24] IN, F., KIM, S., MARISETTY, V. and FAFF, R. Analyzing the performance of managed funds using the wavelet multiscaling method. *Review of Quantitative Finance and Accounting*. 2008, Vol. 31, Iss. 1, pp. 55-70. ISSN 1573-7179.

[25] KIM, S. and IN, F. The relationship between stock returns and inflation: new evidence from wavelet analysis. *Journal of Empirical Finance*. 2005, Vol. 12, Iss. 3, pp. 435-444. ISSN 0927-5398.

[26] KIM, S. and IN, F. A note on the relationship between industry returns and inflation through a multiscaling approach. *Finance Research Letters*. 2006, Vol. 3, Iss. 1, pp. 73-78. ISSN 1544-6123.

[27] KIM, S. and IN, F. On the relationship between changes in stock prices and bond yields in the G7 countries: Wavelet analysis. *Journal of International Financial Markets, Institutions and Money*. 2007, Vol. 17, Iss. 2, pp. 167-179. ISSN 1042-4431.

[28] KOEDIJK, K., CAMPBELL, A.J.R. and KOFMAN, P. Increased correlation in bear markets. *Financial Analysts Journal*. 2002, Vol. 58, Iss. 1, pp. 87-94. ISSN 0015-198X.

[29] LEE, H.S. Price and volatility spillovers in stock markets: A wavelet analysis. *Applied Economics Letters*. 2004, Vol. 11, Iss. 3, pp.197-201. ISSN 1466-4291.

[30] LING, X. and DHESI, G. Volatility spillover and time-varying conditional correlation between

the European and US stock markets. *Global Economy and Finance Journal*. 2010, Vol. 3, Iss. 2, pp. 148-164. ISSN 0972-9496.
[31] LONGIN, F. and SOLNIK, B. Is the correlation in international equity returns constant: 1960–1990? *Journal of International Money and Finance*. 1995, Vol. 14, Iss. 1, pp. 3-26. ISSN 0261-5606.
[32] MALLAT, S.G. and ZHANG, Z. Matching pursuits with time-frequency dictionaries. *IEEE Transactions of Signal Processing*. 1993, Vol. 41, Iss. 12, pp. 3397-3415. ISSN 1053-587X.
[33] MALLIARIS, A.G. and URRUTIA, J.L. The international crash of October 1987: Causality tests. *Journal of Financial and Quantitative Analysis*. 1992, Vol. 27, Iss. 3, pp. 353-364. ISSN 0022-1090.
[34] NECULA, C. Modeling the dependency structure of stock index returns using a copula function. *Romanian Journal of Economic Forecasting*. 2010, Vol. 13, Iss. 3, pp. 93-106. ISSN 1582-6163.
[35] PAKKO, M.R. A spectral analysis of the cross-country consumption correlation puzzle. *Economics Letters.* 2004, Vol. 84, Iss. 3, pp. 341-347. ISSN 0165-1765.
[36] PATEV, P., KANARYAN, N. and LYROUDI, K. Stock market crises and portfolio diversification in Central and Eastern Europe. *Managerial Finance*. 2006, Vol. 32, Iss. 5, pp. 415-432. ISSN 0307-4358.
[37] PERCIVAL, D.B. On the Estimation of the Wavelet Variance. *Biometrika*. 1995, Vol. 82, Iss. 3, pp. 619-631. ISSN 0006-3444.
[38] PERCIVAL, D.B. Analysis of Geophysical Time Series Using Discrete Wavelet Transforms: An Overview. In DONNER, R.V. and BARBOSA, S.M. (Eds.). *Nonlinear Time Series Analysis in the Geosciences. Applications in Climatology, Geodynamics and Solar-Terrestrial Physics*. Berlin/ /Heidelberg: Springer, 2008. pp. 61-79. ISBN 3-540-78937-5.
[39] PERCIVAL, D.B. and MOJFELD, H.O. Analysis of subtidal coastal sea level fluctuations using wavelets. *Journal of the American Statistical Association*. 1997, Vol. 92, Iss. 439, pp. 868-880. ISSN 0162-1459.
[40] PERCIVAL, D.B. and WALDEN, A.T. *Wavelet Methods for Time Series Analysis*. New York: Cambridge University Press, 2000. ISBN 0-521-6406-7.
[41] PINHO, C. and MADALENO, M. *Time frequency effects on market indices: world comovements* [online]. Aveiro/Paris, Finance International Meeting AFFI – EUROFIDAI Paper, 2009 [cit. 2011-30-05]. 46 p. (PDF). Available from: http://www.affi.asso.fr/uploads/Externe/15/CTR_FICHIER_422_125914 7300.pdf.
[42] RAMSEY, J. Regression over Timescale Decompositions: A Sampling Analysis of Distributional Properties. *Economic Systems Research*. 1999, Vol. 11, Iss. 2, pp.163-183. ISSN 0953-5314.
[43] RUA, A. and NUNES, L.C. International comovement of stock market returns: a wavelet analysis. *Journal of Empirical Finance*. 2009, Vol. 16, Iss. 4, pp. 632-639. ISSN 0927-5398.
[44] SCHEICHER, M. The comovements of stock markets in Hungary, Poland and the Czech Republic. *International Journal of Finance & Economics*. 2001, Vol. 6, Iss. 1, pp. 27-39. ISSN 1076-9307.
[45] SCHWENDER, A. *The estimation of financial markets by means of regime-switching model* [online]. St. Gallen, 2010. 147 p. Dissertation. University of St. Gallen, Graduate School of Business Administration, Economics, Law and Social Sciences, No. 3794 [cit. 2011-10-02]. Available from: http://www.iorcf.unisg.ch/de/Forschung/Publikationen/~/media/Internet/Content/Dateien/InstituteUndCenters/IORCF/Abschlussarbeiten/Schwendener%202010%20Diss%20The%20Estimation%20of%20Financial%20Markets%20by%20Means%20of%20a%20Regime%20Switching%20Model.ashx.
[46] SERROUKH, A., WALDEN, A.T. and PERCIVAL, D.B. Statistical Properties and Uses of the Wavelet Variance Estimator for the Scale Analysis of Time Series. *Journal of the American Statistical Association*. 2000, Vol. 95, Iss. 449, pp. 184-196. ISSN 0162-1459.
[47] SERWA, D. and BOHL, M.T. Financial Contagion Vulnerability and Resistance: A Comparison of European Stock Markets. *Economic Systems*. 2005, Vol. 29, Iss. 3, pp. 344-362. ISSN 0939-3625.
[48] SHARKASI, A., RUSKIN, H. and CRANE, M. Interrelationships among international stock market indices: Europe, Asia and the Americas. *International Journal of Theoretical and Applied Finance*. 2005, Vol. 8, Iss. 5, pp. 1-18. ISSN 0219-0249.
[49] SYLLIGNAKIS, M. and KOURETAS, G. *Long And Short-Run Linkages In CEE Stock Markets: Implications For Portfolio Diversification And Stock Market Integration* [online]. William Davidson Institute Working Papers Series, 2006 [cit. 2011-10-02]. 33 p. (PDF). Available from: http://wdi.umich.edu/files/publications/workingpapers/wp832.pdf.
[50] TSE, Y.K and TSUI, A.K. A Multivariate Generalized Autoregressive Conditional Heteroscedasticity Model with Time-Varying Correlations. *Journal of Business and Economic Statistics*. 2002, Vol. 20, Iss. 3, pp. 351-362. ISSN 0735-0015.

[51] VUORENMAA, T.A. *A wavelet analysis of scaling laws and long-memory in stock market volatility* [online]. Helsinki: Bank of Finland Research Discussion Paper 27, 2005 [cit. 2011-10-05]. 44 p. (PDF). Available from: http://www.suomenpankki.fi/en/julkaisut/tutkimukset/keskustelualoitteet/Documents/0527netti.pdf. ISBN 952-462-254-8.
[52] XIAO, L. and DHESI, G. Volatility spillover and time-varying conditional correlation between the European and US stock markets. *Global Economy and Finance Journal*. 2010, Vol. 3, Iss. 2, pp. 148-164. ISSN 1834-5883.
[53] ZHOU, J. Multiscale analysis of international linkages of REIT returns and volatilities. *Journal of Real Estate Financial Economics* [online]. 2011-02-25. Online First [cit. 2011-05-20]. ISSN 0895-5638.

Silvo Dajčman, Ph. D.
University of Maribor
Faculty of Economics and Business
Department of Finance
silvo.dajcman@uni-mb.si

Alenka Kavkler, Ph. D.
University of Maribor
Faculty of Economics and Business
Department of Quantitative Economic Analysis
alenka.kavkler@uni-mb.si

Abstract

WAVELET ANALYSIS OF STOCK RETURN ENERGY DECOMPOSITION AND RETURN COMOVEMENT – A CASE OF SOME CENTRAL EUROPEAN AND DEVELOPED EUROPEAN STOCK MARKETS

Silvo Dajčman, Alenka Kavkler

In this article we investigate comovement of the three Central and Eastern European (CEE) stock markets (Slovenia, the Czech Republic and Hungary) with certain developed European stock markets (Austria, France, Germany and the United Kingdom) through the novel approach of maximal overlap discrete wavelet transform (MODWT). We use two features of MODWT to explore energy decomposition of stock market returns at different time scales and to apply methodology of [29] to study comovement between investigated stock markets. We show that most of the energy (variability) of stock market return series is captured by scale 1 (which correspond to 2–4 days return dynamics) and scale 2 (which correspond to 4–8 days return dynamics) MODWT coefficients. MODWT details are used to show that comovement between stock markets is scale-dependent and declines from raw (daily) return series to first- and second-scale reconstructed return series. The findings of the survey then have important implications for foreign financial investors who already hold international portfolios that exactly replicate those of non-Czech or non-Hungarian stock markets: international investing in the Czech or Hungarian stock markets with investment horizons corresponding to scale 2 (4 to 8 days) brings greater international diversification benefits than shorter (2 to 4 day horizon) international trading diversification strategies. The Slovenian stock market differs from the Czech and Hungarian markets also in this respect, as when the scale is increased the benefits of diversification are reduced. We also find that the volatility of Slovenian stock index returns is less synchronized with other observed stock return series. Interestingly, the Czech and Slovenian stock markets seem to comove with the Austrian stock market to a greater extent than with other developed stock markets.

Key Words: *Central and Eastern Europe, stock market returns, comovement, wavelets.*

JEL Classification: *F21, F36, G11, G15.*

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