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Mean velocity profiles in a boundary layer under the joint action of surface roughness and external turbulent flow

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Abstract

This paper considers the knowledge of the individual action and joint action of surface roughness and external flow turbulence on the mean flow in boundary layer. The experimental evidence of this problem has been reviewed. A lack of results has been ascertain of the investigation on the joint action of the mentioned influences on the development of a boundary layer from the state with laminar flow up to a turbulent boundary layer. The knowledge on the actions of individual effects has been gathered with the regard to the improvement of the evaluation and analysis of the mean flow characteristics of the zero pressure gradient boundary layer developing under the joint action of the uniform roughness of the surface and homogeneous, close to isotropy, free stream turbulence.
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Keywords: boundary layer on rough surface, turbulent flow, the roughness function

1. Introduction

Roughness elements distributed over a surface (WR) and free stream turbulence (SFT) accelerate the laminar turbulent boundary layer transition in comparison with the boundary layer on a smooth surface under non-turbulent flow at otherwise equal conditions. The individual action and the joint action of both effects thus speeds up the boundary layer development from the laminar structure into self preserving turbulent boundary layer. Therefore a deeper understanding of these phenomena may be important in many environmental and technical areas. Experimental investigations of the effects in question are beneficial even if they are individually acting. The authors assembled partial and general knowledge on flow over rough solid surface namely from monographs and severe papers from [1, 2, 3, 15, 21, 26, 28, 29, 30, 33, 36]. Investigations of the effect of free stream turbulence on turbulent boundary layer were very favoured in seventies and eighties of 20$^{\text{th}}$ century, e.g. [5, 9, 10, 11, 14, 18, 22]. The effect of external flow turbulence on boundary layer laminar-turbulent transition is continuously studied since the Second Word War time (e.g. [35]) over countless number of contributions up to now, e.g. [4, 6, 12, 16, 17, 18, 19, 23, 24, 31]. So far the authors are acquainted only with the investigation of the joint action of the mentioned influences (WR and FST) on the laminar layer and its transition published by Gibbins and Al-Shukri [7, 8].

The aim of the contribution is to improve evaluation and analysis of the mean flow characteristics of the zero pressure gradient boundary layer developing under the influence of the individual action and the joint action of the uniform roughness of the surface and homogeneous, close to isotropy, free stream turbulence.

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2. Preliminary analysis

For the sake of simplifying the subsequent considerations we shall handle a boundary layer on a flat plate generally with a rough surface under a zero pressure gradient turbulent flow. Many geometrical forms of the surface roughness are possible but here, the so called K-type roughness will be tested only (e.g. [15]), namely the surface homogeneously covered with roughness elements (sand paper) will be considered. It is very close to the so called *sand roughness* or *standard roughness* characterized by the surface roughness length s or by the roughness Reynolds number R_s

$$R_s = \frac{s\bar{U}_e}{\nu} \tag{1}$$

Many kinds of flow turbulence structure occur in the technical practice however here only the external flow turbulence, homogeneous close to isotropy generated by means of a grid/screen is assumed. The mean velocity $\bar{U}_e$, intensity Iu_e and dissipation length parameter L_e characterise this grid turbulence

$$Iu_e = \frac{\sqrt{\left(\overline{u^2}\right)_e}}{\bar{U}_e}; \qquad L_e = -\frac{\left(\overline{u^2}\right)_e}{\bar{U}_e \frac{\mathrm{d}\left(\overline{u^2}\right)_e}{\mathrm{d}x}} \tag{2}$$

The aim of present analysis is accumulate knowledge beneficial for the improvement of the evaluation of experiments performed by Jonáš et al. [20].

The boundary layer is laminar near its onset $x = 0$ at the leading edge. Roughness grains submerged in the layer cause local pressure distributions, local flow separations- wakes composed of counter-rotating vortex pairs resulting on the one hand in local form drags which act as a part of tangential forces exerted on the surface together with the viscous wall shear stress $\mu(\partial\bar{U}/\partial y)_{y=0}$. On the other hand they cause random flow disturbances inside the laminar layer. Another disturbances penetrate into the layer from the external turbulent flow. Both types of disturbances inside the laminar boundary layer generate two dimensional instability waves, TS waves downstream from the vicinity of the leading edge. Initially the waves are suppressed by the action of viscosity. The waves begin to be amplified and the development of by-pass transition follows in accord with the known scenario after arriving the value of the indifference Reynolds number (minimal coordinate Re of points on the curve of neutral stability, e.g. [33] declare the displacement thickness Reynolds number $(Re_1)_{ind} = 520$ in case of a smooth surface and small Iu_e and a decrease of $(Re_1)_{ind}$ with increasing Iu_e and roughness). Hence, the occurrence of laminar boundary layer on a rough surface in a turbulent flow is basically necessary, at least infinitesimally long laminar layer survives in the vicinity of the leading edge $x = 0$. Blasius solution (e.g. [32]) is describing the velocity field of the assumed layer developing on a smooth plate in the nonturbulent flow. Does the effect of surface roughness and outer stream turbulence modify the mean velocity field?

Usually the effects of surface roughness and outer stream turbulence on laminar boundary layer are investigated separately more often as a part of laminar turbulent transition studies. The joint action of the mentioned effect on the laminar layer and its transition is discussed in [8] only. Gibbins and Al-Shukri performed experimental investigation of laminar boundary layer on smooth surface and two external flow turbulence levels ($Iu_e = 0.8\,\%$, and 2.6 %) and on two rough surfaces (equivalent roughness height $s = 0.105$ mm and 0.130 mm) under external flow with the turbulence level $Iu_e = 1.8\,\%$. The local Reynolds number Re_x was in the limits from $2 \cdot 10^4$ up to $5.5 \cdot 10^5$. They interpolated the measured distributions of the displacement thickness δ_1 and the momentum thickness δ_2 versus Re_x. The distributions retained the Blasius slope of -0.5. Authors derived from the regressions of δ_1 and δ_2 that the increase of both the roughness and the turbulence level reduce the shape factor $H_{12} = \delta_1/\delta_2$ from the Blasius value 2.6. However the plausibility of this conclusion degrades, as the calculated relative differences H_{12} from 2.6 are considerably smaller than the sum of the relative errors σ_i of the interpolations of δ_ι.

Dyban and Epik [5] investigated the effect of external flow turbulence (Iu_e from 0.05 % up to 12.5 %) on laminar boundary layer developing on the smooth surface. They observed an increase of the local wall friction and thickening the layer and qualitatively found a decrease of the shape parameter H_{12} with the increasing turbulence level Iu_e. But an extensive complex of results (see Fig. 50 in [5]) confirms that H_{12} retains the Blasius value in the range of Re_x from $5 \cdot 10^3$ up to $5 \cdot 10^6$ and turbulence level up to 25 %(!).

The same conclusion on the effect of the external flow turbulence level follows from the precise experiments presented in the paper [19]. As a by pass result they ascertained that mean velocity profiles determined upstream from the indifference Reynolds number at a location x are (in the limits of measuring accuracy) *identical* to Blasius profiles but belonging to a little bit larger distance $x' = x + \Delta x$; $\Delta x/x \sim 10^{-2} \div 10^{-1}$.

Kendall [21] found that the most evident result of surface roughness was a displacement of laminar profile outward from the plane, on which are roughness grains stuck, by a distance, which turns out to be significantly greater than the volumetric average thickness of grains creating the roughness. More useful notions were not found on the mean flow characteristics in the laminar boundary layer affected by the actions of both the wall roughness and the external flow turbulence.

The sole action of the effect of the surface roughness appears in an acceleration of the laminar turbulent transition i.e. under otherwise equal conditions, the transition occurs at smaller distance from the onset of boundary layer for a rough surface than for a smooth one. A great ratio of the wall roughness to boundary layer thickness can suppress the value of the critical Reynolds number up to one tenth of that for a smooth wall (e.g. [33]) and thus dramatically shorten the piece of laminar boundary layer occurrence. After finishing the process of laminar turbulent transition boundary layer becomes turbulent. With the regard to the effect of viscosity, a turbulent boundary layer on smooth or rough surface could be divide after universal features of the flow structure in two parts: the *inner layer* ($y \,\tilde{<}\, 0.1\delta$) and the *outer layer* ($y > 50\delta_v$). The inner layer is attached to the surface where molecular together with turbulent momentum transfers act, having the relevant length scale δ_v

$$\tau_w = \mu \left(\frac{\partial \bar{U}}{\partial y} \right)_w ; \qquad u_\tau = \sqrt{\frac{\tau_w}{\rho}}; \qquad \delta_v = \frac{\nu}{u_\tau} \tag{3}$$

where the nomenclature is introduced: $\tau_w(x)$, the local wall shear stress, u_τ, the friction velocity and symbols μ, ν and ρ denote molecular viscosity, kinematics viscosity and density of fluid. The direct effect of molecular viscosity on flow dynamics is negligible in the outer layer and the boundary layer thickness δ is the relevant length scale there

$$\frac{\bar{U}(\delta)}{U_e} = 0.99_5 \tag{4}$$

Components of the inner layer are the viscous sub-layer ($y < 5\delta_v$), buffer layer ($5 < y/\delta_v < 30$) and the overlap region ($50\delta_v < y < 0.1\delta$) if $\delta/\delta_v \gg 1$ at the outer edge. As indicates the label "overlap" that region is an overlap between inner and outer layers, the *inertial sub-layer* after [36], where $y \gg \delta_v$ and simultaneously $y \ll \delta$, the only relevant length scale is then the distance from the wall y. This region is a part of the log-law region ($30\delta_v \tilde{<} y \tilde{<} 0.3\delta$). The important knowledge might be received on mean velocity profiles considering the mean flow momentum equation and physical features of specified layers. Thus the mean velocity in the vicinity of $y = 0$ can be derived (no slip condition is valid at $y = 0$)

$$\frac{\bar{U}}{u_\tau} = u^+ = \frac{y u_\tau}{\nu} = y^+, \qquad 0 \leq y^+ \tilde{<} 5 \tag{5}$$

Farther from the surface at large Reynolds number, the viscosity has little effect on the flow dynamics. The velocity derivative $\mathrm{d}\bar{U}/\mathrm{d}y$ becomes dynamically important quantity as controls the viscous stress and turbulence production. It is a decreasing function with the increasing distance from the surface y. From the dimensional analysis follows that the distance y is the relevant length

$$\frac{\mathrm{d}\bar{U}}{\mathrm{d}y} = \frac{u_\tau}{y} F\left(\frac{y}{\delta_v}, \frac{y}{\delta}, \frac{s}{\delta_v}, \frac{\sqrt{\left(\overline{u^2}\right)_e}}{u_\tau}, \frac{L_e}{\delta}\right) \tag{6}$$

where F is a universal non-dimensional function. Presumably the derivative (6) is a finite monotone function at least in the region $0 \leq y \leq \delta$. Then it must be asymptotically independent of very large or very small parameters. Let us focus on the overlap region.

The estimates of the magnitude of the parameters characterizing the intensity of external turbulence velocity fluctuations are as follows

$$\begin{aligned} &0 < \frac{\sqrt{\left(\overline{u^2}\right)_e}}{u_\tau} = Iu_e \frac{\bar{U}_e}{u_\tau} = O\left(10^{-1} \div 10^0\right) <\sim 3; \\ &Iu_e = O\left(10^{-2} \div 10^{-1}\right); \qquad 0 < \frac{L_e}{\delta < \eta_L} \sim O\left(10^0\right) \end{aligned} \tag{7}$$

if the intensity of external flow turbulent fluctuations ($Iu_e < 0.1$) is not too large by comparison with the friction velocity, the free stream turbulence is indistinguishable from the turbulence generating inside the layer in the inertial sub-layer (e.g. [9]). Similarly it is hard to imagine a direct action of large turbulent eddies (dimensions L_e) from external flow on the inner layer. Thus the external turbulence does not affect the mean flow in the inertial region. This conclusion was certified by numerous experiments e.g. [9, 10, 11, 17, 18].

Comparing the surface roughness length s with the viscous length δ_v three cases can be distinguished for the dimensional estimates

$$\text{(a):} \quad s \ll \delta_v; \qquad \text{(b):} \quad s \gg \delta_v; \qquad \text{(c):} \quad 5\delta_v \tilde{<} s \tilde{<} 70\delta_v \ll \delta \tag{8}$$

The surface behaves as a hydraulically smooth one (a) below the lower limit of s because the roughness grains are submerged in the viscous sub-layer and viscosity suppresses their action. Behind the upper limit of s the surface behaves as fully rough (b) and the roughness grains generate turbulent wakes that cause an inviscid drag on the surface. The case (c) corresponds with the so-called *transient roughness*. A bounded layer cannot exist if the roughness grains are so large that they are not hidden into the layer. In the limits of inequalities (8), the following estimates are valid from the inner layer side

$$
\begin{aligned}
&\text{(a)} \quad \frac{y}{u_\tau}\frac{\mathrm{d}\bar{U}}{\mathrm{d}y} = F_S(y^+) \\
&\text{(b)} \quad \frac{y}{u_\tau}\frac{\mathrm{d}\bar{U}}{\mathrm{d}y} = F_R\left(\frac{y}{s}\right) \\
&\text{(c)} \quad \frac{y}{u_\tau}\frac{\mathrm{d}\bar{U}}{\mathrm{d}y} = F_1(y^+, s^+); \quad \frac{y}{\delta} \ll 1; \quad \frac{y}{\delta_v} = y^+ > 30; \quad \frac{s}{\delta_v} = s^+ \sim O(10^0 \div 10^1)
\end{aligned}
\tag{9}
$$

From the outer layer side, the direct effects of viscosity and surface roughness are negligible but the action of external turbulence must be considered so they are valid estimates (7) together with

$$1 \geq \frac{y}{\delta} = \eta > 0.1, \quad \frac{y}{\delta_v} = y^+ \gg 1, \quad \frac{s}{\delta} \ll 1, \quad \frac{y}{u_\tau}\frac{\mathrm{d}\bar{U}}{\mathrm{d}y} = F_2(\eta, \eta_L, Iu_e) \tag{10}$$

The requirement of identical derivatives (9) and (10) must be executed in the overlap region. Because the functions F_S, F_R, F_1 and F_2 have no joint independent variables, the equations can be satisfied (matching) only if the functions adopt a constant value $1/\kappa$. The equations (9) (a, b, c) integrate to the log law

$$
\begin{aligned}
&\text{(a)} \quad \frac{\bar{U}}{u_\tau} = \frac{1}{\kappa}\ln(y^+) + B_S; \qquad B_S = 5.0 \\
&\text{(b)} \quad \frac{\bar{U}}{u_\tau} = \frac{1}{\kappa}\ln\left(\frac{y}{s}\right) + B_R; \qquad B_R = 8.0 \\
&\text{(c)} \quad \frac{\bar{U}}{u_\tau} = \frac{1}{\kappa}\ln(y^+) + B_1\left(\frac{s}{\delta_v}\right)
\end{aligned}
\tag{11}
$$

The values of constants κ, B_S and B_R were determined from experiments for the limiting cases of wall roughness, e.g. [33]. Another authors use little different values e.g. $\kappa = 0.4$, $B_S = 5.1$, $B_R = 8.5$ in [15] and $\kappa = 0.41$, $B_S \cong 5$, $B_R = 8.5$ after [2 and 38].

Subtracting velocities in limiting cases (11) at the same coordinate y^+ we receive

$$(u^+)_R - (u^+)_S = -\frac{1}{\kappa}\ln s^+ + B_R - B_S < 0 \text{ if } s > 4.2\delta_v, \quad s^+ = \frac{s}{\delta_v}, \tag{12}$$

Apparently, the surface roughness causes a downward shift in the log-law. According to this it is customary write the case (c) of (11) in the form

$$\frac{\bar{U}}{u_\tau} = \frac{1}{\kappa}\ln(y^+) + B_S - \Delta u^+(s^+) \tag{13}$$

Once the function of the roughness Δu^+ is known for the given surface it can be used for the friction loses calculations of any surface with the same roughness, [29].

The equation (10) integrates (from $y = \delta$ up to y) to the mean velocity defect law

$$\frac{\bar{U}_e - \bar{U}(y)}{u_\tau} = -\frac{1}{\kappa}[\ln(\eta) + B_2(\eta, \eta_L, Iu_e)] \tag{14}$$

The overlap needs to match the formulae (14) with the log-law (13). We receive in the inertial region

$$\frac{\bar{U}_e - \bar{U}(y)}{u_\tau} = -\frac{1}{\kappa}\ln(\eta) + \left(\frac{\bar{U}_e}{u_\tau} - \frac{1}{\kappa}\ln\delta^+ - B_S + \Delta u^+\right) \tag{15}$$

Perturbations of the velocity defect law from the log-law distribution occur farther from the overlap region in external stream direction. Thus already Coles tabulated the wake function, assumed to be the same for all boundary layers on smooth surfaces in non turbulent flows with arbitrary pressure gradients. From the experimental evidence follows: the velocity defect profiles measured on rough surface in non turbulent flow differ weakly from those *universal* on the smooth surface if normalized by the friction velocity (15), e.g. [2]. However the wake functions in boundary layers on smooth surface under turbulent flow display a strong dependence on the external turbulence scales, e.g. [9, 17, 27]. Thus the introduction of the generalized form of the wake function f is necessary

$$\frac{\bar{U}_e - \bar{U}(y)}{u_\tau} = -\frac{1}{\kappa}\ln(\eta) + \left(\frac{\bar{U}_e}{u_\tau} - \frac{1}{\kappa}\ln\delta^+ - B_S + \Delta u^+\right) - \frac{\Pi}{\kappa}[2 - f(\eta, \eta_L, Iu_e)] \tag{16}$$

where $\Pi(x)$ is called the wake strength parameter and f is the generalized wake function (usually denoted by W if it is depending on η only). The function f must undertaken conditions

$$\begin{gathered} f(0, \eta_L, Iu_e) = 0, \quad (\mathrm{d}f/\mathrm{d}\eta)_{\eta=0} = 0, \quad f(1, \eta_L, Iu_e) = 2, \\ (\mathrm{d}f/\mathrm{d}\eta)_{\eta=1} = 0, \quad \int_0^1 f(\eta, \eta_L, Iu_e)\,\mathrm{d}\eta = 1 \end{gathered} \tag{17}$$

Next the Coles' law of the wake will be derived after some formal arrangement

$$u^+ = \frac{1}{\kappa}\ln y^+ + B_S - \Delta u^+ - \frac{\Pi}{\kappa} f(\eta, \eta_L, Iu_e) \tag{18}$$

This law holds from the inertial layer up to the periphery of the boundary layer. It should be noted that no approximation of the wake function $f(\eta, \eta_L, Iu_e)$ is so far known for the investigated boundary conditions. Substituting the first member on the right hand side in (14) by means of (13) (process of matching) the local skin friction coefficient C_f is derived

$$C_f = 2\left(\frac{u_\tau}{\bar{U}_e}\right)^2, \quad \sqrt{\frac{2}{C_f}} = \frac{\bar{U}_e}{u_\tau} = \frac{1}{\kappa}\ln Re_1 + B_S - \Delta u^+(s^+) + B_2(\eta_L, Iu_e) \tag{19}$$

Fruitful discussion of the effect of external turbulence can start from this expression. Let us assume a given value of Re_1 and start from the simplest configuration, from the case with a smooth wall under non turbulent flow. Then the equation (19) represents a linear relation on $\ln Re_1$and the skin friction coefficient has the value $(C_f)_0$. The surface roughness will influence the value of C_f as follows: from the estimates (12) it is known that Δu^+ is increasing with roughness s^+ and thus the skin friction coefficient will increase $C_f(s^+)/(C_f)_0 > 1$. Published results [5, 9, 11, 17, 18] etc. on the effect of external turbulence on turbulent boundary layer proved that the skin friction coefficient increases with the turbulence level e.g. [18]

$$\frac{C_f(\eta_L, Iu_e)}{(C_f)_0} = 10.9\Phi(\eta_L, Iu_e)(1 \pm 0.03), \qquad \Phi(\eta_L, Iu_e) = \frac{2Iu_e}{\frac{L_e}{\delta + 5}} \tag{20}$$

where $\Phi(\eta_L, Iu_e)$ is the modification of the parameter originally proposed by [10]. From (20) we can deduce that the parameter B_2 in formulas (14) up (19) must decrease with the increasing $\Phi(\eta_L, Iu_e)$ and the effect of the external turbulence on B_2 may be described by Φ only.

The weak point of all presented formulations of mean velocity profiles in a layer on a rough surface is the necessity determine the effective zero-plane displacements e.g. from the level of upper parts of the biggest grains, Δy in addition to the evaluation of u_τ and Δu^+. The shift Δy on the level, where the mean velocity equals zero, is a small fraction of the roughness height s

$$0.15 < \frac{\Delta y}{s} < 0.3 \rightarrow \bar{U}(-\Delta y) = 0 \tag{21}$$

The limits of the inequalities are borrowed from the paper [2]. Thence the ratio of the shift Δy to the boundary layer thickness δ (position y_δ) is of the order few hundredth. Therefore the effective zero-plane displacement is of a small importance for evaluation of some flow characteristics.

3. Experimental set-up and primary flow characteristics

The flat plate boundary layer was investigated in the close circuit wind tunnel IT AS CR, Prague $(0.5 \times 0.9)\,\mathrm{m}^2$. The boundary layer develops on an aerodynamically smooth plate (2.75 m long and 0.9 m wide) made from a laminated wood-chip board 25 mm thick in the primary configuration. The scheme of the working section and the introduction of the orthogonal co-ordinate system $[x, y, z]$ are shown in Figure 1. Rough plate was made from a thin plywood plate (7 mm thick) with sandpaper stuck on its surface. It has the elliptic leading edge L.E. ($a \times b = 60\,\mathrm{mm} \times 20\,\mathrm{mm}$) and it is attached to the surface of the primary plate so as cover the primary L. E. Rough surface starts 33 mm downstream from the nose of L.E. The maximum size of grains on sandpaper was chosen as the representative length of roughness s. The height of peaks of roughness grains is s (grits 80) $= (0.343 \pm 0.009)\,\mathrm{mm}$. Square mesh ($M$) plane grids — screens with cylindrical rods (D) across the external flow in the distance x_G upstream of the leading edge ($x = 0$) of the plate with the investigated boundary layer were producing the external flow turbulence. For more details on the experimental facility and characteristics of the generated turbulence see e.g. [18, 19, 20].

So far experiments were limited only to the measurement of mean velocity profiles $U(y)$ at the external flow mean velocity magnitude $U_e \approx 5$ m/s. High accuracy and sensitivity of the available pressure transducers, in particular BARATRON, enabled us replace the laborious hot-wire measurements [19] with measurements by means of a flattened Pitot probe. The couple of the flattened Pitot probe $(0.18 \times 2.95\,\mathrm{mm}^2)$ and round nosed static pressure probe ($\Phi = 0.18\,\mathrm{mm}$) are outlying 55 mm in the y-direction. They are connected to the pressure transducer

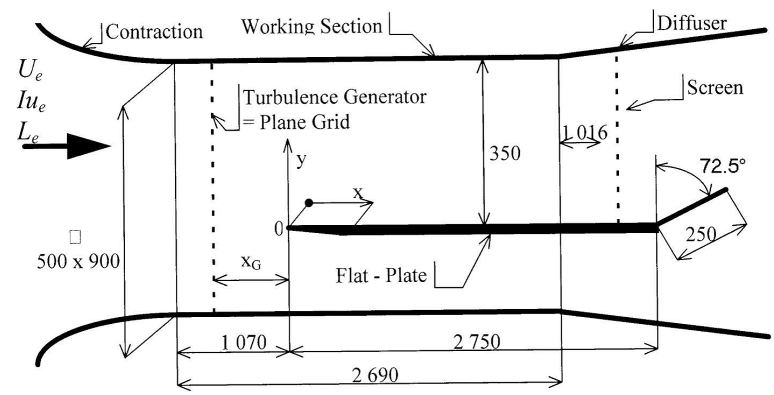

Fig. 1. Working section of the wind tunnel $(0.5 \times 0.9)\,\mathrm{m}^2$

BARATRON (special order on high accuracy, max 1 kPa; $\pm 0.02\,\%$ of reading above 20 Pa). The profiles of the local dynamic pressure $q'(x, y) = (P_0 - P)$ were measured in the plane $z = 0$. Homogeneity in the spanwise direction was checked formerly [19].

Representative pressure q_r [Pa] and absolute static pressure P [Pa] were measured using Pitot-static tube (diameter = 6 mm, the nose located in the point $[0.225\,\mathrm{m}, 0.13\,\mathrm{m}, -0.36\,\mathrm{m}]$) connected to the pressure transducer OMEGA Techn. Ltd. (max 1.2 kPa; $\pm 0.25\,\%$ FS). Static holes are connected to the transducer Druck DPI 145 (max 100 kPa; $\pm 0.005\,\%$ FS). The readings of q_r and P were done simultaneously with the measurement of the local dynamic pressure $q'(x, y)$ [Pa] as to avoid errors caused by small and slow variations of the external flow velocity U_e. Following correction of this effect was applied

$$q(x, y) = 0.5\rho U^2(x, y) = \frac{q'(x, y)\tilde{q}_r}{q_r}; \quad \tilde{q}_r = \text{Average of } (q_r)(1 \pm 0.006) \tag{22}$$

The additional correction of the total pressure P_o (Pitot tube) reading was made after MacMillan. Thermometer Pt 100 connected to the Data Acquisition/Switch Unit HP 34970A was measuring the flow temperature t [°C]. Output signals proportional to the mean values of P, q_r and t were read by means of the unit HP 34970A just after start of the observation in each point $[x, y, 0]$. Afterwards the simultaneous reading by the HP unit and 30 s averaging of signals proportional to q_r and $q'(x, y)$ followed. The data were recorded in a personal computer after the end of every observation.

Estimates of upper limits of relative measurement errors of the fundamental characteristics were derived from the accuracy of devices and with the regard to the scatter of repeated observations

$$\frac{\Delta q_r}{q_r} \leq \pm 0.02 \text{ at } U_r \cong 5\,\mathrm{m/s}; \quad \frac{\Delta q}{q} \leq \pm 0.02 \text{ at } U(x, y) \geq 0.6\,\mathrm{m/s}; \quad \Delta P \simeq \pm 5\,\mathrm{Pa} \tag{23}$$

The absolute error of the local dynamic pressure at higher local velocity $U(x, y)$ remains constant, about ± 0.005 Pa, i.e. on the level at $U \approx 0.6$ m/s. The analysis of results is based on integral characteristics — displacement thickness δ_1, momentum thickness δ_2 and shape parameter $H_{12} = \delta_1/\delta_2$. The necessary integrations were done using the trapezium rule. Errors estimates of the mean velocity, displacement ($i = 1$) and momentum ($i = 2$) thickness and the shape factor follow from the estimates (23)

$$\frac{\Delta U_r}{U_r} \simeq 0.015, \quad \frac{\Delta U}{U(x, y)} \tilde{\leq} 0.01, \quad \frac{\Delta \delta_i}{\delta_i} \approx 0.015, \quad \frac{\Delta H_{12}}{H_{12}} \approx 0.03 \tag{24}$$

4. Determination of the wall friction from the mean flow profile

Boundary layer experimental investigation in a long enough region (in the stream-wise direction) is very desirable. “Long enough” means start the measurement in the location with laminar boundary layer, $Re_1 \sim O(10^2)$ and finish the measurements in the location with fully developed — self sustaining turbulent boundary layer, $Re_1 \sim O(10^3)$.

Often the wall shear stress $\tau_w(x)$ [Pa] can be determined from the slope of the profile $\bar{U}(x, y)$ interpolated very near the surface. An example is shown in Figure 2.

$$y = 0 = y'_0 + \Delta y \rightarrow \bar{U}(y) = 0 \tag{25}$$

where y' is the reference distance of the probe from the wall, y'_0 the probe touch with tops of the roughness grains and Δy includes the probe dimensions together with the shift of the “zero

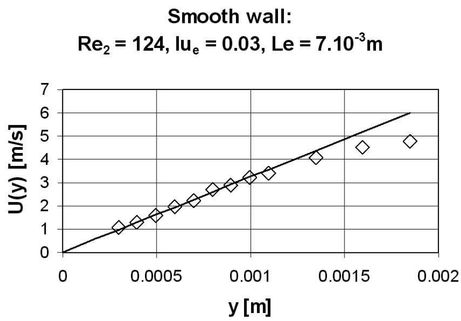

Fig. 2. Example of the interpolation near thesurface

level" — already mentioned effect of roughness [28]. This procedure works quite satisfactory in regions of pseudo-laminar flow, not far from the transition start and in turbulent layers with relatively thick viscous sublayer. It proved itself generally better in case of a smooth surface.

The value of shape parameter $H_{12} \cong 2.6$ is the first indication that we have to do with a flat plate zero pressure gradient boundary layer in the section $x = \text{const}$. Next the values of the Blasius coordinate η_k, $k = 1, 2, \ldots, n$ are calculated, corresponding to the evaluated values of the normalized mean velocity $\bar{U}_k/\bar{U}_e$ (i.e. 1st derivative of Blasius function) in the region $\sim 0.3\delta < y'_k < \delta$. The linear interpolation of η_k vers. y'_k validates the assumption on laminar boundary layer profile and determines of Δy giving the best correlation

$$\varsigma = a + by' = b\left(\frac{a}{b} + y'\right) = \sqrt{\frac{U_e}{\nu x}}(y' + \Delta y) \tag{26}$$

A satisfactory small error of interpolation confirms the assumption on laminar boundary layer profile. The regression parameters a and b allow determine the "zero level" Δy, link the effective streamvise coordinate x_{ef} with the investigated cross-section x and calculate the local wall shear stress

$$\tau_w(x) = \mu\left(\frac{\partial \bar{U}}{\partial y}\right)_w = 0.332\mu U_e\sqrt{\frac{U_e}{\nu x}} \tag{27}$$

A bad statistical pertinence of (26) signifies that the assumption on laminar boundary layer profile measured nearest the leading edge must be denied and another measuring method is necessary to determine the wall friction.

The value of shape parameter $H_{12} \sim 1.3 \div 1.5$ is an indication that we have to do with a turbulent boundary layer in the investigated section $x = \text{const}$. Our aim is the evaluation of three unknowns i.e. the zero level shift Δy, the roughness function Δu^+ and the friction velocity u_τ from the mean velocity profile in the investigated configuration, generally rough surface and turbulent external flow. Various methods for the evaluation of only the wall friction u_τ are known in case of canonical boundary layers (e.g. the Clauser chart method). Having in mind that the external flow turbulence remarkably modifies the shape of the wake function (most cogently shown in [9]) the evaluation of our unknowns is impossible support with the velocity defect law. Thus only the logarithmic law of the wall (13) is available for the evaluation of the three unknowns Δy, Δu^+ and u_τ. A generalized least-squares regression technique must be used with successive approximations of Δy and with specifying the range from $\min y'$ to

$\max y'$. The theoretical regression function has the form

$$\begin{array}{ll} Y = a + bX, & X = \ln(y' + \Delta y_j), \\ Y = \frac{\bar{U}(y')}{\bar{U}_e}, & j = 0, 1, 2, \ldots \end{array} \tag{28}$$

where the coefficients a and b have the meaning

$$\left(\frac{u_\tau}{\bar{U}_e}\right)_j = \kappa b_j, \quad \left(\Delta u^+\right)_j = \frac{1}{\kappa}\left[\ln\left(\frac{\bar{U}_e}{\upsilon}\right) + \kappa B_S + \ln(\kappa b_j) - \frac{a_j}{b_j}\right] \tag{29}$$

The estimates of interpolation errors are the criterion of the regression accuracy. The range of coordinates y' must be very carefully chosen.

Examples of the application of the described procedures for the evaluation of the *zero level* shift Δy, the friction velocity $u_\tau/\bar{U}_e$, wall friction τ_w [Pa] and the roughness function Δu^+ are shown in Figures 2, 3, 4. The mean velocity profile measured in the boundary layer on smooth surface in external flow with grid turbulence with the shape factor $H_{12} = 2.55$ is shown on the Figure 2. The linear interpolation near the wall allowed determine the slope $(\mathrm{d}\bar{U}/\mathrm{d}y)_{y=0}$ with a high accuracy ($\sim 1.5\,\%$). The values were determined: $u_\tau/\bar{U}_e = 0.042\,7$ and τ_w [Pa] $= 0.059\,3$.

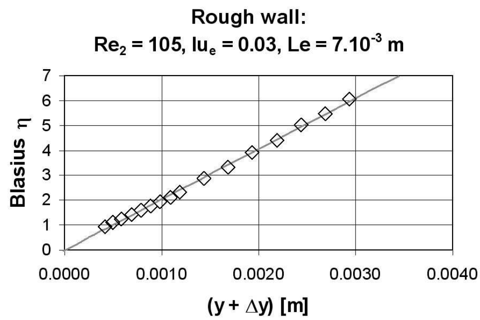

Fig. 3. The correlation of the measured ve-locity profile with the Blasius solution

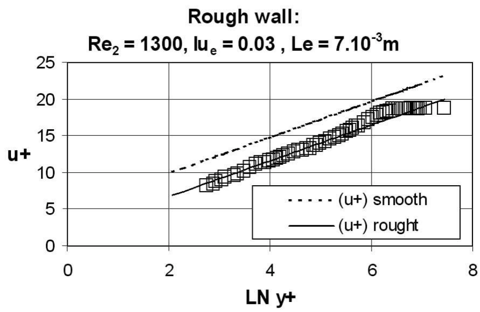

Fig. 4. Comparison of the measured veloc-ity profile with the log law on smooth wall

The second example represents the evaluation of the mean velocity profile with the shape factor $H_{12} = 2.61$ (laminar shape) measured in the boundary layer on rough surface in external flow with grid turbulence. The correlation of the Blasius variable and the probe distance from the zero level at the same ratio $\bar{U}(y + \Delta y)/\bar{U}_e$ is plot on the Figure 3. They were determined: the zero level shift $\Delta y = 6 \cdot 10^{-5}$ m, the values of $u_\tau/\bar{U}_e = 0.046\,3$ and τ_w [Pa] $= 0.061\,8$ and the effective distance from the onset of boundary layer $x_{ef} = 0.077$ m (the true $x = 0.05$ m).

The fully turbulent mean velocity profile ($H_{12} = 1.39$) measured in the same boundary layer as the preceding one is the last example shown in the Figure 4. Following estimates follow from regressions (28) with (29): $\Delta y = 0.202$ mm, $u_\tau/\bar{U}_e = 0.053\,4$ and τ_w [Pa] $= 0.082\,9$ and the value of the roughness function $\Delta u^+ = 3.17$.

5. Conclusion

Three modes of the evaluation of the mean velocity profiles measured in the flat plate boundary layer with generally rough surface under turbulent external stream were derived on the basis of published and the authors personal experiences.

The estimated errors of the determination of the wall friction, the shift of the zero level and the roughness function are broadly satisfactory (about $\pm 2\,\%$, $\pm$ several hundredth of millimeter and less ± 0.1 respectively). However the evaluation is very laborious and it could be influenced by personal errors.

The development of a proper direct wall friction measuring method is very necessary for the processing of great sets measurements in boundary layers on rough surfaces under external turbulent flow.

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