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Estimation of the nuclear fuel assembly eigenfrequencies in the probability sense

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Abstract

The paper deals with upper and lower limits estimation of the nuclear fuel assembly eigenfrequencies, whose design and operation parameters are random variables. Each parameter is defined by its mean value and standard deviation or by a range of values. The gradient and three sigma criterion approach is applied to the calculation of the upper and lower limits of fuel assembly eigenfrequencies in the probability sense. Presented analytical approach used for the calculation of eigenfrequencies sensitivity is based on the modal synthesis method and the fuel assembly decomposition into six identical revolved fuel rod segments, centre tube and load-bearing skeleton linked by spacer grids. The method is applied for the Russian TVSA-T fuel assembly in the WWER1000/320 type reactor core in the Czech nuclear power plant Temelín.

Keywords: nuclear fuel assembly, eigenfrequency, random parameters, sensitivity

1. Introduction

Deterministic vibration of the nuclear fuel assembly exists only if there is perfect control over all design and operational parameters. The vibrational response of the fuel assembly depends on many parameters that cannot be precisely predicted. Every parameter p_j, magnitude of which cannot be precisely predicted, is taken as a random variable characterised by a mean value $\bar{p_j}$ and a range of values $\Delta p_j = p_{j\max} - p_{j\min}$. If the random variables have Gaussian distribution, then the probability of p_j lying inside the interval $\bar{p_j} \pm 3\sigma_j$ is 0.997 3 [4], where σ_j is the standard deviation of p_j calculated according to the condition

$$\sigma_j = \frac{1}{6}\Delta p_j\,, \quad j = 1, \ldots, s\,. \tag{1}$$

The goal of the paper, in direct sequence at deterministic interpretation of fuel assembly modal analysis [5], is a presentation of the probabilistic method for the estimation of the upper and lower limits of fuel assembly eigenfrequencies.

2. Conservative mathematical model of the fuel assembly

For the modelling purposes, the fuel assembly of hexagonal type, Fig. 1, is divided into subsystems — six identical rod segments $(s = 1, \ldots, 6)$, centre tube (CT) and load-bearing skeleton (LS), see the cross-section shown in Fig. 2. Each rod segment of the Russian TVSA-T fuel assembly (in Fig. 2, it is drawn in the lateral cross section and circumscribed by triangles) is

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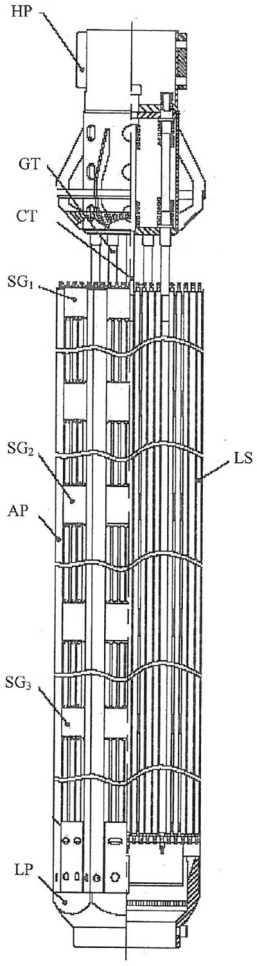

Fig. 1. Fuel assembly

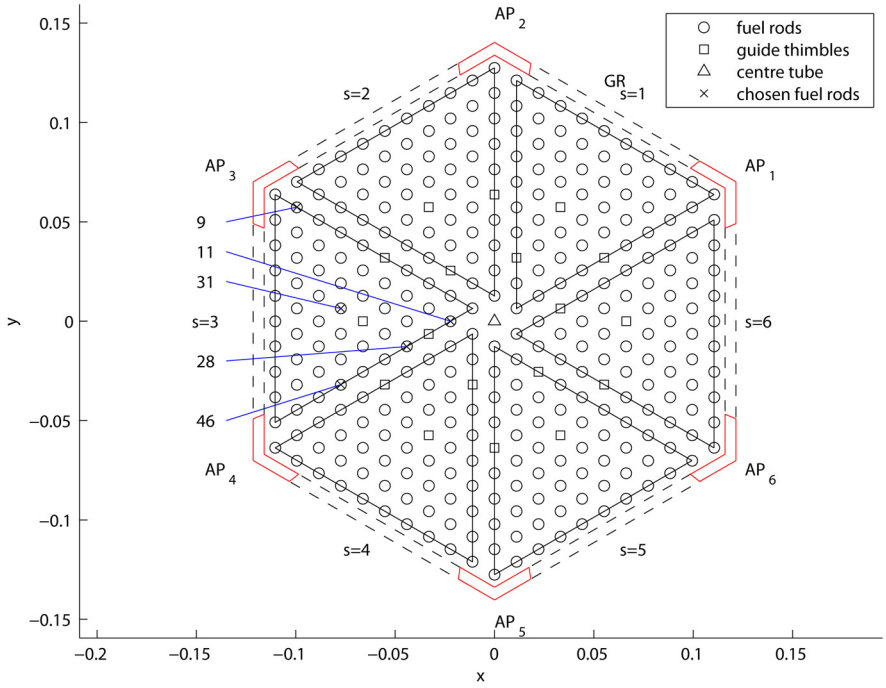

Fig. 2. Cross-section of fuel assembly

composed of 52 fuel rods with fixed bottom ends in lower piece (LP) and 3 guide thimbles (GT) fully restrained in lower and head pieces (HP). The fuel rods and guide thimbles are linked by transverse spacer grids $(g = 1, \ldots, 8)$ of three types (SG1-SG3) inside the segments. All fuel assembly components are modelled as one-dimensional continuum of beam type with nodal points located in corresponding gravity centres of their cross-section on the level of the spacer grids. The mathematical models of the six segments $(s = 1, \ldots, 6)$ are identical in consequence of radial $\xi_{r,g}^{(s)}$ and orthogonal $\eta_{r,g}^{(s)}$ fuel rods and guide thimbles lateral displacements and bending angles $\vartheta_{r,g}^{(s)}$, $\psi_{r,g}^{(s)}$ around these lateral displacements on the level of the spacer grid g. Then, the fuel assembly conservative model in the configuration space

$$\boldsymbol{q} = [\boldsymbol{q}_1^T, \ldots, \boldsymbol{q}_6^T, \boldsymbol{q}_{CT}^T, \boldsymbol{q}_{LS}^T]^T \tag{2}$$

of dimension $n = 6n_S + n_{CT} + n_{LS}$ can be written [5] as

$$\boldsymbol{M}\ddot{\boldsymbol{q}} + (\boldsymbol{K} + \boldsymbol{K}_C)\boldsymbol{q} = \boldsymbol{0}\,. \tag{3}$$

The block diagonal mass and stiffness matrices appearing in Eq. (3) and given as

$$\boldsymbol{M} = \mathrm{diag}[\boldsymbol{M}_S, \ldots, \boldsymbol{M}_S, \boldsymbol{M}_{CT}, \boldsymbol{M}_{LS}]\,, \quad \boldsymbol{K} = \mathrm{diag}[\boldsymbol{K}_S, \ldots, \boldsymbol{K}_S, \boldsymbol{K}_{CT}, \boldsymbol{K}_{LS}]$$

correspond to a fictitious fuel assembly that is divided into mutually uncoupled subsystems — the six rod segments (subscript S), the centre tube (subscript CT) and the load-bearing skeleton (subscript LS). The couplings between the subsystems are expressed by means of a coupling stiffness matrix

$$\boldsymbol{K}_C = \boldsymbol{K}_{S,S} + \boldsymbol{K}_{S,CT} + \boldsymbol{K}_{S,LS}\,, \tag{4}$$

where the matrices $\boldsymbol{K}_{S,S}$, $\boldsymbol{K}_{S,CT}$ and $\boldsymbol{K}_{S,LS}$ express the interaction between all rod segments ($\boldsymbol{K}_{S,S}$) and other appropriate subsystems as denoted by the subscripts. Note that all matrices appearing in Eqs. (3) and (4) are derived in the monograph [3].

Because the fuel assembly model given by Eq. (3) has too many degrees of freedom (DOF), that is the DOF number n is too large, it is appropriate to compile the condensed conservative model of the fuel assembly by using the modal synthesis method. The global vector of generalised coordinates defined in Eq. (2) is transformed by means of modal matrices (submatrices) of the subsystems into the following form:

$$\begin{bmatrix} \boldsymbol{q}_1 \\ \boldsymbol{q}_2 \\ \vdots \\ \boldsymbol{q}_6 \\ \boldsymbol{q}_{CT} \\ \boldsymbol{q}_{LS} \end{bmatrix} = \begin{bmatrix} {}^m\boldsymbol{V}_S & \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & {}^m\boldsymbol{V}_S & \dots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \vdots & & & & & \\ \boldsymbol{0} & \boldsymbol{0} & \dots & {}^m\boldsymbol{V}_S & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{V}_{CT} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \dots & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{V}_{LS} \end{bmatrix} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \\ \vdots \\ \boldsymbol{x}_6 \\ \boldsymbol{x}_{CT} \\ \boldsymbol{x}_{LS} \end{bmatrix} \Rightarrow \boldsymbol{q} = {}^m\boldsymbol{V}\boldsymbol{x}\,. \tag{5}$$

The matrix ${}^m\boldsymbol{V}_S \in R^{n_S,m_S}$ is the modal submatrix of the isolated rod segment S corresponding to its m_S low-frequency eigenvectors and $\boldsymbol{V}_{CT} \in R^{n_{CT},n_{CT}}$, $\boldsymbol{V}_{LS} \in R^{n_{LS},n_{LS}}$ are the modal matrices of the mutually isolated subsystems CT and LS, respectively. All modal matrices (submatrices) fulfil the orthogonality condition

$${}^m\boldsymbol{V}_S^T\boldsymbol{M}_S\,{}^m\boldsymbol{V}_S = \boldsymbol{E}_{m_S}, \quad \boldsymbol{V}_{CT}^T\boldsymbol{M}_{CT}\boldsymbol{V}_{CT} = \boldsymbol{E}_{n_{CT}}, \quad \boldsymbol{V}_{LS}^T\boldsymbol{M}_{LS}\boldsymbol{V}_{LS} = \boldsymbol{E}_{n_{LS}}\,, \tag{6}$$

where $\boldsymbol{E}$ denotes a unit matrix of dimension given by its subscripts. The application of the transformation (5) to the fuel assembly model given by Eq. (3) yields the condensed conservative model of the fuel assembly in the form

$$\ddot{\boldsymbol{x}} + [{}^m\boldsymbol{\Lambda} + {}^m\boldsymbol{V}^T(\boldsymbol{K} + \boldsymbol{K}_C)^m\boldsymbol{V}]\boldsymbol{x} = \boldsymbol{0}\,. \tag{7}$$

The diagonal matrix

$${}^m\boldsymbol{\Lambda} = \mathrm{diag}[{}^m\boldsymbol{\Lambda}_S, \dots, {}^m\boldsymbol{\Lambda}_S, \boldsymbol{\Lambda}_{CT}, \boldsymbol{\Lambda}_{LS}]$$

is compiled from spectral matrices of mutually isolated subsystems. The condensed model has the DOF number of $m = 6m_S + n_{CT} + n_{LS}$, where $m_S \ll n_S$. The eigenfrequencies Ω_i and eigenvectors $\boldsymbol{x}_i$, $i = 1, \dots, m$ of the condensed model (7) are used below for the sensitivity analysis of the fuel assembly eigenfrequencies.

3. Upper and lower limits of the fuel assembly eigenfrequencies

Let us assume that the matrices in the fuel assembly model (3) are dependent on stochastic independent random parameters $\boldsymbol{p} = [p_j] \in R^s$. These parameters are defined by their mean values $\bar{\boldsymbol{p}}$ and the diagonal covariance matrix $\boldsymbol{\Sigma}_p = \mathrm{diag}[\sigma_j^2] \in R^{s,s}$. With respect to Eq. (1), each diagonal element of the covariance matrix

$$\sigma_j^2 = \frac{1}{36}\mathrm{diag}[\Delta p_j^2],\ j = 1, \dots, s \tag{8}$$

corresponds to the range Δp_j of the j-th random parameter. By using the gradient method and on the condition that the standard deviations σ_j are relatively small, we can write the approximate relation for the covariance matrix of the fuel assembly eigenfrequencies [2] in the form

$$\boldsymbol{\Sigma}_\Omega \doteq \left(\frac{\partial \boldsymbol{\Omega}(\boldsymbol{p})}{\partial \boldsymbol{p}^T}\right)_{\boldsymbol{p}=\bar{\boldsymbol{p}}} \boldsymbol{\Sigma}_p \left(\frac{\partial \boldsymbol{\Omega}(\boldsymbol{p})}{\partial \boldsymbol{p}^T}\right)^T_{\boldsymbol{p}=\bar{\boldsymbol{p}}}, \tag{9}$$

where $\boldsymbol{\Omega}(\boldsymbol{p}) = [\Omega_i(\boldsymbol{p})]$, $i = 1, \ldots, m$ is the fuel assembly low-eigenfrequency vector of dimension m. The mean values of fuel assembly eigenfrequencies are approximated as

$$\bar{\Omega}_i(\boldsymbol{p}) \doteq \Omega_i(\bar{\boldsymbol{p}}),\ i = 1, \ldots, m\,. \tag{10}$$

The diagonal elements $\sigma^2_{\Omega_i}$ of the covariance matrix $\boldsymbol{\Sigma}_\Omega$ can be used for the calculation of upper and lower limits of the fuel assembly eigenfrequencies

$$\Omega_{i\min} = \bar{\Omega}_i - 3\sqrt{\sigma^2_{\Omega_i}}, \qquad \Omega_{i\max} = \bar{\Omega}_i + 3\sqrt{\sigma^2_{\Omega_i}} \tag{11}$$

according to the 'three sigma criterion' in the probability sense.

4. Sensitivity analysis of the fuel assembly eigenfrequencies

Elements $\frac{\partial \Omega_i}{\partial p_j}$ of the sensitivity matrix $\frac{\partial \boldsymbol{\Omega}(\boldsymbol{p})}{\partial \boldsymbol{p}^T}$ in Eq. (9) express the absolute sensitivity of the eigenfrequency Ω_i with respect to the parameter p_j. The differentiation of the fuel assembly eigenvalue problem

$$(\boldsymbol{K} + \boldsymbol{K}_C - \Omega_i^2 \boldsymbol{M})\boldsymbol{v}_i = \boldsymbol{0}\,, \quad i = 1, \ldots, m$$

with respect to design or operation parameter p_j and its multiplication by the transposed eigenvector $\boldsymbol{v}_i^T$ yields

$$\frac{\partial \Omega_i}{\partial p_j} = \frac{1}{2\Omega_i} \boldsymbol{v}_i^T \left(\frac{\partial \boldsymbol{K}}{\partial p_j} + \frac{\partial \boldsymbol{K}_C}{\partial p_j} - \Omega_i^2 \frac{\partial \boldsymbol{M}}{\partial p_j}\right) \boldsymbol{v}_i\,, \quad i = 1, \ldots, m,\ j = 1, \ldots, s\,, \tag{12}$$

where according to Eq. (5), $\boldsymbol{v}_i = {}^m\boldsymbol{V}\boldsymbol{x}_i$. The eigenfrequencies Ω_i and eigenvectors $\boldsymbol{x}_i$ are calculated from the eigenvalue problem of the condensed model (7)

$$[{}^m\boldsymbol{\Lambda} + {}^m\boldsymbol{V}^T(\boldsymbol{K} + \boldsymbol{K}_C){}^m\boldsymbol{V} - \Omega_i^2 \boldsymbol{E}]\boldsymbol{x}_i = \boldsymbol{0}\,, \quad i = 1, \ldots, m\,. \tag{13}$$

Here, the eigenvectors $\boldsymbol{x}_i$ fulfil the condition $\boldsymbol{x}_i^T \boldsymbol{x}_i = 1$.

The sensitivity matrix appearing in Eq. (9) can be written in the form

$$\left[\frac{\partial \boldsymbol{\Omega}(\boldsymbol{p})}{\partial \boldsymbol{p}^T}\right]_{\boldsymbol{p}=\bar{\boldsymbol{p}}} = \left[\begin{array}{ccc} \frac{\partial \Omega_1}{\partial p_1} & \cdots & \frac{\partial \Omega_1}{\partial p_s} \\ \vdots & & \\ \frac{\partial \Omega_m}{\partial p_1} & \cdots & \frac{\partial \Omega_m}{\partial p_s} \end{array}\right]_{\boldsymbol{p}=\bar{\boldsymbol{p}}} \in R^{m,s}\,. \tag{14}$$

According to Eq. (12), the elements of the eigenfrequency sensitivity matrix are given as

$$\left[\frac{\partial \Omega_i}{\partial p_j}\right]_{\boldsymbol{p}=\bar{\boldsymbol{p}}} \doteq \frac{1}{2\Omega_i(\bar{\boldsymbol{p}})} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \left(\frac{\partial \bar{\boldsymbol{K}}}{\partial p_j} + \frac{\partial \bar{\boldsymbol{K}}_C}{\partial p_j} - \Omega_i^2(\bar{\boldsymbol{p}}) \frac{\partial \bar{\boldsymbol{M}}}{\partial p_j}\right) {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}})\,, \tag{15}$$

where

$${}^m\bar{\boldsymbol{V}} = {}^m\boldsymbol{V}(\bar{\boldsymbol{p}})\,, \quad \frac{\partial \bar{\boldsymbol{K}}}{\partial p_j} = \frac{\partial \boldsymbol{K}(\bar{\boldsymbol{p}})}{\partial p_j}\,, \quad \frac{\partial \bar{\boldsymbol{K}}_C}{\partial p_j} = \frac{\partial \boldsymbol{K}_C(\bar{\boldsymbol{p}})}{\partial p_j}\,, \quad \frac{\partial \bar{\boldsymbol{M}}}{\partial p_j} = \frac{\partial \boldsymbol{M}(\bar{\boldsymbol{p}})}{\partial p_j}\,.$$

5. Application

The methodology presented above was applied to the Russian TVSA-T fuel assembly used in the Czech nuclear power plant Temelín [5]. For the purpose of this paper, we will introduce the mean values and the limits of the fuel assembly eigenfrequencies, both calculated according to Eqs. (10) and (11), respectively. The chosen random parameters t_{AP} (thickness of the angle pieces sheet), k_g (lateral stiffness of the couplings between fuel rods by means of spacer grig cells on all levels), ϱ_x (mass density), E_x (Young's modulus) of the fuel rods cladding from zirconium ($x = C$), fuel pellets ($x = P$) and guide thimbles ($x = GT$) and F (pressure axial load acting on the fuel assembly head piece) are characterised by a mean value $\bar{p}_j$ and a relative range of values $\frac{\Delta p_j}{\bar{p}_j}$, $j = 1, \ldots, 9$ listed in Table 1.

Table 1. Mean values and relative value range of chosen random parameters

p_j	t_{AP}	k_g	ϱ_C	ϱ_P	ϱ_{GT}	E_C	E_P	E_{GT}	F
SI-units	mm	Nm^{-1}	$\mathrm{kg\,m}^{-3}$			GPa			N
$\bar{p}_j$	0.65	89 500	6 550	10 500	6 350	75	216	75	9 760
$\frac{\Delta p_j}{\bar{p}_j}$	0.02	0.1	0.1	0.2	0.1	0.1	0.2	0.1	0.2

With respect to each of the chosen random parameter, the elements of the eigenfrequency sensitivity matrix (14) are expressed according to Eq. (15) as

$$
\begin{aligned}
\frac{\partial \Omega_i}{\partial t_{AP}} &= \frac{1}{2\Omega_i(\bar{\boldsymbol{p}})} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \left(\frac{\partial \bar{\boldsymbol{K}}}{\partial t_{AP}} - \Omega_i^2(\bar{\boldsymbol{p}}) \frac{\partial \bar{\boldsymbol{M}}}{\partial t_{AP}} \right) {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}}) \,, \\
\frac{\partial \Omega_i}{\partial k_g} &= \frac{1}{2\Omega_i(\bar{\boldsymbol{p}})} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \frac{\partial \bar{\boldsymbol{K}}_C}{\partial k_g}\, {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}}) \,, \\
\frac{\partial \Omega_i}{\partial \varrho_x} &= -\frac{\Omega_i(\bar{\boldsymbol{p}})}{2} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \frac{\partial \bar{\boldsymbol{M}}}{\partial \varrho_x}\, {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}}) \,, \quad x = C, P, GT \,, \\
\frac{\partial \Omega_i}{\partial E_x} &= \frac{1}{2\Omega_i(\bar{\boldsymbol{p}})} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \left(\frac{\partial \bar{\boldsymbol{K}}}{\partial E_x} + \frac{\partial \bar{\boldsymbol{K}}_C}{\partial E_x} \right) {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}}) \,, \quad x = C, P, GT \,, \\
\frac{\partial \Omega_i}{\partial F} &= \frac{1}{2\Omega_i(\bar{\boldsymbol{p}})} \boldsymbol{x}_i^T(\bar{\boldsymbol{p}})\, {}^m\bar{\boldsymbol{V}}^T \frac{\partial \bar{\boldsymbol{K}}}{\partial F}\, {}^m\bar{\boldsymbol{V}} \boldsymbol{x}_i(\bar{\boldsymbol{p}})
\end{aligned}
\tag{16}
$$

for $i = 1, \ldots, m$. The mean values and limits of the 20 lowest fuel assembly eigenfrequencies at the temperature 350 °C are summarised in Table 2. Note that in this table, the pairs of eigenfrequencies corresponding to flexural (F) and breathing (B) mode shapes are presented only once and that the unpaired single eigenfrequencies correspond to torsion (T) mode shapes.

Selected typical mode shapes of the fuel assembly sampled from particular mode shape families and calculated for mean values of parameters are shown in Figs. 3–5. From these figures, it can be noted that while the flexural mode shapes are characterised by inphase deformations of all fuel assembly components, the spacer grid cells remain practically non-deformed. Also the torsion mode shapes are characterised by maximal deformations of the outside fuel rods, whereas the breathing modes are characterised by large deformations of the spacer grid cells. All mode shapes are visualised on the cross section of the fuel assembly at the level of the chosen (in Figs. 3–5 on the fourth-central) spacer grid. The visualised abscissae represent the lateral deformations of all fuel assembly components (fuel rods, guide thimbles, centre tube and skeleton angle pieces) on equal scale.

Table 2. Mean values and limits of the fuel assembly eigenfrequencies

i	$\bar{f}_i$[Hz]	mode shape	f_i limits	
			lower	upper
1,2	2.46	F1	2.37	2.56
3	3.37	T1	3.25	3.49
4,5	5.69	F2	5.46	5.91
6	7.74	T2	7.45	8.02
7,8	9.87	F3	9.47	10.28
9	11.68	T3	11.23	12.13
10,11	13.03	B1	12.47	13.60
12,13	14.01	B2	13.42	14.59
14,15	15.16	F4	14.52	15.79
16	16.40	T4	15.73	17.07
17,18	16.43	B3	15.77	17.10
19,20	18.16	B4	17.37	18.95

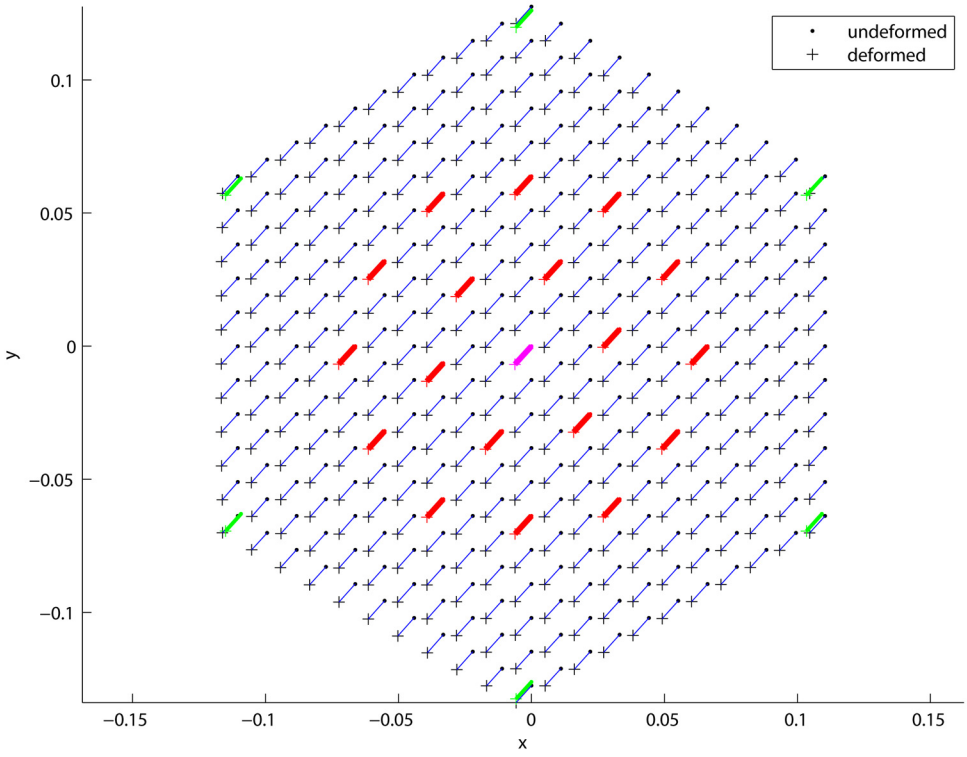

Fig. 3. The first flexural mode shape corresponding to $\bar{f}_1$

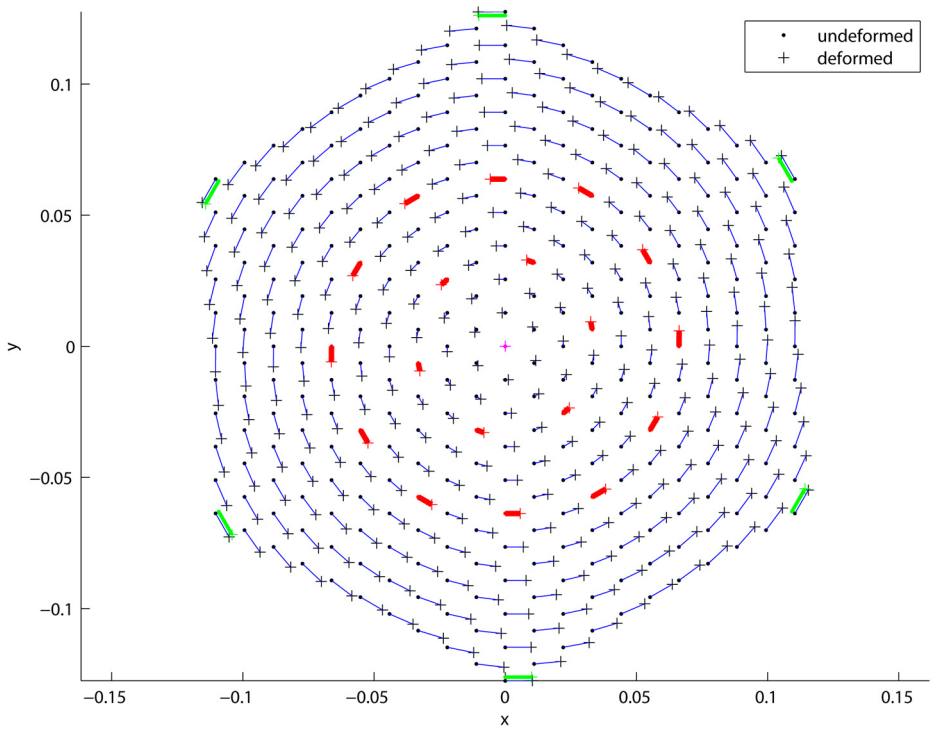

Fig. 4. The first torsional mode shape corresponding to $\bar{f}_3$

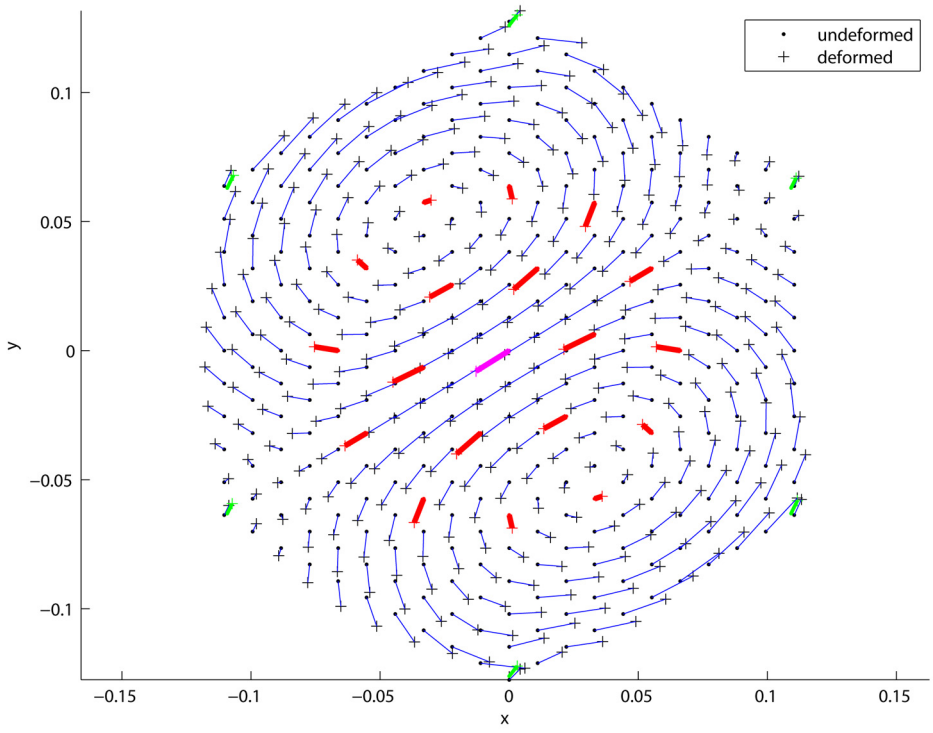

Fig. 5. The fourth breathing mode shape corresponding to $\bar{f}_{19}$

6. Conclusion

The described analytical-numerical method enables to investigate the eigenfrequencies limits of linear vibrating systems with a large number of DOF. The presented approach based on the system decomposition into smaller subsystems, the modal synthesis and the gradient methods enable an efficient calculation of the upper and lower limits of eigenfrequencies in the probability sense. In cases, where an analytical calculation of the partial derivation of eigenfrequencies with respect to design or operation parameters is impossible, a numerical procedure for the calculation of the sensitivity and covariance matrices of the eigenfrequencies can be applied instead. The developed methodology was used for the calculation of eigenfrequencies limits in the nuclear fuel assembly of hexagonal type. The assembly design and operation parameters taken as random variables were chosen on the basis of their possible change during the reactor operation. The principles of the developed method are generally applicable to various mechanical systems.

Acknowledgements

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