An evaluation of the stress intensity factor in functionally graded materials

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Abstract

Functionally graded materials (FGM) are characterised by variations in their material properties in terms of their geometry. They are often used as a coating for interfacial zones to protect the basic material against thermally or mechanically induced stresses. FGM can be also produced by technological process for example butt-welding of polymer pipes. This work is focused on a numerical estimation of the stress intensity factor for cracks propagating through FGM structure. The main difficulty of the FE model creation is the accurate description of continual changes in mechanical properties. An analysis of the FGM layer bonded from both sides with different homogenous materials was performed to study the influence of material property distribution. The thickness effect of the FGM layer is also discussed. All analyses are simulated as a 2D problem of an edge cracked specimen. In this paper, the above effects are quantified and conclusions concerning the applicability of the proposed model are discussed.

1. Introduction

Functionally graded materials (FGM) are composites where the composition varies from place to place in order to effect the best performance of the structure. The development of FGM has demonstrated its possible uses in a wide range of thermal and structural applications such as thermal barrier coatings, corrosion and wear resistant coatings and metal/ceramic joining. Mechanical properties gradation offers ways of optimizing structure and achieving high performance and material efficiency. At the same time, this optimization can result in numerous mechanical problems including estimation of effective properties and crack propagation behaviors in the final structure [3].

Functionally graded materials commonly occur in nature. The human body contains many examples of complex FGM parts, such as bones or teeth. Another example of naturally occurring FGM is in bamboo [16], see Fig. 1. The cross section of bamboo resembles a fibre-reinforced composite material with continuous change of fibre density. This configuration leads to a continuous change in material properties which is, in fact, also the philosophy of modern FGM materials.

Approaches towards how to study fracture behaviors of FGM structures are available in the literature. Many of them focus on analytical solutions, see [4, 5, 9]. The comprehensive

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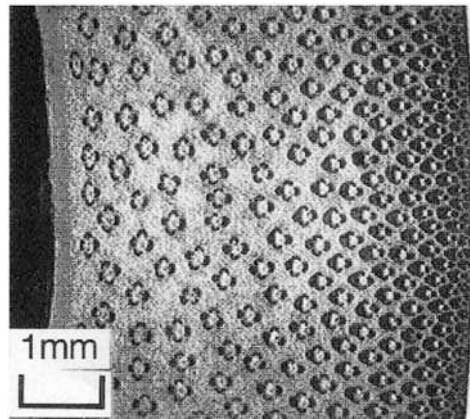*Corresponding author. Tel.: +420 541 212 362, e-mail: sevcik@ipm.cz.

Fig. 1. Cross section of bamboo culm [16]

review concerning with the use of weight method for studying the crack propagation in FGM was published in [2]. The FGM structures have also been studied by numerical approaches such as the extended finite element method (X-FEM) [6, 7] or the finite element method (FEM) [3, 13, 14, 15].

Recent numerical simulations focus mainly on two dimensional (2D) analyses of crack propagation in FGM structures. This is due to difficulties occurring during FE model creation. The aim of this work is to develop a suitable discretization method for definition of the mechanical properties of the FGM applicable in common FE codes. This method should have greater accuracy and a lower computational time requirement which will prove beneficial during three dimensional analyses (3D) of much more complicated problems.

As the simulations performed in this work take into account the continual change of Young's modulus the linear elastic fracture mechanics approach is used to study crack behavior.

2. Estimation of stress intensity factor

In the Williams expansion (see e.g. [1]) for a linear elastic crack-tip stress field, the stress intensity factor corresponds to the first singular term:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + T\delta_{1i}\delta_{1j} \ldots \tag{1}$$

Here K_I is a stress intensity factor (only the normal mode of loading is considered), T is a T-stress, and $f_{ij}(\theta)$ is a known function of the polar angle θ. The stress intensity factor is widely used in many applications and the quality of this solution has been confirmed by numerous experimental results.

The validation of the use of the stress intensity factor in FGM layers has been achieved by many authors, e.g. [8] or [12]. Generally speaking, there are numerous ways how to estimate the stress intensity factor. One of the most common is by using quarter-point singular elements around the crack tip. This distortion of the FE mesh causes singularity $r^{-1/2}$. Due to the distortion, the stress field at the vicinity of the crack tip is better described and a very fine mesh is not necessary.

3. Coupled model of FGM

3.1. Material properties

In reality, the FGM structure is often connected with other materials. FGM layers often serve as a connection between two homogenous materials. In such cases, we need to simulate a complete configuration consistent with homogeneous parts connected to FGM, see [11]. The corresponding model of this set is shown in Fig. 2, where M1 indicates material number 1 (e.g. Al_2O_3) and M2 indicates material number 2 (e.g. Ni).
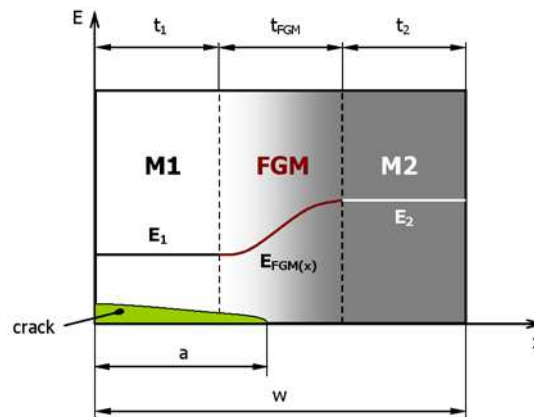

Fig. 2. Crack in the FGM layer

The Young's modulus E is constant in both materials M1 and M2. Due to the fact that only elastic material properties are used for presented simulations the linear elastic fracture mechanics approach is used for description of the crack behavior. In the FGM layer, the continuous change of E_{FGM} from E_1 (corresponding to material M1) to E_2 (material M2) is described as a function of the coordinates. In this general numerical study, the exact values of material properties are not necessarily known. The variable material property studied here is a ratio of Young's modulus E_2/E_1. In the following it is assumed that $E_1 = 1\,000$ MPa. The value of Young's modulus of material M2 is given by the ratio E_2/E_1. In this work, two values of the ratio E_2/E_1 are studied, namely $E_2/E_1 = 0.1$ – the crack spreads to the softer material and $E_2/E_1 = 10$ – the crack spreads to the tougher material. To describe the material properties distribution the following functions are commonly used in the literature [3, 10, 17]:

a) exponential function

$$
\begin{aligned}
f_{FGM}(x) &= f_1 \mathrm{e}^{[\beta(x/w)]}, \\
\beta &= \ln(f_2/f_1),
\end{aligned}
$$

(2)

where f_i, $i = 1, 2$, is an arbitrary material property specified for material 1 or 2, β is constant of non-homogeneity, x is Cartesian coordinate, w is the width of the specimen, see Fig. 2

b) power-law function

$$
f_{FGM}(x) = f_1 + (f_2 - f_1)(x/w)^g,
$$

(3)

where g is a constant describing the gradient of material changes

c) double power-law function

$$h_1\left(x\right) = \tfrac{1}{2}\left(\tfrac{x}{w/2}\right)^p \qquad 0 \le x \le w/2$$

$$h_2\left(x\right) = 1 - \tfrac{1}{2}\left(\tfrac{w-x}{w/2}\right)^p \qquad w/2 \le x \le w, \tag{4}$$

$$f_{FGM}(x) = h_1(x)f_2 + [1 - h_1(x)]f_1 \qquad 0 \le x \le w/2$$

$$f_{FGM}(x) = h_2(x)f_2 + [1 - h_2(x)]f_1 \qquad w/2 \le x \le w, \tag{5}$$

where p is a degree of the polynomial, $h(x)$ determines volume fraction of components

d) rule of mixture

$$f_{FGM}(x) = v_f(x)E_2 + [1 - v_f(x)]E_1, \tag{6}$$

where $v_f(x)$ is local volume fraction of material.

The exponential change is often used for analytical approaches because of its easy numerical manipulation. The linear shape can be determined if $g = 1$ in case of the power-law function or for $p = 1$ in the case of a double-power law function. The rule of mixture is useful for those cases where the volume fraction of the material is known. In this study material properties distribution is determined by the power-law function (b) for all analyses.

3.2. Discretization of the material properties

In this paper, the new type of discretization has been developed. The structure is divided into a certain number of strips of different thickness t^i depending on the material properties distribution. The idea of this approach is shown in Fig. 3. The parameter controlling the number of the strips is a step factor s. The choice of its optimal values is studied in this paper.
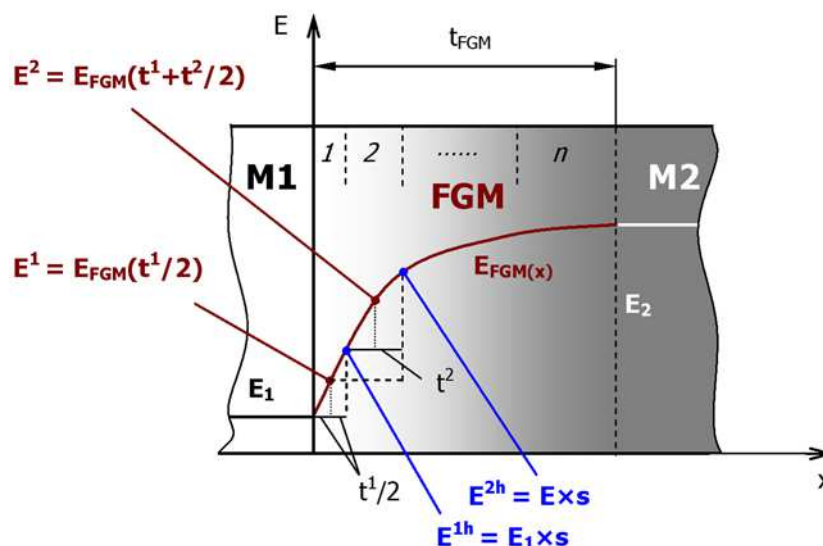

Fig. 3. The proposed discretization method

The nonhomogenous discretization of the continuous change of the material properties leads to n strips, which are perpendicular to the crack propagation direction. The advantage of this approach is a better description of the material property changes in positions where a strong

gradient is presented. In these positions, more strips with smaller thickness are created and thus better accuracy is achieved. The dependence between a step factor s and the number of strips n corresponds to the geometry studied and the material changes used in the FGM layer is given in Table 1.

Table 1. Number of the strips n as a function of the step factor s

s	n	
	$E_2/E_1 = 10$	$E_2/E_1 = 0.1$
1.05	46	44
1.1	24	22
1.2	13	11
1.5	6	6
2	4	4
5	2	2

3.3. Boundary conditions

The boundary conditions (see Fig. 4) correspond to displacement loading. The top nodes are loaded by tensile stress $\sigma = 1$ MPa and the nodes are coupled so that the vertical displacements are identical. Because of the model symmetry, only one half is modelled. The problem is considered as 2D under plane strain conditions.
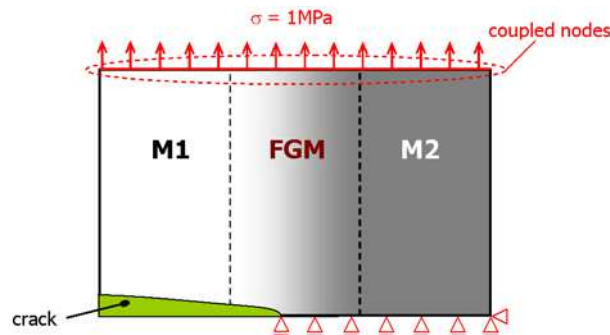

Fig. 4. Boundary conditions of coupled model of FGM – displacement loading

4. Results and discussion

4.1. Estimation of the step factor value

The aim of the first analysis was to find a suitable value for the step factor s. This parameter controls the number of the strips and the accuracy of the results. For that reason, an analysis of the influence of s was performed. The values of $s = 1.05$–5 and the ratios $E_2/E_1 = 0.1$ and $E_2/E_1 = 10$ were considered and analysed. The change in Young's modulus across the FGM layer was assumed to be linear. The results are presented in Fig. 5 and 6.

This analysis gives the estimation of the strip numbers for describing the FGM layer. For $n = 44$ and $n = 22$ the values of stress intensity factor are almost identical, the difference is less than 0.5 %. In case of the number $n = 11$ the difference is less then 1.5 %, which is a
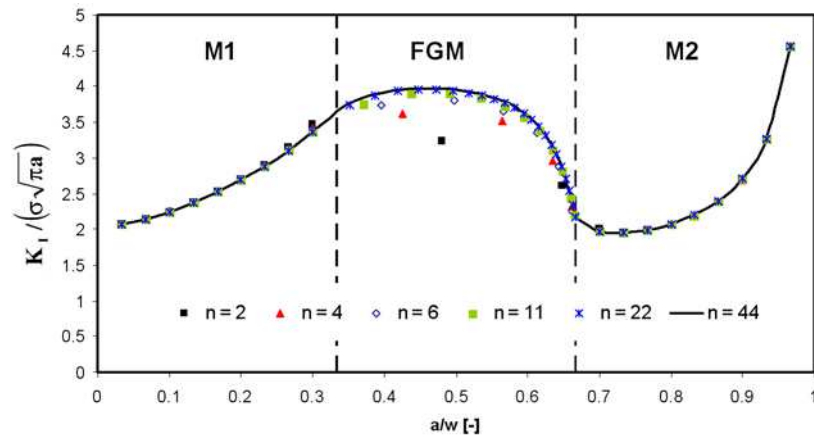
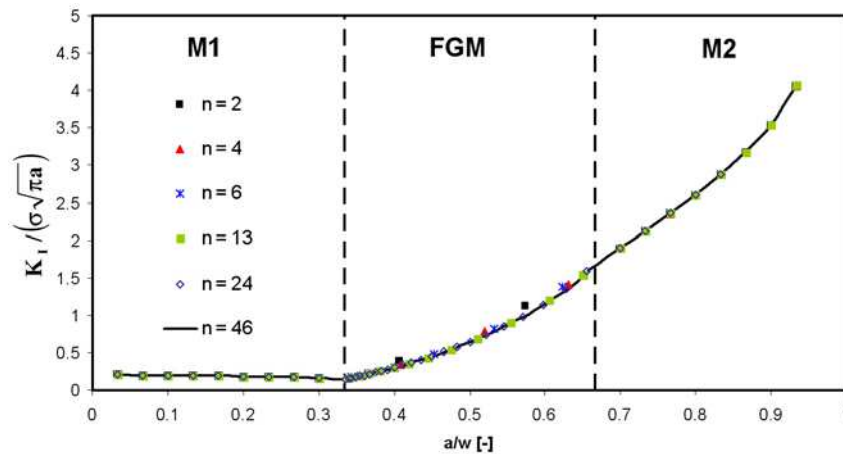Fig. 5. Sensitivity of the stress intensity factor to a number of the areas – $E_2/E_1 = 0.1$

Fig. 6. Sensitivity of the stress intensity factor to a number of areas – $E_2/E_1 = 10$

good approximation. However, for $n = 6$, $n = 4$ and $n = 2$ the results are incorrect because the values of the stress intensity factor are rapidly underestimated. A similar analysis to the previous was performed but the ratio $E_2/E_1 = 10$ was assumed, see Fig. 6.

In comparison with the previous analysis ($E_2/E_1 = 0.1$), the stress intensity factor reaches significantly different values for $E_2/E_1 = 10$. The stress intensity factor is considerably lower in case of $E_2/E_1 = 10$. Another event is that the stress intensity factor is for ratio $E_2/E_1 = 10$ almost constant in material M1. This is caused by stiffer material in front of the crack. However, for the ratios $E_2/E_1 > 1$ the number of the strips in FGM layer can be very low. For example, the maximal difference between results for $n = 46$ and $n = 2$ was approximately 15 %. To conclude usage of 13–24 strips (corresponding to value of step factor $s = 1.2$–1.1) is in this case sufficient for correct simulation of the stress intensity factor in the FGM layers.

4.2. Influence of graded material properties on fracture parameters

Knowledge of the material properties distribution is necessary for simulating FGM of real structures. In general, the material here is described by a power function (see eq. 3). The distributions are described by the parameter g – gradient index, see Figs. 7a) and 7b). The initial size of the defect corresponded to the ratio $a/w = 0.03$. The crack propagates in material M1 in 9 steps until the crack tip reaches the FGM layer.
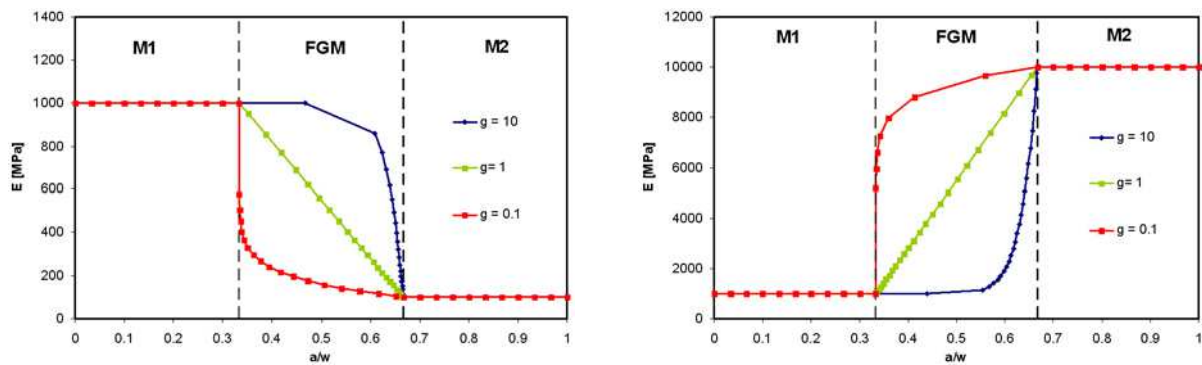
Fig. 7. Studied Young's modulus distributions a) $E_2/E_1 = 0.1$ b) $E_2/E_1 = 10$
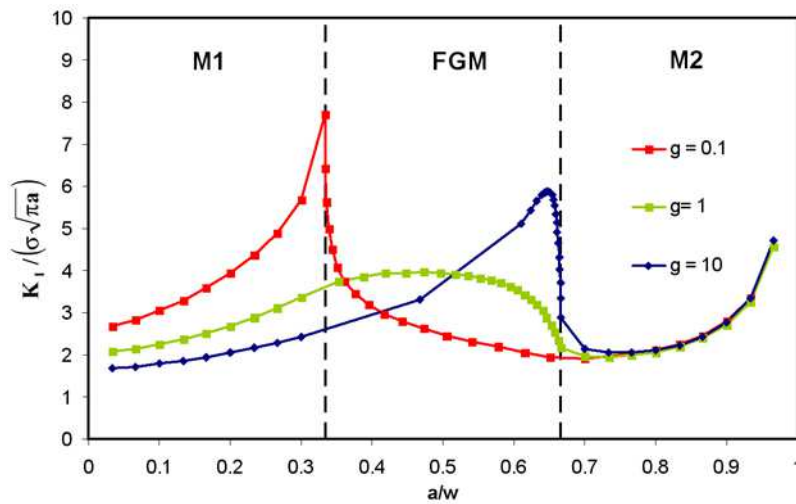

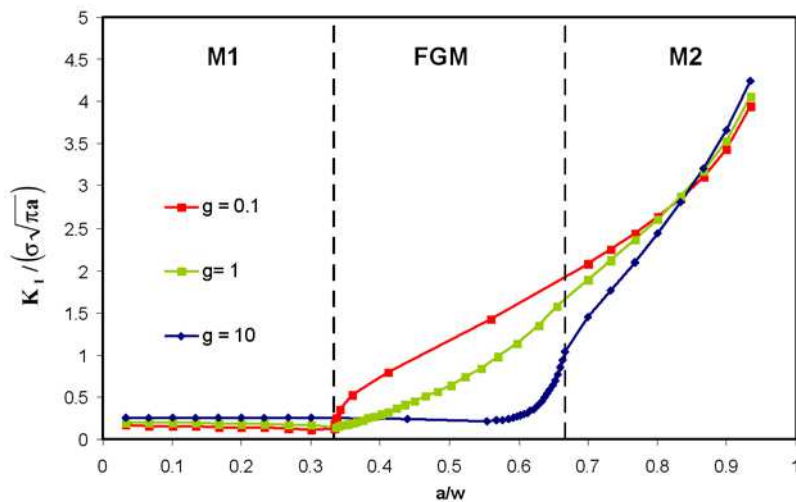Fig. 8. Stress intensity factors $E_2/E_1 = 0.1$, displacement loading

Fig. 9. Stress intensity factors $E_2/E_1 = 10$, displacement loading

Then the material properties of the FGM layer are divided by the discretization described into $n = 20$ strips in the case of $E_2/E_1 = 0.1$ and into $n = 24$ strips in the case of $E_2/E_1 = 10$. The results of the analyses are shown in Figs. 8 and 9. Significant differences in stress intensity factor values are evident. For the FGM layer with a prompt decrease in Young's modulus ($g = 0.1$) the dependence of the stress intensity factor looks similar to the crack penetrating
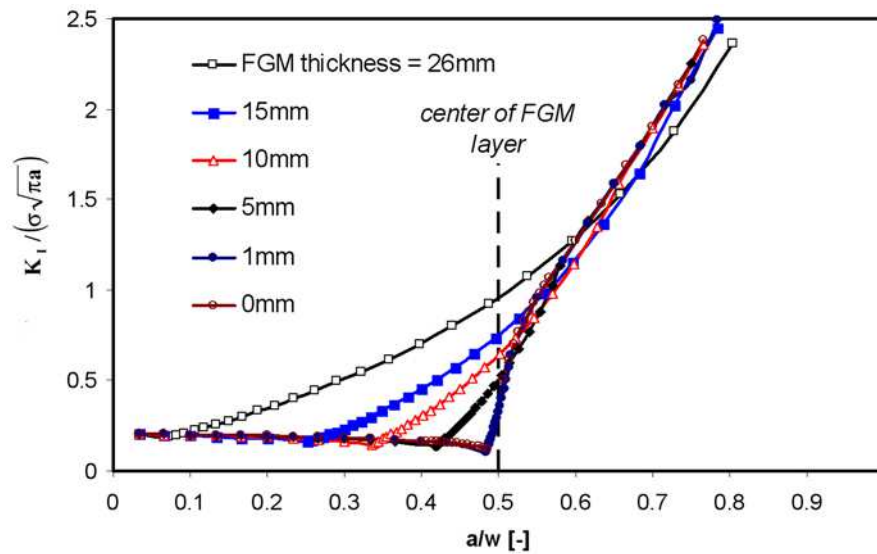
Fig. 10. Influence of the thickness of the FGM layer, $E_2/E_1 = 10$, displacement loading

the sharp interface. A very rapid decrease in Young's modulus appears immediately after the interface which caused the increase of the stress intensity factor. However, in the FGM layer the stress intensity factor rapidly decreases and starts to grow as far as material M2. A similar situation occurs for $g = 10$ where a strong change in the stress intensity factor value occurs near the interface of the FGM and material M2. The peak is nearly at the interface but the analysis showed a decrease of the stress intensity factor in the region in front of the FGM/M2 interface, see Fig. 8. In the case of linear change ($g = 1$) there is continuous smooth dependence. This is due to a gradual change in the Young's modulus.

As mentioned earlier, the stress intensity factor is almost constant for ratio $E_2/E_1 = 10$ in the material M1. Then the slow increase is present even though its values are not too high. This combination, i.e. $E_2/E_1 = 10$ and displacement loading, produces the lowest stress intensity factor of all the configurations studied independently of the value of gradient index g.

The previous analyses showed the possibility to study the FGMs by the method presented. The influence of the material properties distribution is significant and should be taken into account in the design of FGM structures. A suitable configuration is able to prolong the residual lifetime of the cracked structure and to assist in safe service.

4.3. Influence of the thickness of the FGM layer

A parameter which is often considered during the design of an FGM structure is the thickness of the FGM layer. For the thickness of the FGM, $t_{FGM} = 0$, the step change of material properties is assumed and this interface corresponds to the connection of two materials which do not allowed any diffusion. A connection of this type produces significant shear stresses, both positive and negative. The greater the difference in material properties, the greater the step in shear stress. In order to minimize these shear stresses the thickness of the FGM layer should be $t_{FGM} > 0$. The analysis of the influence of the t_{FGM} has been performed to show the effect on the stress intensity factor. The geometry studied is the same as in previous analyses, see Fig. 2. The thickness of the FGM layer t_{FGM} varied from 0 to 26 mm. In case of $t_{FGM} = 0$ the bi-material interface is modelled. The total thickness of the specimen is $w = 30$ mm. The linear change of the Young's modulus had been assumed. The results of the simulations are shown in Fig. 10. Even though the thicker FGM layer causes a decrease in the shear stresses

it negatively affects the fracture behaviors of the structure. However, the positive effect of the decrease in shear stress ultimately proves to be an advantage that more significantly contributes to the longer lifetime of the structure than a low stress intensity factor by itself.

5. Conclusions

The aim of this paper was to study complex FGM structures from a fracture mechanics point of view. The idea of the discretization of the material properties is proposed here. The advantage of this principle is the fine FE mesh in locations where the gradient of the material properties is higher. The sensitivity analysis of the step factor s has been carried out. It was found that for $s = 1.05$ and $s = 1.1$ the results are identical. A good accuracy was also found for $s = 1.2$. The use of $s = 1.1$ for ratios $E_2/E_1 > 1$ and $s = 1.2$ for ratios $E_2/E_1 < 1$ can be recommended. The analyses of the influence of the material properties distribution were performed on the basis of this discretization principle. Three types of shapes were studied – linear change, prompt change from material M1 to material M2, and strong change closer M2, see Fig. 7.

The values of the stress intensity factor were calculated for boundary conditions corresponding to displacement loading. In general, the material properties distributions where the strong change closer to M2 was assumed showed a radical increase in the stress intensity factor in the vicinity of the interface between FGM layer and material M2. This effect positively influence the lifetime of the structure.

The last analysis was focused on simulations of various thicknesses of the FGM layers. Even though the thicker FGM layer causes a decrease in shear stress, it negatively affects the fracture behaviours of the structure. The analyses proved that with an increase in t_{FGM} the stress intensity factor is even greater. However, the ultimate positive effect of a decrease in shear stress is an advantage that contributes to the longer lifetime of the structure more significantly than a low stress intensity factor on its own.

The principle proposed here can be useful for the FE simulation of 3D FGM structures to minimize computational time and for effective simulations. The results can also help in the design process of FGM structures as well as in the educational field.

Acknowledgements

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