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A Global Optimization Algorithm for Rotation Alignment of Spherical Surfaces

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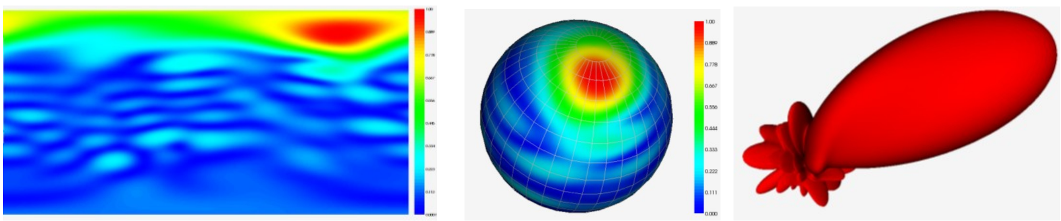
Abstract. We propose a new approach to global optimization algorithm based on controlled random search techniques for rotational alignment of spherical surfaces with associated scalar values. To reduce the distortion in correspondence and increase efficiency, the spherical surface is first re-sampled using a geodesic sphere. The rotation in space is represented using the modified Rodrigues parameters. Correspondence between two spherical surfaces is implemented in the parametric domain. We applied the methods to the alignment of beam patterns computed from the outer ear shapes of bats. The proposed method is compared with other approaches such as alignment based on principal component analysis (PCA), exhaustive search in the discrete space of rotations defined by Euler angles and direct search using uniform samples over the rotation group SO(3). Experimental results demonstrate that the proposed rotation alignment obtained using the proposed algorithm has a high degree of precision and give the best result among the other four approaches.

Keywords. spherical surface; alignment; global optimization; beam pattern

1. Introduction

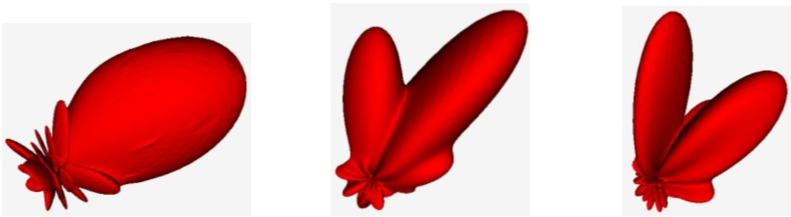
Spherical surfaces with associated scalar values are often used in physics and biometrics. For example, in sonar systems, a beam pattern is scalar gain values as a function of directions and frequencies, which is commonly defined over a spherical surface enclosing the acoustic source. These functions are usually determined by a measurement system or calculated from the geometric shape by numerical methods. A beam pattern can be considered as a real valued function $f(\theta,\phi)$ with its parameterization over the azimuth angle $\theta \in [0, 2\pi]$ and the elevation angle $\phi \in [0, \pi)$ (Figure 1(a)). The surface of a unit sphere with scalar attribute data (gain values) can also represent a beam pattern, as shown in Figure 1(b). Alternatively, a beam pattern can be regarded as a 3D surface transformed from spherical coordinate system (f,θ,ϕ) into Cartesian coordinate system (x, y, z) by $x = f\sin\phi\cos\theta$, $y = f\sin\phi\sin\theta$ and $z = f\cos\phi$ (Figure 1(c)).



(a) function of two variables (b) sphere with attribute data (c) 3D surface

Figure 1 Different beam pattern representations

Beam patterns from bat noseleaf and ear shapes have been used to understand bat echolocations [1-3]. The beam patterns calculated from bat ear shapes present a variety of shapes. Figure 2 gives beam patterns with different lobes. To classify and compare beam patterns, it is important to align beam patterns across various frequencies and shapes into spatially equivalent positions. Aligning two beam patterns requires finding the optimal rotation $R \in SO(3)$, the special orthogonal group of rotations in 3D space, that minimizes the dissimilarity measure between them. Obviously, an exhaustive search over all possible rotations to find the optimal alignment is prohibitively slow. In this paper, we propose a global optimization algorithm based on controlled random search techniques for finding the globally optimal 3D rotation that best aligns spherical surfaces with scalar attribute data. This algorithm is able to find a global minimum of the dissimilarity measure with arbitrary resolutions.



(a) (b) (c)

Figure 2 beam patterns with different lobes
(a) one main lobe with many small side lobes (b) one main lobe with one strong side lobe (c) two lobes of approximately equal strength

2. Related Work

Shape alignment is a well-studied area of research in computer graphics and vision. Iterative Closest Point (ICP) is a popular and well-known algorithm for 3D shape alignment [4][5]. It iteratively revises the translation and rotation transformation needed to minimize the distance between the points of two shapes. However, different points of one shape are possibly mapped to the same point of another shape according to the closest point criteria. ICP algorithm cannot be directly applied in the beam pattern alignment problem which requires one-to-one correspondence between the points of beam patterns.

Another popular approach to computation of the alignment of 3D objects is based on principal component analysis (PCA), which aligns the objects so that their three principal axes computed with PCA are aligned together [6][7]. Experiences show that PCA alignment works well for models for which the principal axes are well distinguished. As most beam patterns show a pattern of lobes at various angles, PCA alignment is often imprecise and can lead to poor alignments for beam patterns.

By transforming the alignment problem into a correlation problem and substituting the real valued function on a 2-sphere with their spherical harmonics expansions, efficient signal processing techniques can be used to find the optimal rotation [9]. However, the aligning result and running time is affected by the band-width of expansion. It would be computationally expensive to achieve high accuracy while using efficient signal processing techniques [8].

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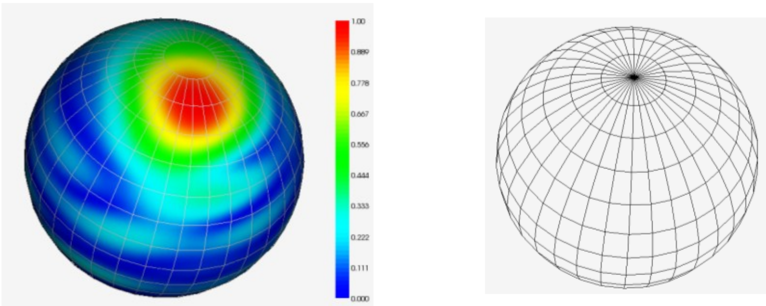
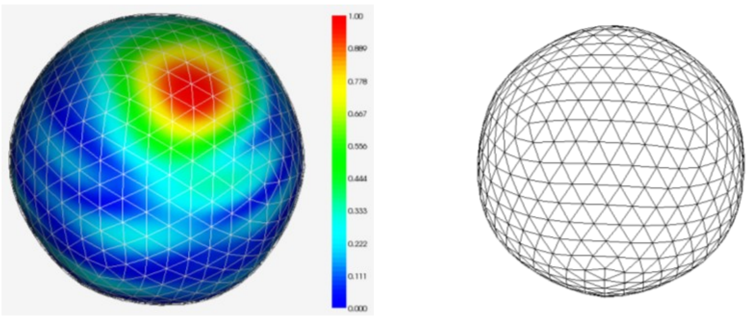
The proposed method is based on a heuristic global optimization algorithm. The global optimization algorithm belongs to the class of controlled random search methods which are very efficient and reliable for the global minimization of nonlinear multivariate functions of several variables [10][11]. This approach was already adopted in the structural alignment of protein surfaces [12]. However, their dissimilarity measure is based on the solution to a costly asymmetric assignment problem on a bipartite graph. The best isometric transformation between two finite sets of points corresponding to protein structures is obtained using a controlled random search method. In contrast, we propose a new correspondence method and dissimilarity measure between two spherical functions.

3. Surface Correspondence and Dissimilarity Measure

A spherical surface with attribute data is usually measured or calculated uniformly in the parametric domain (Figure 3(a)). To reduce the distortion in one-to-one correspondence between the sample points of two spherical surfaces, we first resample the points uniformly according to geodesic distance (Figure 3(b)). Each vertex of an icosphere mesh can be transformed from Cartesian coordinate system (x, y, z) into parametric domain (θ, φ) by $\theta = \tan^{-1}(y/x)$ and $\phi = \cos^{-1}(z)$, whose gain value is then computed using bilinear interpolation in original real valued function $f(\theta, \phi)$.



(a) uniformly sampling in the parametric domain



(b) uniformly sampling according to geodesic distance

Figure 3 Spherical surface sampling

Given two spherical surfaces, a re-sampled query $P=\{p_1,\dots,p_n\}$ and a reference $Q=\{q_1,\dots,q_m\}$, we associate a point q of Q to every sample point p of P using the the closest point criteria. Although finding the nearest point can be substantially optimized by employing special search strategies [13], it is still too slow for millions of points. Instead, we transform the query point p into parametric domain and search in the real valued function representation of Q.

The dissimilarity measure $dist(P,Q)$ between two sets of points, $P=\{p_1,\ldots,p_n\}$ and $Q=\{q_1,\ldots,q_m\}$, is based on the attribute values other than the closest point distance,

$$dist(P,Q)=\frac{1}{n}\sum_{i=1}^{n}\left(g(p_i)-g(\psi(p_i))\right)^2$$

where $p_i \in P$ is mapped to exact one point $\psi(p_i)\in Q$ and $g(p)$ is the gain value at point p. Although we use mean squares metric, other metrics can be used according to the application.

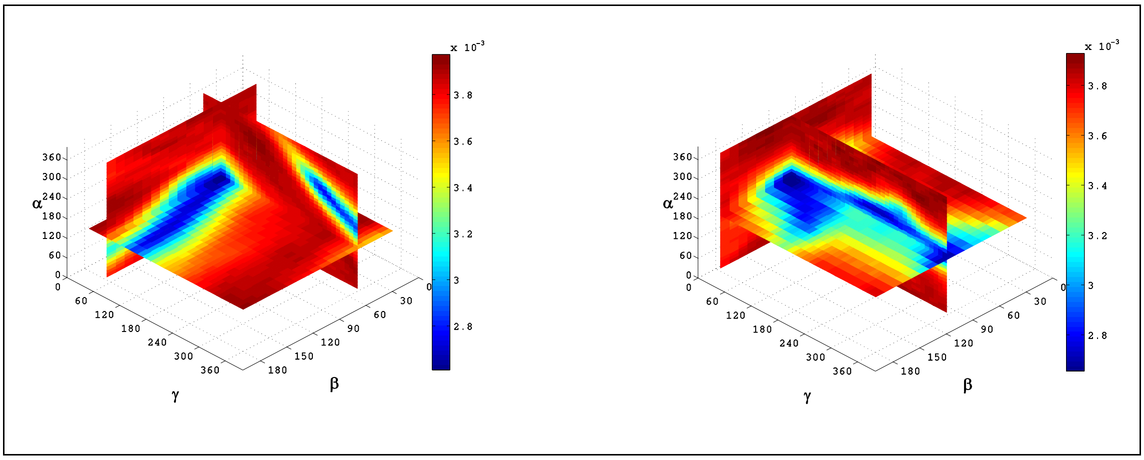
To detect the behavior of dissimilarity measure over all possible rotations, we test possible rotations with an accuracy of 10° using Euler angles. The space of rotations $SO(3)$ can be parameterized using the Euler notation in zyz-convention denoted by the angles γ,β,α with $\gamma,\alpha\in[0,2\pi]$ and $\beta\in[0,\pi]$ [14]. The corresponding rotation matrix is as follows:

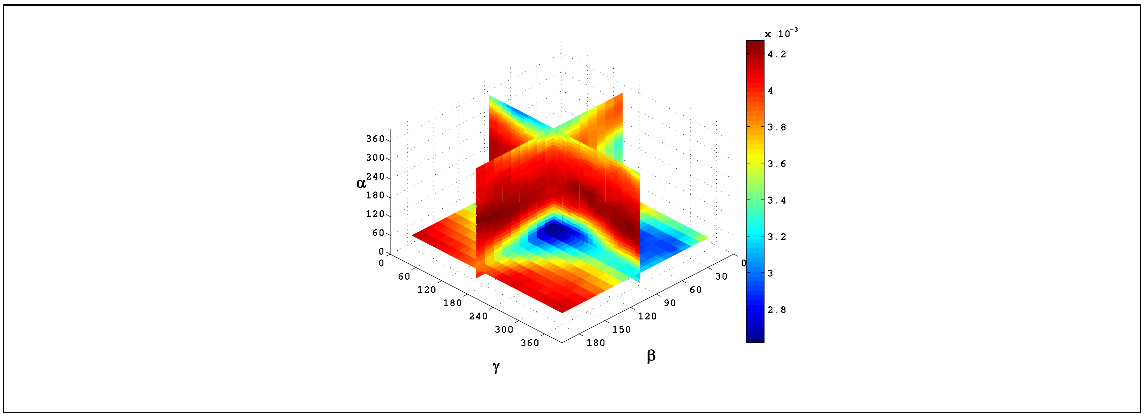
$$R(\gamma,\beta,\alpha)=R_{z,\gamma}R_{y,\beta}R_{z,\alpha}=\begin{bmatrix} c_\gamma & -s_\gamma & 0\\ s_\gamma & c_\gamma & 0\\ 0 & 0 & 1\end{bmatrix}\begin{bmatrix} c_\beta & 0 & s_\beta\\ 0 & 1 & 0\\ -s_\beta & 0 & c_\beta\end{bmatrix}\begin{bmatrix} c_\alpha & -s_\alpha & 0\\ s_\alpha & c_\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$$

$$=\begin{bmatrix} -s_\gamma s_\alpha + c_\gamma c_\beta c_\alpha & -s_\gamma c_\alpha - c_\gamma c_\beta s_\alpha & c_\gamma s_\beta\\ c_\gamma s_\alpha + s_\gamma c_\beta c_\alpha & c_\gamma c_\alpha - s_\gamma c_\beta s_\alpha & s_\gamma s_\beta\\ -s_\beta c_\alpha & s_\beta s_\alpha & c_\beta\end{bmatrix}$$

where $c_\alpha=\cos\alpha$ and $s_\alpha=\sin\alpha$, etc.

For the three beam patterns in Figure 2, their dissimilarity measures are shown in Figure 4. The slices are put at the minimizers. Some dissimilarity measures have long low-value valleys due to imperfect rotational symmetry of beam patterns presented in Figure 2(a).





(a) Figure 2(a) and Figure 2(b) (b) Figure 2(a) and Figure 2(c) (c) Figure 2(b) and Figure 2(c)

Figure 4 dissimilarity measures between surfaces in Figure 2

4. The optimization algorithm

In a naïve method, if we would like to achieve an accuracy of 1°, it would have to test possible rotations of $360\times180\times360\approx2.3\times10^7$. This would be computationally expensive even if we are only considering discrete integer angles. In this section we discuss a continuous global optimization algorithm for the rotation alignment of spherical surfaces. This algorithm is able to find a global minimum of the dissimilarity measure with arbitrary precision.

To obtain arbitrary resolutions of the rotation parameters, we draw our inspiration from controlled random search algorithms for solving global optimization problems [10][11][12]. The controlled random search is a population based optimization algorithm. The algorithm starts by randomly choosing r points over the feasible domain. During the optimization process, the population of r points is iteratively updated by adding a new better point and discarding the worst one. When the number of iterations grows, the population clusters around a neighborhood of a global minimum point.

To parameterize 3D rotation in the optimization process, Euler angles is probably the most widely used technique. However, Euler angles is a redundant representation of rotations and many combinations of Euler angles correspond to the same rotation matrix. In the controlled random search algorithm, exploring the whole range $[0,2\pi]\times[0,2\pi]\times[0,\pi]$ will waste computation time on some duplicate rotation matrices. To overcome the disadvantage, we employ the modified Rodrigues parameters to represent 3D rotations. The modified Rodrigues parameters can be defined by $\boldsymbol{\sigma}=\mathbf{n}\tan(\theta/4)$ where $\boldsymbol{\sigma}$ is a 3×1 vector, $\mathbf{n}$ is a unit vector corresponding to the axis of rotation and θ is the angle of rotation [15]. In order to represent all rotation matrices, the following range for the three components of $\boldsymbol{\sigma}$ is needed: $[-1,1]\times[-1,1]\times[-1,1]$. $\mathbf{n}$=$\{a,b,c\}$ and θ can be computed from $\boldsymbol{\sigma}$. The rotation matrix corresponding to the modified Rodrigues parameters is as follows [14]:

$$\begin{bmatrix} a^2k+\cos\theta & abk-c\sin\theta & ack+b\sin\theta \\ abk+c\sin\theta & b^2k+\cos\theta & bck-a\sin\theta \\ ack-b\sin\theta & bck+a\sin\theta & c^2k+\cos\theta \end{bmatrix}$$

where $k=1-\cos\theta$.

The procedure described below is a modification of Price's algorithm[10]:

Input:

(a) Two sets of points, $P=\{p_1,\ldots,p_n\}$ and $Q=\{q_1,\ldots,q_m\}$

(b) Gain values on the points, $\{g(p_1),\ldots,g(p_n)\}$ and $\{g(q_1),\ldots,g(q_m)\}$

Step 1: initialization

a) Set k=3, the number of parameters of $SO(3)$.

b) Choose an integer $r \gg k$. A commonly used value for that is r=25k.

c) Set a small positive value for ε. $\varepsilon=10^{-6}$ has been used in our experiments.

d) Randomly choose a set $S=\{s_1,\ldots,s_r\}$ of r triples from the feasible domain

$[-1,1]\times[-1,1]\times[-1,1]$

e) Evaluate the value of the objective function over S,

$$dist(s_i)=dist(R(s_i)P,Q)=\frac{1}{n}\sum_{j=1}^{n}\left|g(p_j)-g(\psi(Rp_j))\right|,\ i=1,\ldots,r$$

Step 2: stopping criterion

a) Select $s_{\min}\in S$ and $s_{\max}\in S$ that yield the minimum and maximum values of the objective function correspondingly. Set

$dist_{\min}=dist(s_{\min})$, $dist_{\max}=dist(s_{\max})$

b) If $dist_{\max}-dist_{\min}<\varepsilon$, **return** $s_{\min}$ as the discovered global minimum and stop.

Step 3: new search

a) Randomly select a reduced set $T=\{t_1,\ldots,t_{k+1}\}$ from S

b) Compute the weighted centroid s_c of the first k triples, $s_c=\frac{1}{k}\sum_{j=1}^{k}t_j$

c) Compute a trial triple $s_{trial}=2s_c-t_{k+1}$

d) If $s_{trial}\notin[-1,1]\times[-1,1]\times[-1,1]$, modulate it to the domain.

Step 4: updating population

a) If $dist(s_{trial})<dist_{\max}$, take $S=S\cup\{s_{trial}\}-\{s_{\max}\}$

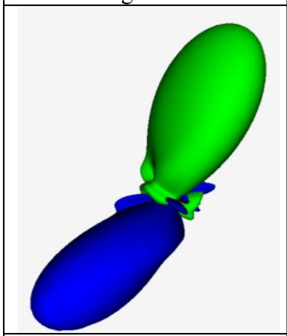
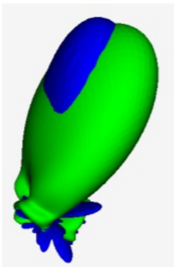
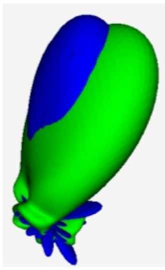
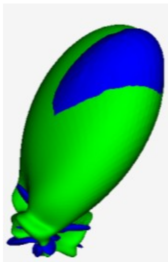
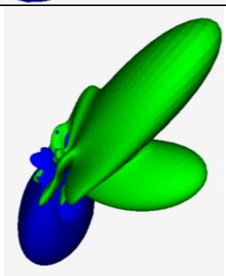
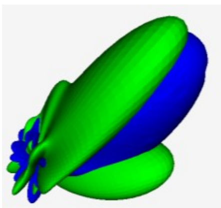
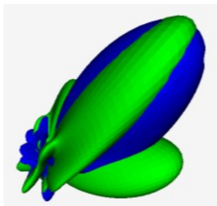
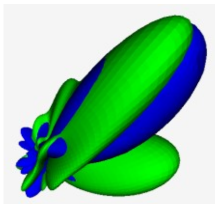
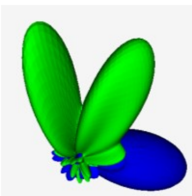
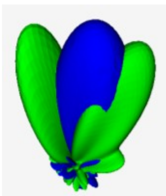
b) Go to Step 2.

5. Experimental Results

We applied the proposed method to align beam patterns computed from the outer ear shapes of bats. The controlled random search algorithm is compared with the method using discrete uniform samples over the rotation group $SO(3)$ through various beam patterns including one main lobe, strong side lobes and split lobes. 10000 uniform samples are generated using the method described in [16].

We also compared our algorithm with the method based on PCA axes alignment. However, PCA does not give directions for principal axes and heuristics are needed to determine positive directions of the axes. We employ axis flip technique to solve the orientation problem. Four possible axis flips are evaluated and the one with minimum dissimilarity is chosen.

Some of our experimental results are shown in Figure 5. For visual effects, the beam patterns are shown as 3D surfaces. The first number under the figures is the best *dist distance* and the second is the number of evaluating *dist*. It is obvious that the controlled random search algorithm obtains better results in less time.

Beam pattern before alignment	PCA based alignment	Alignment using uniform sampling	Proposed controlled random search
	 (0.0639286, 4)	 (0.0644953, 10000)	 (0.063458, 5637)
	 (0.0727219, 4)	 (0.0699109, 10000)	 (0.069130, 4928)
	 (0.0788119, 4)	 (0.0711404, 10000)	 (0.070991, 5103)

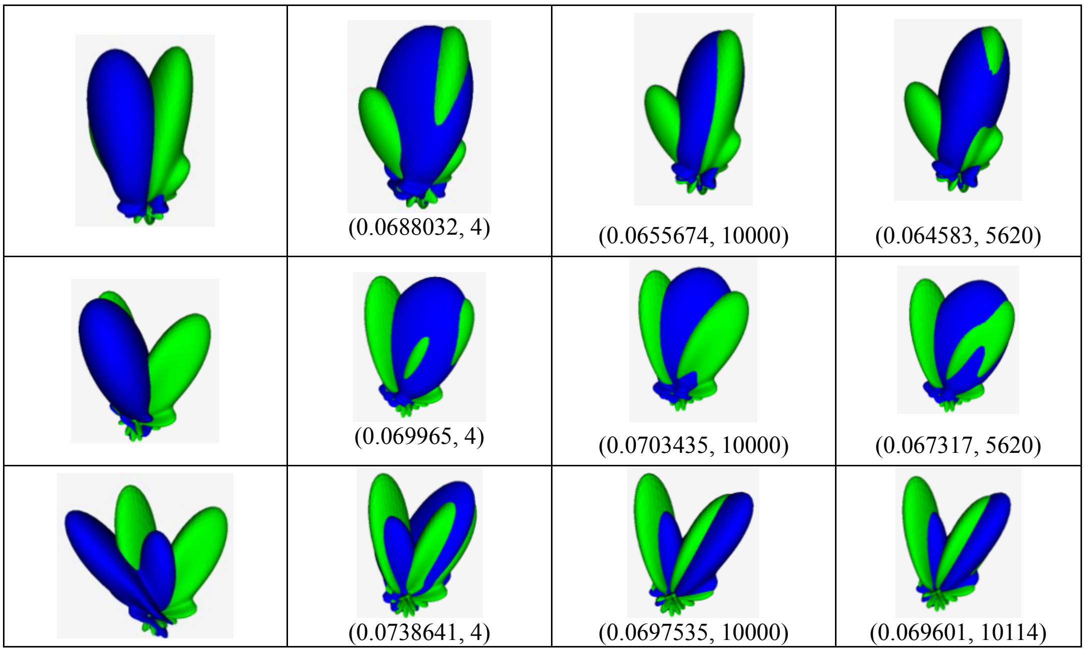


Figure 5. Comparison between PCA, uniform samples over the rotation group and controlled random search algorithm..

Experiments were made using Matlab system on xxGHz and XX GB RAM. Typical time of computation is xxx [s] for PCA, zzz[s] for uniform sampling and xxx[s] for the proposed approach.

Figure 6 shows three beam patterns from the same geometric shape corresponding to multiple frequencies. We select the first one as the reference and align the other beam patterns to it using controlled random search algorithm.

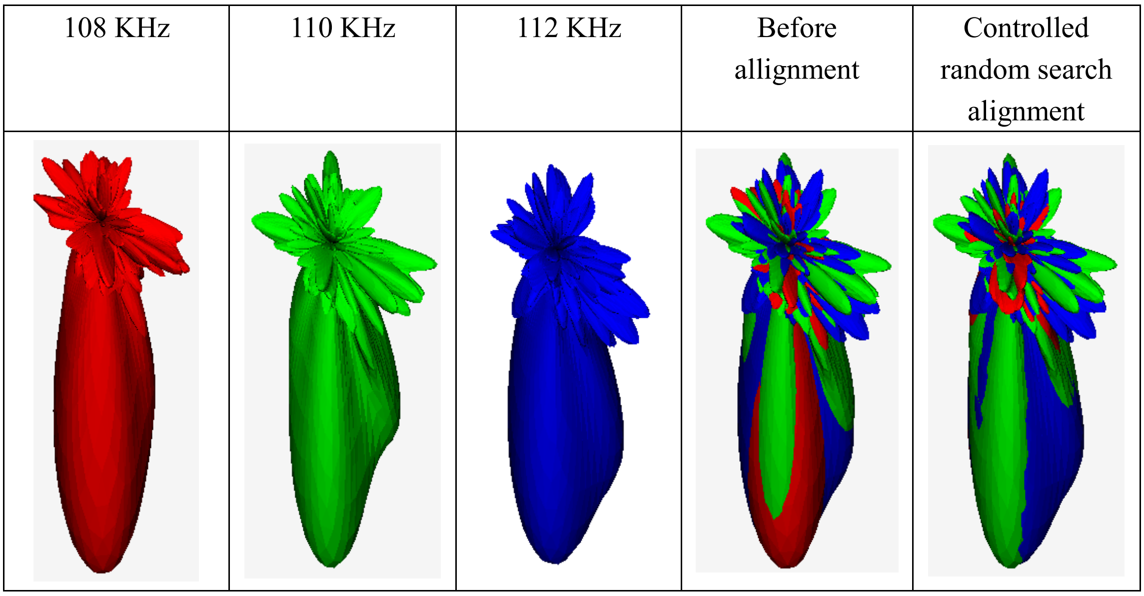


Figure 6. Beam patterns from different frequencies are aligned together. From left to right: beam patterns corresponding to 108KHz, 110KHz and 112KHz, before alignment, alignment using controlled

random search algorithm

6. Conclusion

This paper presents a new method for finding a globally optimal 3D rotation that best aligns spherical surfaces based on controlled random search techniques. The matching distance is based on the gain values at the corresponding orientations. The experiments made proved significantly higher reliability and accuracy at a reasonable computational cost.

Acknowledgments

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