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Numerical simulation of airflow through the model of oscillating vocal folds

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Abstract

This work deals with numerical simulation of flow in time-dependent 2D domains with a special interest in medical applications to airflow in human vocal folds. The mathematical model of this process is described by the compressible Navier-Stokes equations. For the treatment of the time-dependent domain, the Arbitrary Lagrangian-Eulerian (ALE) method is used. The discontinuous Galerkin finite element method (DGFEM) is used for the space semidiscretization of the governing equations in the ALE formulation. The time discretization is carried out with the aid of a linearized semi-implicit method with good stability properties. We present some computational results for flow in a channel with a prescribed periodic motion of a part of the channel walls.

Keywords: compressible flow, Navier-Stokes equations, Arbitrary Lagrangian-Eulerian method, discontinuous Galerkin finite element method, voice source biomechanics

1. Introduction

The simulation of compressible flow in time dependent domains plays an important role in many areas, for example development of aircrafts and turbines, in civil engineering, car industry or medicine. The presented work is concerned with medical applications by modelling the airflow in human vocal folds, where the airflow energy is transferred into the acoustic energy generating a voice source signal. The mechanism of such energy transfer is not properly known. The airflow in the glottal region has been modelled mostly as incompressible (see, e.g. [8]). The model of the compressible flow was used in [7] for numerical simulation of the flow field in the glottal region by the finite volume method.

We describe here the numerical technique for the solution of the compressible Navier-Stokes equations written in the ALE (Arbitrary Lagrangian-Eulerian) form, using the discontinuous Galerkin finite element method (DGFEM). This work also creates the basis of the further work in the direction of fluid-structure interaction, when the flow problem is coupled with elastic behaviour of vocal folds.

2. Governing equations

We deal with compressible flow in a bounded domain $\Omega_t \subset \mathbb{R}^2$ depending on time $t \in [0, T]$. We assume that the boundary of Ω_t consists of three disjoint parts: $\partial\Omega_t = \Gamma_I \cup \Gamma_O \cup \Gamma_{W_t}$, where Γ_I and Γ_O represent the inlet and outlet and Γ_{W_t} represents moving impermeable walls.

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We consider the Navier-Stokes equations in the conservative form [3]:

$$\frac{\partial \boldsymbol{w}}{\partial t} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{f_s}(\boldsymbol{w})}{\partial x_s} = \sum_{s=1}^{2} \frac{\partial \boldsymbol{R}_s\left(\boldsymbol{w}, \nabla \boldsymbol{w}\right)}{\partial x_s} \quad \text{in } \Omega_t,\ t \subset [0,T]\,, \tag{1}$$

where

$$\begin{aligned} \boldsymbol{w} &= (\rho, \rho v_1, \rho v_2, E)^T \in \mathbb{R}^4, \\ \boldsymbol{f}_s(\boldsymbol{w}) &= (\rho v_s, \rho v_1 v_s + \delta_{1s} p, \rho v_2 v_s + \delta_{2s} p, (E+p) v_s)^T, \quad s = 1,2, \\ \boldsymbol{R}_s\left(\boldsymbol{w}, \nabla \boldsymbol{w}\right) &= \left(0, \tau_{s1}, \tau_{s2}, \tau_{s1} v_1 + \tau_{s2} v_2 + k \frac{\partial \theta}{\partial x_s}\right)^T, \quad s = 1,2, \\ \tau_{ij} &= \lambda \delta_{ij} \operatorname{div} \boldsymbol{v} + 2\mu d_{ij}(\boldsymbol{v}), \quad d_{ij}(\boldsymbol{v}) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \quad i,j = 1,2. \end{aligned} \tag{2}$$

We use the following notation: $\boldsymbol{n}$ — the unit outer normal to $\partial\Omega_t$, $\boldsymbol{z}$ – wall velocity, ρ – density, p – pressure, E – total energy per unit volume, $(v_1, v_2)^T$ – velocity vector, θ – absolute temperature, $c_v > 0$ – specific heat at constant volume, $\gamma > 1$ – Poisson adiabatic constant, $\mu > 0$, λ – viscosity coefficients, $k > 0$ – heat conduction coefficient. We set $\lambda = -2\mu/3$.

System (1) is completed by the thermodynamical relations

$$p = (\gamma - 1)(E - \rho \left|\boldsymbol{v}\right|^2 /2), \tag{3}$$

$$\theta = \left(\frac{E}{\rho} - \frac{1}{2}\left|\boldsymbol{v}\right|^2\right)/c_v, \tag{4}$$

initial condition $\boldsymbol{w}(\boldsymbol{x}, 0) = \boldsymbol{w}^0(\boldsymbol{x}),\ \boldsymbol{x} \in \Omega_t$, and boundary conditions:

$$\text{Inlet } \Gamma_I: \quad \begin{aligned} &\rho|_{\Gamma_I \times (0,T)} = \rho_D, \\ &\boldsymbol{v}|_{\Gamma_I \times (0,T)} = \boldsymbol{v}_D = (v_{D1}, v_{D2})^T, \\ &\sum_{j=1}^{2}\left(\sum_{i=1}^{2} \tau_{ij} n_i\right) v_j + k \frac{\partial \theta}{\partial \boldsymbol{n}} = 0 \quad \text{on } \Gamma_I \times (0,T); \end{aligned} \tag{5}$$

$$\text{Moving wall } \Gamma_W: \quad \boldsymbol{v}_{\Gamma_W \times (0,T)} = \boldsymbol{z}, \quad \frac{\partial \theta}{\partial \boldsymbol{n}} = 0; \tag{6}$$

$$\text{Outlet } \Gamma_O: \quad \sum_{i=1}^{2} \tau_{ij} n_i = 0, \quad \frac{\partial \theta}{\partial \boldsymbol{n}} = 0, \quad j = 1,2. \tag{7}$$

The vector functions $\boldsymbol{f}_s$ are inviscid fluxes of the quantity $\boldsymbol{w}$ in the directions x_s and $\boldsymbol{R}_s$ represent viscous terms, $s = 1,2$. In the sequel, we shall use the following relations for the fluxes $\boldsymbol{f}_s$:

$$\boldsymbol{f}_s(\boldsymbol{w}) = \mathbb{A}_s(\boldsymbol{w})\boldsymbol{w}, \quad \text{where } \mathbb{A}_s(\boldsymbol{w}) = \frac{D\boldsymbol{f}_s}{D\boldsymbol{w}},\ s = 1,2. \tag{8}$$

Furthermore, the term $\boldsymbol{R}_i(\boldsymbol{w}, \nabla\boldsymbol{w})$ can be expressed in the form

$$\boldsymbol{R}_i(\boldsymbol{w}, \nabla \boldsymbol{w}) = \sum_{j=1}^{2} \mathbb{K}_{ij}(\boldsymbol{w}) \frac{\partial \boldsymbol{w}}{\partial x_j}, \tag{9}$$

where $\mathbb{K}_{ij}$ are 4×4 matrices dependent on $\boldsymbol{w}$ and independent of $\nabla\boldsymbol{w}$ (cf. [3]).

For viscous flows we define the so-called *Reynolds* and *Mach numbers*, defined as

$$Re = \frac{U^\star L^\star \rho^\star}{\mu^\star}, \quad M = \frac{|\boldsymbol{v}|}{a}, \tag{10}$$

where $U^\star$ is the characteristic velocity, $L^\star$ is the characteristic length, $\rho^\star$ is the characteristic density, $\mu^\star$ is the characteristic viscosity and $a = \sqrt{\gamma p/\varrho}$ is the speed of sound.

3. ALE formulation

In order to simulate flow in a time-dependent domain, we employ the Arbitrary Eulerian-Lagrangian method. Let us denote *reference configuration* by $\Omega_{ref} = \Omega_0$ for the computational domain at the initial time (see Fig. 1). A smooth, one-to-one mapping of the reference configuration onto the computational domain Ω_t at time t (the so-called *current configuration*) is denoted by $\mathcal{A}_t$ (cf. [5]), i.e.

$$\mathcal{A}_t : \bar{\Omega}_{ref} \longrightarrow \bar{\Omega}_t, \text{ i.e. } \mathcal{A}_t : \boldsymbol{X} \longmapsto \boldsymbol{x} = \boldsymbol{x}(\boldsymbol{X}, t) = \mathcal{A}_t(\boldsymbol{X}). \tag{11}$$

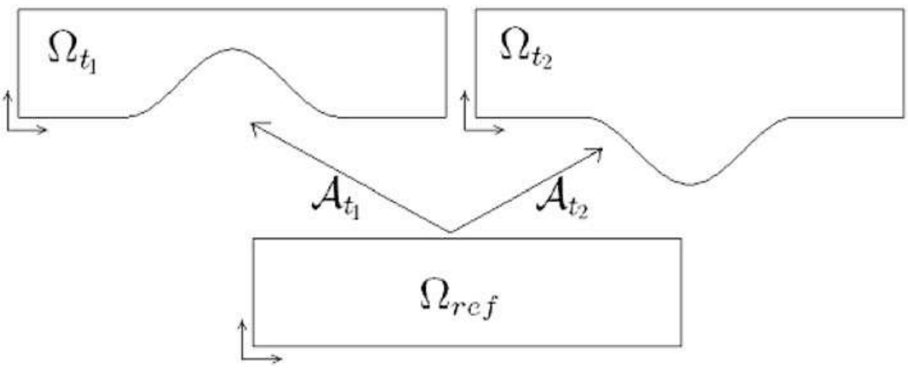

Fig. 1. ALE mapping scheme

Based on this mapping we define the ALE velocity:

$$\begin{aligned} \tilde{\boldsymbol{z}}(\boldsymbol{X}, t) &= \frac{\partial}{\partial t}\mathcal{A}_t(\boldsymbol{X}), \quad t \in [0, T], \ \boldsymbol{X} \in \Omega_0, \\ \boldsymbol{z}(\boldsymbol{x}, t) &= \tilde{\boldsymbol{z}}(\mathcal{A}^{-1}(\boldsymbol{x}), t), \quad t \in [0, T], \ \boldsymbol{x} \in \bar{\Omega}_t. \end{aligned} \tag{12}$$

Moreover for a function $f = f(\boldsymbol{x}, t)$ defined for $\boldsymbol{x} \in \Omega_t$ and $t \in [0, T]$, we introduce the ALE derivative

$$\frac{D^{\mathcal{A}}}{Dt} f(\boldsymbol{x}, t) = \frac{\partial \tilde{f}}{\partial t}(\boldsymbol{X}, t), \quad \text{where } \tilde{f}(\boldsymbol{X}, t) = f(\mathcal{A}_t(\boldsymbol{X}), t), \ \boldsymbol{X} \in \Omega_0. \tag{13}$$

By the chain rule we obtain

$$\frac{D^{\mathcal{A}} f}{Dt} = \frac{\partial f}{\partial t} + \boldsymbol{z} \cdot \nabla f = \frac{\partial f}{\partial t} + \operatorname{div}(\boldsymbol{z} f) - f \operatorname{div} \boldsymbol{z}, \tag{14}$$

which yields the ALE form of the Navier-Stokes equations:

$$\frac{D^{\mathcal{A}} \boldsymbol{w}}{Dt} + \sum_{s=1}^{2} \frac{\partial \boldsymbol{g}_s(\boldsymbol{w})}{\partial x_s} + \boldsymbol{w} \operatorname{div} \boldsymbol{z} = \sum_{s=1}^{2} \frac{\partial \boldsymbol{R}_s(\boldsymbol{w}, \nabla \boldsymbol{w})}{\partial x_s}, \tag{15}$$

where $\boldsymbol{g}_s, \ s = 1, 2$, are modified inviscid fluxes

$$\boldsymbol{g}_s(\boldsymbol{w}) = \boldsymbol{f}_s(\boldsymbol{w}) - z_s \boldsymbol{w}, \ s = 1, 2. \tag{16}$$

4. Space semidiscretization by the discontinuous Galerkin method

Let $\mathcal{T}_{ht}$ be a partition of $\overline{\Omega}_t$ formed by a finite number of triangles, whose interiors are mutually disjoint. Let $I \subset \mathbb{Z}^+$ be a numbering of triangles in $\mathcal{T}_{ht}$. If two different elements $K_i, K_j \in \mathcal{T}_{ht}$ share a common face, we call them neighbours and set $\Gamma_{ij} = \partial K_i \cap \partial K_j$. For $i \in I$ we define $s(i) = \{j \in I; K_j \text{ is a neighbour of } K_i\}$. We denote all boundary faces by S_j, where $j \in I_b \subset \mathbb{Z}^- = \{-1, -2, \ldots\}$ and set $\gamma(i) = \{j \in I_b; S_j \text{ is a face of } K_i\}$. Furthermore, we define $S(i) = s(i) \cup \gamma(i)$ and $\gamma_D(i) = \{j \in I_b; \text{ Dirichlet boundary condition prescribed on } \Gamma_{ij}\}$. The diameter of Γ_{ij} will be denoted by $d(\Gamma_{ij})$. By $\boldsymbol{n}_{ij}$ we shall denote the unit outer normal to the boundary of K_i on the face Γ_{ij}.

We shall seek the approximate solution in the finite dimensional space

$$S_{ht} = S^{r,-1}(\Omega_t, \mathcal{T}_{ht}) = \{v; v|_K \in P_r(K)\ \forall K \in \mathcal{T}_{ht}\}^4, \tag{17}$$

where $P_r(K)$ is the space of all polynomials on K of degree $\leq r$. For $v \in S_{ht}$ we set $v_{ij} = v|_{\Gamma_{ij}} =$ trace of $v|_{K_i}$ on Γ_{ij}, $\langle v \rangle_{ij} = \frac{1}{2}(v_{ij} + v_{ji})$, $[v]_{ij} = v_{ij} - v_{ji}$, the average and the jump on an edge, respectively. By $(\cdot, \cdot)$ we denote the $L^2(\Omega)$-scalar product.

In the derivation of the discrete problem we proceed in the following way. The system of governing equations (15) is multiplied by a test function $\boldsymbol{\varphi} \in S_{ht}$ and integrated over each $K_i \in \mathcal{T}_{ht}$. We apply Green's theorem, sum over all $i \in I$.

4.1. Discretization of convective terms

The discretization of convective terms is carried out as in [2, 4]. We define the convective form

$$\begin{aligned} b_h(\boldsymbol{w}, \boldsymbol{\varphi}) \;=\; & -\sum_{K_i \in \mathcal{T}_{ht}} \int_{K_i} \sum_{s=1}^{2} \boldsymbol{g}_s(\boldsymbol{w}) \cdot \frac{\partial \boldsymbol{\varphi}}{\partial x_s}\, \mathrm{d}\boldsymbol{x} \,+ \\ & \sum_{K_i \in \mathcal{T}_{ht}} \sum_{j \in S(i)} \int_{\Gamma_{ij}} \boldsymbol{H}_g(\boldsymbol{w}|_{\Gamma_{ij}}, \boldsymbol{w}|_{\Gamma_{ji}}, \boldsymbol{n}_{ij}) \cdot \boldsymbol{\varphi}\, \mathrm{d}S. \end{aligned} \tag{18}$$

Here $\boldsymbol{H}_g(\cdot, \cdot, \cdot)$ is an apropriate numerical flux, which approximates the physical flux through an edge Γ_{ij}. In practice, we use the Vijayasundaram numerical flux, which has a suitable form for linearization:

$$\boldsymbol{H}_g(\boldsymbol{w}_L, \boldsymbol{w}_R, \boldsymbol{n}) = \mathbb{P}^+ \left((\boldsymbol{w}_L + \boldsymbol{w}_R)/2, \boldsymbol{n}\right) \boldsymbol{w}_L + \mathbb{P}^- \left((\boldsymbol{w}_L + \boldsymbol{w}_R)/2, \boldsymbol{n}\right) \boldsymbol{w}_R. \tag{19}$$

Here $\mathbb{P}(\boldsymbol{w}, \boldsymbol{n}) := \sum_{s=1}^{2} \left(\mathbb{A}_s(\boldsymbol{w}) - z_s \mathbb{I}\right) n_s$, where $\mathbb{A}_s(\boldsymbol{w}) = \frac{D\boldsymbol{f}_s(\boldsymbol{w})}{D\boldsymbol{w}} = \left(\frac{\partial f_{si}(\boldsymbol{w})}{\partial w_j}\right)_{i,j=1}^{m}$. $\mathbb{I}$ is the unit matrix and $\mathbb{P}^+$, $\mathbb{P}^-$ denote the positive and negative parts of $\mathbb{P}$ — see [3], Section 3.3.4.

4.2. Discretization of viscous terms

For the treatment of second order viscous nonlinear terms the *incomplete interior penalty Galerkin* (IIPG) scheme [1] is used. We proceed similarly as in the case of convective terms.

After some manipulation we obtain the following viscous form

$$\begin{aligned} a_h(\boldsymbol{w},\boldsymbol{\varphi}) &= \sum_{i\in I}\int_{K_i}\sum_{s=1}^{2}\boldsymbol{R}_s(\boldsymbol{w},\nabla\boldsymbol{w})\cdot\frac{\partial\boldsymbol{\varphi}}{\partial x_s}\,\mathrm{d}x - \\ &\quad \sum_{i\in I}\sum_{\substack{j\in s(i)\\ j<i}}\int_{\Gamma_{ij}}\sum_{s=1}^{2}\langle\boldsymbol{R}_s(\boldsymbol{w},\nabla\boldsymbol{w})\rangle_{ij}n_{ij}^{(s)}\cdot[\boldsymbol{\varphi}]_{ij}\,\mathrm{d}S - \\ &\quad \sum_{i\in I}\sum_{j\in\gamma_D(i)}\int_{\Gamma_{ij}}\sum_{s=1}^{2}\boldsymbol{R}_s(\boldsymbol{w},\nabla\boldsymbol{w})n_{ij}^{(s)}\cdot\boldsymbol{\varphi}\,\mathrm{d}S. \end{aligned} \tag{20}$$

4.3. Penalty terms

To ensure good properties of the resulting numerical scheme we introduce the interior and boundary penalty form

$$J_h(\boldsymbol{w},\boldsymbol{\varphi}) = \sum_{i\in I}\sum_{\substack{j\in s(i)\\ j<i}}\int_{\Gamma_{ij}}\sigma[\boldsymbol{w}]_{ij}\cdot[\boldsymbol{\varphi}]_{ij}\,\mathrm{d}S + \sum_{i\in I}\sum_{j\in\gamma_D(i)}\int_{\Gamma_{ij}}\sigma\boldsymbol{w}\cdot\boldsymbol{\varphi}\,\mathrm{d}S, \tag{21}$$

where the weight σ is defined as $\sigma|_{\Gamma_{ij}} = \frac{C_W}{Re\, d(\Gamma_{ij})}$, with a constant $C_W > 0$ and the Reynolds number Re. Integrals over the Dirichlet boundary are balanced by terms enforcing the Dirichlet boundary conditions:

$$l_h(\boldsymbol{w},\boldsymbol{\varphi}) = \sum_{i\in I}\sum_{j\in\gamma_D(i)}\int_{\Gamma_{ij}}\sigma\boldsymbol{w}_B\cdot\boldsymbol{\varphi}\,\mathrm{d}S. \tag{22}$$

The state $\boldsymbol{w}_B$ is determined on the basis of the Dirichlet boundary data and extrapolation.

We can finally write the discrete (IIPG) formulation of system (15). Find $\boldsymbol{w}_h(t) \in [S_{ht}]^4$ such that

$$\begin{aligned} &\sum_{K_i\in\mathcal{T}_{ht}}\int_{K_i}\frac{D^{\mathcal{A}}\boldsymbol{w}_h(t)}{Dt}\cdot\boldsymbol{\varphi}\,\mathrm{d}\boldsymbol{x} + b_h(\boldsymbol{w}_h(t),\boldsymbol{\varphi}) + \sum_{K_i\in\mathcal{T}_{ht}}\int_{K_i}\mathrm{div}\boldsymbol{z}\,(\boldsymbol{w}_h(t)\cdot\boldsymbol{\varphi})\;\mathrm{d}\boldsymbol{x} + \\ &\quad a_h(\boldsymbol{w}_h(t),\boldsymbol{\varphi}) + J_h(\boldsymbol{w}_h(t),\boldsymbol{\varphi}) = l_h(\boldsymbol{w}_h,\boldsymbol{\varphi}), \quad \forall\boldsymbol{\varphi}\in S_{ht}. \end{aligned} \tag{23}$$

5. Time discretization

Scheme (23) represents a system of ordinary differential equations, which must be discretized with respect to time. We use the method developed in [2]. A backward Euler method is used and the nonlinear terms in the scheme are linearized.

We consider a partition $0 = t_0 < t_1 < t_2 \ldots$ of the interval $(0, T)$ and set $\tau_k = t_{k+1} - t_k$. We use the symbol $\boldsymbol{w}_h^k$ for the approximation of $\boldsymbol{w}_h(t_k)$. The ALE time derivative is approximated by

$$\frac{D^{\mathcal{A}}\boldsymbol{w}_h}{Dt}(\boldsymbol{x}, t_{k+1}) \approx \frac{\boldsymbol{w}_h^{k+1}(\boldsymbol{x}) - \hat{\boldsymbol{w}}_h^k(\boldsymbol{x})}{\tau_k}, \quad \boldsymbol{x}\in\Omega_{ht_{k+1}}, \tag{24}$$

where $\hat{\boldsymbol{w}}_h^j(\boldsymbol{x}) = \boldsymbol{w}^j\left(\mathcal{A}_{t_j}\left(\mathcal{A}_{t_{k+1}}^{-1}\right)(\boldsymbol{x})\right), \quad \boldsymbol{x}\in\Omega_{ht_{k+1}}$. The convective form in (23) is linearized using the homogeneity of the Euler fluxes and the Vijayasundaram numerical flux:

$$b_h(\boldsymbol{w}_h^{k+1}, \boldsymbol{\varphi}) \approx \tilde{b}_h(\hat{\boldsymbol{w}}_h^k, \boldsymbol{w}^{k+1}, \boldsymbol{\varphi}) = -\sum_{i \in I} \int_{K_i} \sum_{s=1}^{2} \frac{D\boldsymbol{g}_s(\hat{\boldsymbol{w}}_h^k)}{D\boldsymbol{w}} \boldsymbol{w}_h^{k+1} \cdot \frac{\partial \boldsymbol{\varphi}_h}{\partial x_s} \, \mathrm{d}\boldsymbol{x} + \tag{25}$$

$$\sum_{i \in I} \sum_{j \in S(i)} \int_{\Gamma_{ij}} \left\{ \mathbb{P}^+ \left(\langle \hat{\boldsymbol{w}}_h^k \rangle_{ij}, \boldsymbol{n}_{ij} \right) \boldsymbol{w}_{hij}^{k+1} + \mathbb{P}^- \left(\langle \hat{\boldsymbol{w}}_h^k \rangle_{ij}, \boldsymbol{n}_{ij} \right) \boldsymbol{w}_{hji}^{k+1} \right\} \cdot \boldsymbol{\varphi} \, \mathrm{d}S.$$

Viscous terms (20) are linearized in a similar fashion using the fact that the viscous terms $\boldsymbol{R}_i(\boldsymbol{w}, \nabla \boldsymbol{w})$ can be expressed in the form (9). Thus, we linearize the nonlinearities in the form $a_h(\boldsymbol{w}_h^{k+1}, \varphi)$ in the following way:

$$\boldsymbol{R}_i(\boldsymbol{w}_h^{k+1}, \nabla \boldsymbol{w}_h^{k+1}) \approx \sum_{j=1}^{2} \mathbb{K}_{ij}(\hat{\boldsymbol{w}}_h^k) \frac{\partial \boldsymbol{w}_h^{k+1}}{\partial x_j}. \tag{26}$$

Finally, the interior and boundary penalty jump terms J_h are linear with respect to $\boldsymbol{w}_h^{k+1}$ and can be treated implicitly. Hence, we obtain a numerical scheme which requires the solution of only one large sparse linear system per time level. This system is solved by the block-Jacobi preconditioned GMRES method.

6. Numerical experiments

We consider compressible flow in a channel, whose geometry is inspired by the shape of vocal folds and a part of supraglottal spaces as shown in Figure 2. The considered sizes of the domain are summarised in Table 1. The lower channel wall between points A and B is changing the shape according to the given function of time and axial coordinate:

$$\begin{aligned} y(x,t) &= (a_1 + a_t) \left[\sin\left(\frac{3\pi}{2} + \pi \frac{x - x_A}{x_C - x_A} \right) + 1 \right] + d, \ x \in [x_A, x_C], \\ y(x,t) &= 2(a_1 + a_t) \cos\left(\frac{\pi}{2} \frac{x - x_C}{x_B - x_C} \right) + d, \ x \in [x_C, x_B], \\ a_t &= a_2 \sin(2\pi f t), \ t \in [0, T]; \ a_1 = 0.18, \ a_2 = 0.015, \end{aligned} \tag{27}$$

where $f = 5.38 \cdot 10^3$ is dimensionless frequency of the vocal folds oscillation corresponding to the real frequency 100 Hz. The motion of the upper wall of the channel is treated in a similar way. This movement is interpolated to the rest of the domain resulting in the ALE mapping $\mathcal{A}_t$. The computation for the same computational domain and input data can be found in [7], where similar computations were carried out by the finite volume method and assuming the symmetry of the flow field. We considered the following input parameters and boundary conditions for the airflow: inlet flow velocity 4 m/s, viscosity $15 \cdot 10^{-6}$ Pa $\cdot$ s, density $1.225 \mathrm{kg/m^3}$, outlet pressure 9 7611 Pa, $\mathrm{Re} = 10\,453$, $k = 2.428 \cdot 10^{-2}$, $c_v = 721.428$.

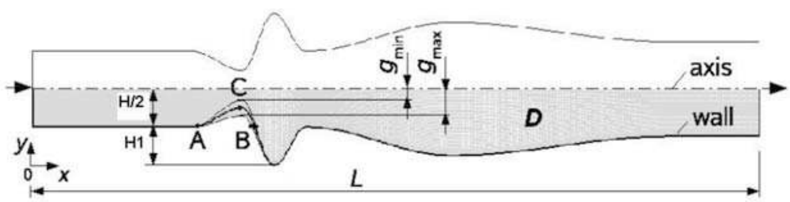

Fig. 2. Computational domain (cf. [7])

Table 1. Dimensions of the computational domain

	x [–]	y [-]	x [mm]	y [mm]
A	1.75	0.4	35	8
B	2.4	0.4	48	8
C	2.3	$y(x_C, t)$	46	$20y(x_C, t)$
g_{min}	–	0.01	–	0.2
g_{max}	–	0.07	–	1.4
L	8	–	160	–
H/2	–	0.4	–	8
H1	–	0.4	–	8

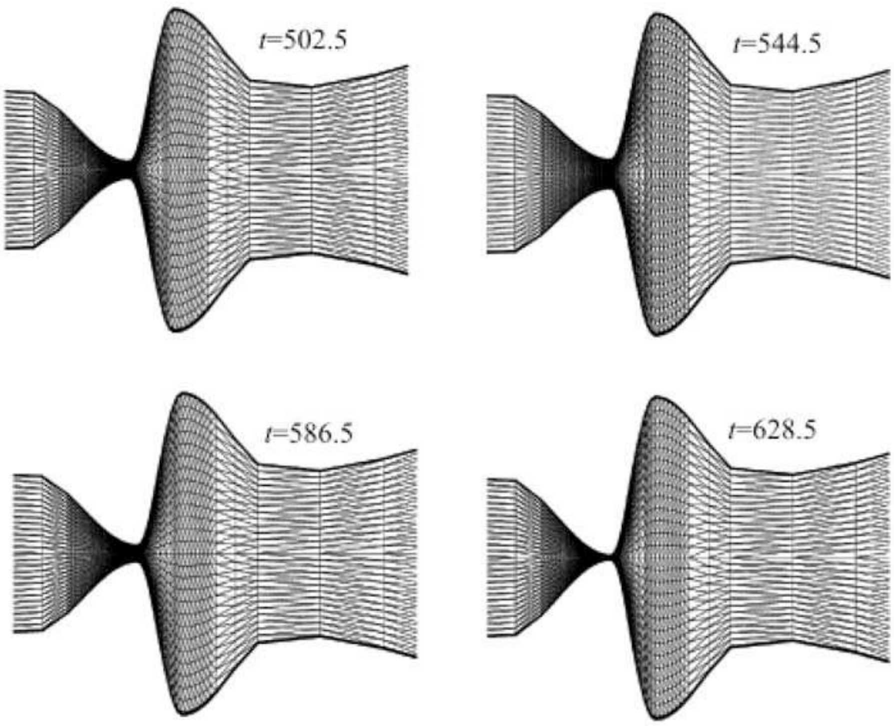

Fig. 3. Detail of triangulation in several time instants

Figures 3, 4, 5 show the used triangulation (consisting of 1829 vertices and 3480 elements) and computed streamlines and flow velocity vectors at different dimensionless time instants $t = 502.5$, 544.5, 586.5, 628.5 during the fourth period of the motion. In Figures 4 and 5 we can observe large vortex formations. These vortices are slowly convected downstream through the domain. The velocity flow field pattern is not periodic and not axisymmetric in spite the computational domain is axisymmetric and the motion of the channel walls is periodic.

Numerical experiments with the formulated problem were also carried out on meshes with different structure having 2 804 vertices and 5 325 elements or 3 469 vertices and 6 645 elements. In all cases quadratic elements were used. It is possible to say that the quality of the results obtained on these meshes is comparable.

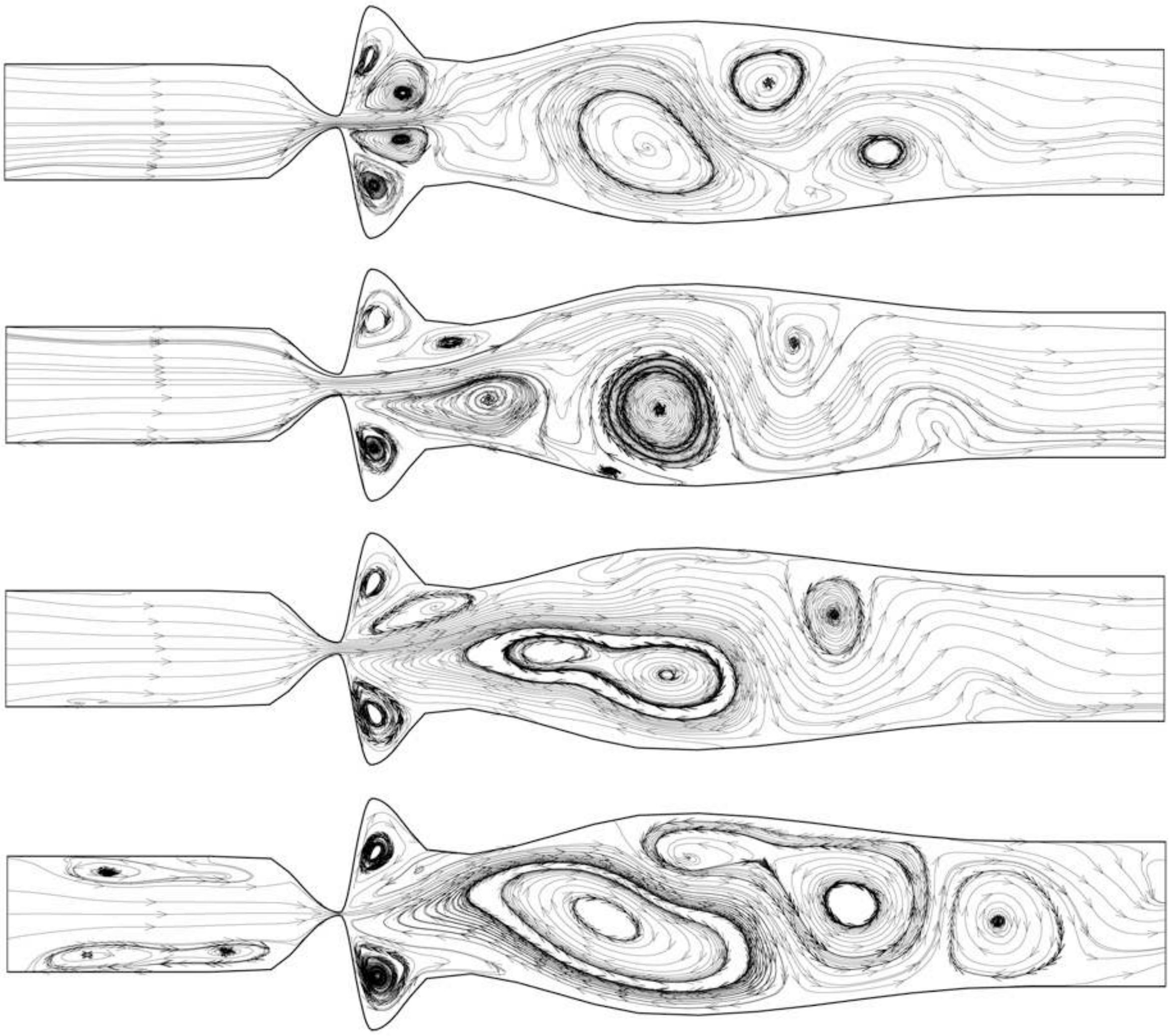

Fig. 4. Streamlines at several time instants ($t = 502.5$, 544.5, 586.5, 628.5)

7. Conclusion

The numerical technique based on the use of the discontinuous Galerkin method and the special program code for solving the 2D unsteady Navier-Stokes equations for viscous compressible flow in time-dependent domains has been developed. This method has been applied to the numerical solution of the airflow in a simplified model of the human glottis geometry with prescribed oscillations of the vocal folds. Computations show that it is not possible to simplify the mathematical model supposing an axisymmetry of the solution, because the nonsymmetric flow structure is developed even in the axisymmetric computational domain.

Future work will be focused on a more complex modelling of the real geometry of the glottis and the vocal tract and mainly on the application of the fluid-structure interaction consisting in the solution of coupled system describing flow and structure behaviour.

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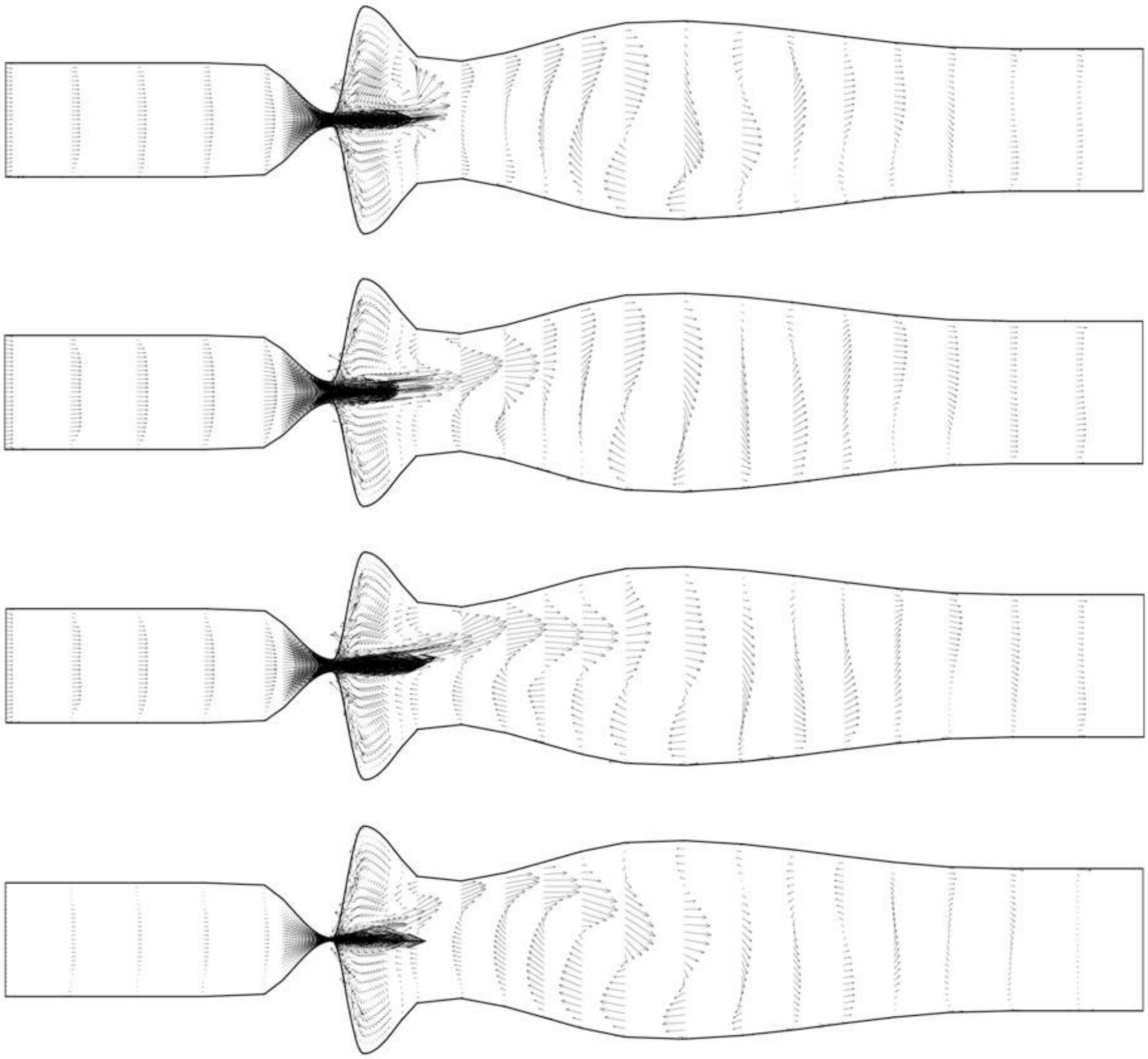

Fig. 5. Velocity vectors at several time instants ($t = 502.5$, 544.5, 586.5, 628.5)

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