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Finite element based investigation of buckling and vibration behaviour of thin walled box beams

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Abstract

Thin-walled box type conventional and composite structures are having wide applications for building the structural system which are used in advanced ships, aerospace, civil, construction equipment and etc. Often these structures are subjected to vibration and buckling due to the environmental effect such as mechanical, thermal, electrical, magnetic, and acoustic or a combination of these. Also damping material and structural stiffness plays an important role for the improvement of vibration, noise control, fatigue and bulking resistance of these structures. So it is important to know the dynamic and buckling characteristics of these structures. Pre-stress in a structure affects the stiffness, which modifies the dynamic and stability characteristics of the structure. So it is also important to know the influence of pre-stress on the vibration and buckling character. In this paper, buckling and dynamic characteristics of the thin-walled box type structures are analyzed using finite element software ANSYS.

Keywords: thin-walled box, vibration, buckling, composite, FEM

1. Introduction

Stiffened thin-walled structures like box sections are widely used as components of structural systems in civil, aerospace, marine industries, construction equipment and etc. In all cases of buckling, critical loads are proportional to the flexural rigidity of the plates. Stability of the plate increases with the increase in the thickness of the plate but a more economical solution is obtained by keeping the thickness of the plate as small as possible and increasing the stability by introducing stiffeners. Being a thin walled structure the design of stiffened plates is governed both by stability and strength criterion. In other words, stiffeners are commonly attached to such structures along the major load-carrying directions to achieve higher stiffness/weight and strength/weight ratios. To increase further, laminated composites have been first introduced in the aerospace industry, and currently being used in many applications. Sarawit et al. [10] discussed analysis types, material models, elements and initial conditions that are taken into account in the FE analysis of thin-walled structures. Guo et al. [2] studied the buckling behaviour of laminated panels with one stiffener subjected to uniaxial compression by using a layerwise finite element formulation. Rikards et al. [9] developed a triangular finite element for buckling analysis of laminated composite stiffened plates and shells. For a stiffened structure subjected to uniaxial compression, there are usually two distinct modes can be observed such as overall buckling and local plate buckling. However, there are specific geometric dimensions when

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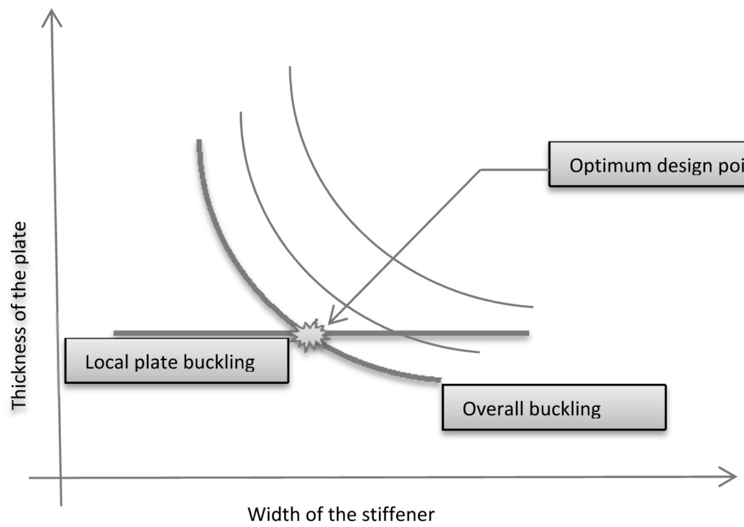


Fig. 1. Simplified optimum design curve for local and overall buckling

these two modes occur together and interact very closely. The following figure shows a simplified design space with only two design variables, plate thickness and width of the stiffener web.

Fig. 1 shows the constraints against local plate buckling and overall panel buckling, and it is evident that the optimum design would be at the junction of these two constraints. Such an optimum panel would have the highest bifurcation buckling stress in its class of panels of equal weight per unit width. It is useful to have a structural parameter by which one can determine which mode of buckling would occur first in a given panel.

If, in addition, exposed to a dynamic situation, the composite materials offer better dynamic characteristics than any other conventional materials. The weight saving is an important consideration for high performance applications. Also, for the dynamic problem of thin-walled structures, it is often necessary to minimize the maximum deflections without introducing any considerable weight penalty. This can be achieved by adding stiffeners to the host structure. When a structure is in contact with a fluid, there exists a possible feedback mechanism between the structure and fluid which leads to a coupling between structure and fluid responses. This is termed as Fluid Structure Interaction (FSI) in literature. It is well known that the dynamics of a container or vessel is affected by the type and amount of liquid surrounding and/or within it. Dynamic response of fluid filled structures is of great important in many engineering applications such as fuel tanks of space vehicles and refrigeration compressors. A major reason for this is an increase in the number of structures that have been optimized to minimize weight and to avoid failure by dynamic forces. The natural frequencies of vibration of a fluid-filled vessel are lower than its empty counterpart due to added mass effect of the fluid. This coupling effectively increases the mass of the fluid filled structures with its stiffness remaining unchanged. The apparent increase in the mass of these fluid filled structures can be determined using techniques such as the added mass principle, the Rayleigh-Ritz method and finite element method. Mazuch et al. [4] calculated the added mass by solving the generalized eigenvalue problem with its mass term consisting of both the mass of the plate and an added mass term from the fluid-structure coupling. The added mass term was derived from the pressure field determined from Laplace equation in an incompressible in-viscid fluid with boundary conditions consisting of the vibrating plate and a free surface. The Rayleigh-Ritz method has been successfully used by Vaillette [11] to predict mode shapes and associated frequencies of submerged plates. Finite element method has been used successfully by several researchers to analyze the free vibration characteristics of fluid filled containers. For example, Mazuch et al. [4] used FEM to predict the natural frequencies of various shaped vessels containing fluid. A number of papers on stiffened beam, shell and plate structures have been proposed. However, the papers on the buckling and vibration analysis of the stiffened conventional/laminated composite thin walled structures on square box-section with multiple stiffeners have rarely been published. Thus the work concentrates on the buckling and vibration analyses of stiffened conventional/laminated composite thin-walled structures with different boundary conditions. Several researchers investigated the effect of fluid structure interaction in box type fluid filled containers. Pal et al. [6] have presented the finite element coupled slosh analysis of rectangular liquid filled laminated composite tanks. Ashok [1] have investigated the free vibration analysis of fluid filled rectangular and circular containers. Recently, Santanu Mitra and Sinhamahapatra [5] studied the coupled slosh dynamics of fluid filled containers using pressure based FE method. Temperature effect has not been included in the previous studies. Hence in the present work, free vibration behavior of fluid filled box type containers under a thermal environment made up of isotropic and composite materials are analyzed using finite element method. In addition, studies are carried out to analyze the effect of temperature on the forced vibration behavior of liquid filled rectangular containers.

2. Buckling and vibration study

2.1. Numerical modeling and validation

2.1.1. Numerical modeling and analysis of stiffened thin-walled structure

The linear analysis of the Finite Element program, ANSYS 8.1 is used for prediction of both buckling and frequency behaviors of stiffened conventional/composite thin walled plate like structures. Eigenvalue method of analysis is the basic concept in this package. In order to evaluate the natural frequencies a linear eigenvalue problem is solved

$$[K]\{u\} - \lambda[M]\{u\} = 0. \tag{1}$$

Here K and M is a global stiffness and mass matrices, u is global displacement vector and $\lambda = \omega^2$, where $\omega = 2\pi f$ is a circular frequency and f is a frequency. Different methods are employed to estimate buckling loads. The simplest is linearized buckling analysis, where the critical load parameter λ can be estimated by solving the equation

$$[K]\{u\} - \lambda[K_G]\{u\} = 0. \tag{2}$$

Here $[K_G]$ is a geometric stiffness matrix. SHELL 63 and SHELL99 elements are used for modeling and analysis of isotropic and composite box structure, respectively. Four nodes linear elastic element SHELL 63 has the capability to simulate both membrane and flexural behavior. It has six degrees of freedom along and about their axes. The layered linear element SHELL99 for modeling composite material has eight nodes with six degrees of freedom at each node. The stiffened section is divided into sufficient number of elements and some trial runs were carried out to study the convergence of the results. Modelling laminated stiffened structures needs care in defining the properties of the plate and stiffener, number of layers, symmetric or un-symmetric, thickness and fiber orientations of each layer.

Boundary conditions are defined accordingly in the modeling procedure and for the stability issue; a uniformly distributed axial load is applied along the length of the section. Fig. 2, shows a typical stiffened model of thin-walled structures used in the present study. In order to verify the accuracy of the method of analysis, a series of pre-selected cases are modeled and analyzed using ANSYS 8.1.

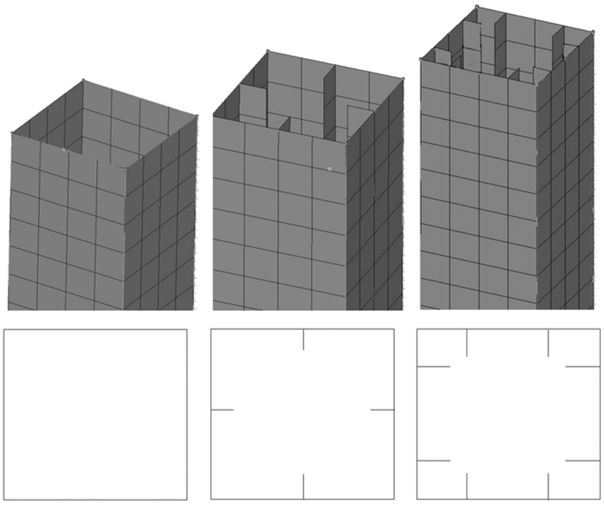


Fig. 2. Thin-walled structures with and without stiffeners

Also an appropriate conventional plate finite element which was used by Ramkumar et al. [8] has been used to showcase the result correlation part and carry out few parametric studies too.

2.1.2. Validation

To validate the present finite element ANSYS model, results are compared with the results reported by Peng et al. [7] for the vibration of stiffened L-sectioned plate and Dong-Min Lee et al. [3] for the vibration of stiffened composite plate. Also the current ANSYS APDL code has been compared with conventional plate finite element formulation of Ramkumar et al. [8].

Vibration of the isotropic stiffened L-sectioned cantilevered plate

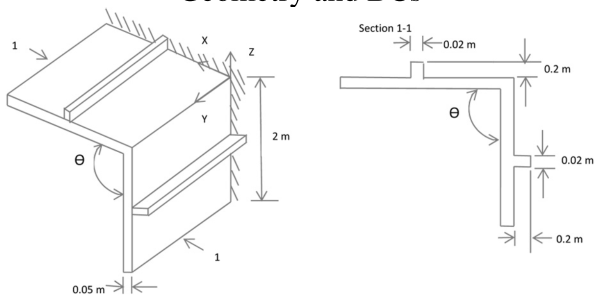
To validate present method or approach, the cantilevered isotropic L-shaped eccentrically stiffened plate with two stiffeners, which was reported by Peng et al. [7], has been selected as an example. The stiffeners are fitted along $y = 1$ m and $z = -1$ m, respectively (the midline of the two flat plates). The stiffeners are made from the same material as the stiffened L-plate. The Young's modulus E is taken to be 3×10^7 Pa, the Poisson's ratio μ is 0.3 and the density ρ is 1 000 kg/m^3. The frequencies of the five modes of the stiffened L-shaped eccentrically stiffened plate are listed in Table 1 alongside the results from Peng et al. [7] for comparison.

Vibration of the laminated stiffened plate

Next, the vibration of laminated composite plate with a single rib is examined. The reference solution is taken from Dong-Min Lee et al. [3] in order to verify the present ANSYS model for a plate with composite stiffener. Numerical results are obtained for the clamped stiffened plate made of a carbon/ epoxy composite material (AS1/3501-6).

The properties of that material are listed in Table 2. The layer stacking sequence of the skin plate is $[0/\pm 45/90]$s. The ply thickness is 0.13 mm and at all in the skin there are eight

Table 1. Natural frequencies (Hz) of a stiffened L-sectioned isotropic cantilevered plate



Geometry and BCs	mode	Present (ANSYS)	Ramkumar et al. [8]	Peng et al. [7]
	1	0.77	0.79	0.77
	2	0.84	0.89	0.84
	3	2.05	2.16	2.05
	4	2.58	2.63	2.58
	5	3.27	3.51	3.27

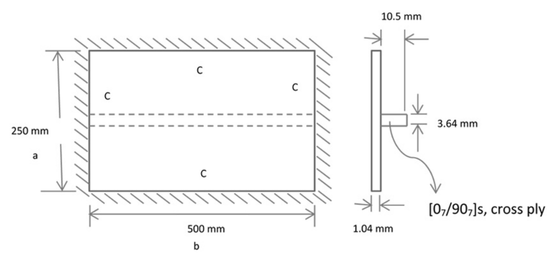
layers. The rib is made from the cross-ply laminate with 28 layers. The layer stacking sequence throughout the rib thickness is $[0_7/90_7]$s. The plate parameters are as follows:

$$a = 250\ \text{mm}; \quad b = 500\ \text{mm}; \quad t_p = 1.04\ \text{mm}; \quad h_s = 10.5\ \text{mm}; \quad t_s = 3.64\ \text{mm}.$$

Table 2. Material properties of the composite plate with cross-ply stiffener

Material	E_1 [GPa]	E_2 [GPa]	$G_{12} = G_{13}$ [GPa]	G_{23} [GPa]	v_{12}	ρ [kg/m^3]
Carbon/epoxy	128	11	4.48	1.53	0.25	1 500

Table 3. Validation of natural frequency (Hz)



	Mode No.	Dong-Min Lee et al. [3]	Ramkumar et al. [8]	Present (Ansys)
	1	213.6	211.6	208.72
	2	222.2	219.2	218.08
	3	270.2	269.2	267.15
	4	312.4	311.3	306.86
	5	356.4	354.8	352.51

The composite stiffened plate was calculated by the shell model, i.e., the rib also was assembled by using the shell elements SHELL99. The mesh was 24×24 elements. Numerical results for the first five frequencies are presented in Table 3. The results are shown in Table 1 and 3 shows excellent correlation of results.

2.2. Isotropic thin-walled structures

2.2.1. Thermal buckling

Thermal buckling analysis of un-stiffened/stiffened thin-walled box structures subjected to uniform temperature rise is studied. The critical buckling temperature of thin-walled structures

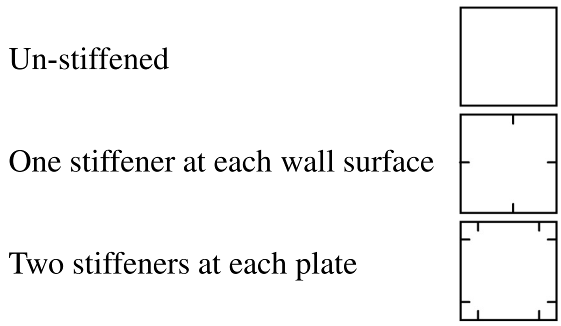


Fig. 3. Typical cross section of un-stiffened/stiffened thin-walled structures

having different cross-sections considered in this present study are shown in Fig. 3 with different stiffener configurations such as one stiffener at each plate and two stiffeners at each plate. Coefficient of thermal expansion is taken as $12.33 \cdot 10^{-6}\,^{\circ}\mathrm{C}^{-1}$.

Mechanical properties and geometrical dimensions considered are given as follows:

$$L = 0.750 \text{ m}, \quad b = 0.150 \text{ m}, \quad tp = 0.004 \text{ m}, \quad ts = 0.004 \text{ m}$$

$$\text{and } E = 210.0 \cdot 10^{9} \text{ Pa}, \quad \nu = 0.3, \quad \alpha = 12.6 \cdot 10^{-6} \text{ C}^{-1}.$$

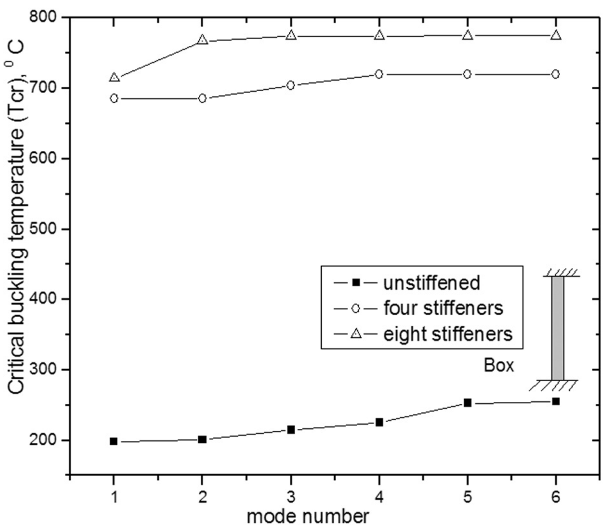


Fig. 4. Thermal buckling temperature of Box structures ($L = 0.75$ m) with and without internal stiffeners

Where L is the length of the column, b is the width of the column; tp is the thickness of the each plate and ts is the thickness of the stiffener. Width of the stiffener is taken as 0.025 m. The variation of thermal buckling temperature of un-stiffened and stiffened thin-walled structures of different sections with respect to its mode numbers are shown in the Fig. 4. Boundary condition considered in the thermal buckling section is clamped-clamped. From the Fig. 4, it is seen that the critical buckling temperature of thin-walled structures having stiffeners is more when compared to structures are not having stiffeners. Also it was found that the critical buckling temperature is high for such structures having two stiffeners at each plate when compared to single stiffener at each plate. Next, the similar analysis has been carried out on the thin walled structures having double in length i.e., length of the column or panel is taken as 1.5 m to study

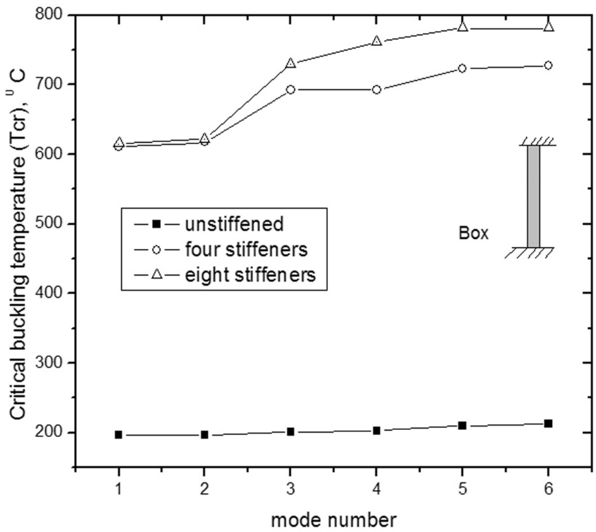


Fig. 5. Thermal buckling temperature of Box long structures ($L = 1.5$ m) with and without internal stiffeners

the influence of length of the column. The remaining material and geometrical (thickness and width of the plate/stiffeners) parameters considered are same. The variation of thermal buckling temperature of un-stiffened and stiffened long thin-walled structures of different sections with respect to its mode numbers are shown in the Fig. 5.

From the Fig. 5, it shows that there is a reduction in the thermal buckling temperature when the length of the section increased from L to $2L$ and the effect of stiffeners are same even when length of the section increased from L to $2L$.

2.2.2. *Free vibration*

The free vibration analysis on the isotropic (steel) thin walled box structures having three different stiffeners configuration and without stiffeners is carried out. To know the influence of the length of such sections and boundary condition on the frequency behaviour, two different lengths and boundary conditions were considered and examined. Initially to start with the vibration behavior is investigated on the box sections having the same geometric parameters and material properties that are used in section 2.2.1.

From the Fig. 6, it was found that, the influence of stiffeners on the cantilevered thin walled structures is less when compared to both ends clamped thin walled structures. It is because of the effect of stiffness, inertia terms and the modal behavior. This situation can also be explained by the mode shapes. As we observed from the previous subsection, the effect of stiffeners on the frequencies of long cantilevered structures is less than the both ends clamped case. It implies that the mode shape of such structures varies as the number of stiffeners and boundary conditions varies during vibration. In order to know the influence of stiffeners on the mode shapes, free vibration study is carried out on the four different cross sectioned thin walled structures with and without stiffeners. The natural frequencies and its vibration mode shapes has been calculated by ANSYS for the steel thin walled structures ($0.75 \times 0.15 \times 0.004$ m) and are shown in the Table 4. From the Table 4, it is seen that for the cantilevered short un-stiffened box column, the first two vibration modes correspond to column bending and have the same natural frequency. This is expected, as the structure is symmetric. For the third mode, column torsion takes place. Fourth mode is a first plate bending mode in which each plate of the column undergoes first plate bending. The rest of the modes are higher plate bending modes.

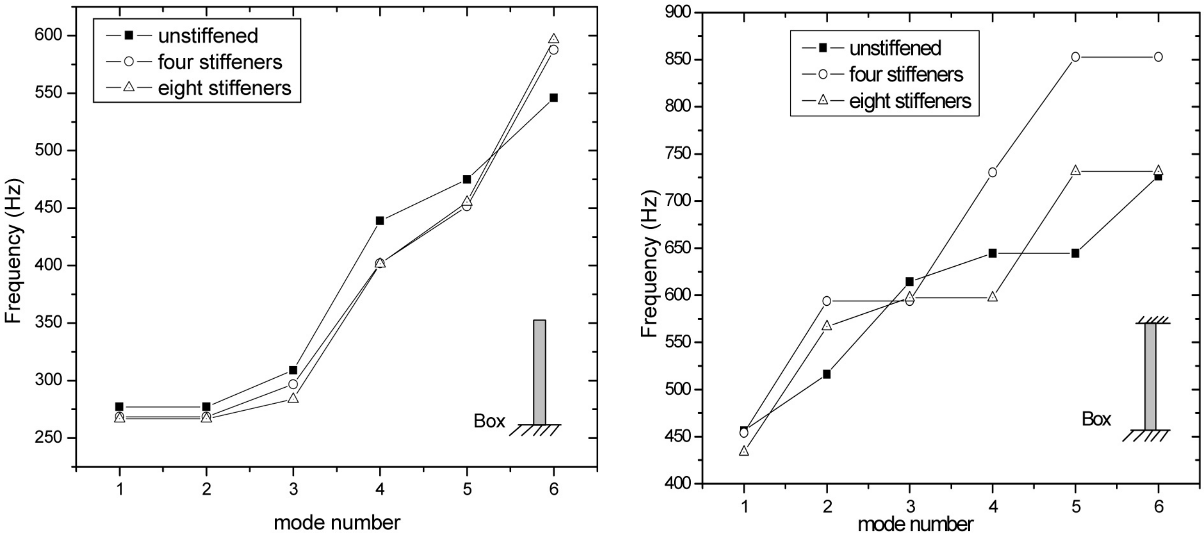
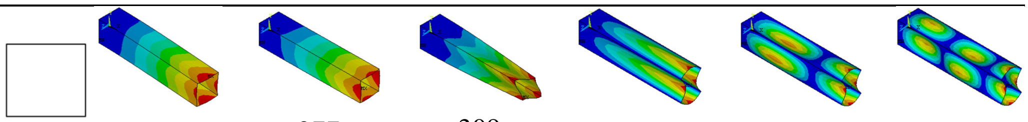
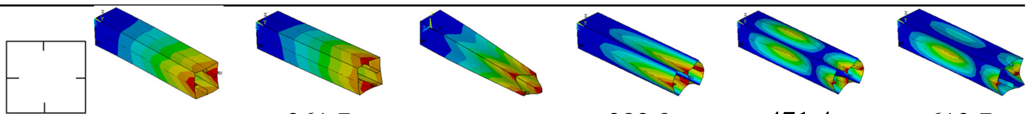
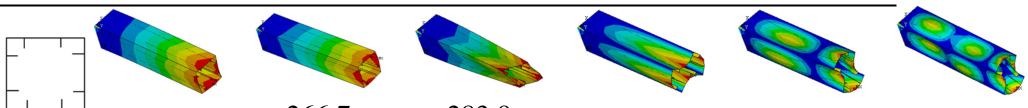
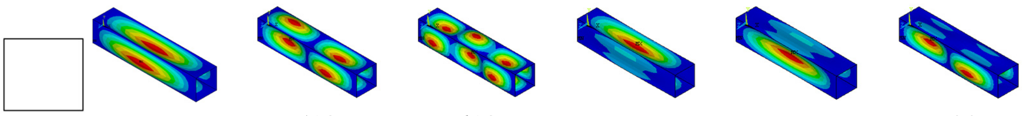
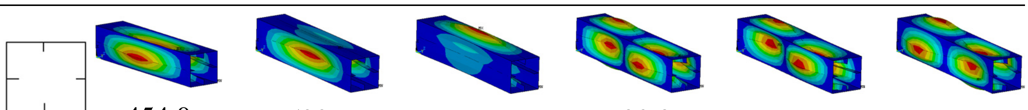
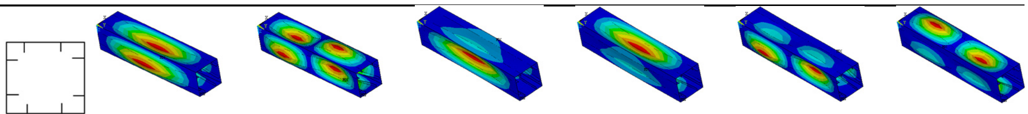


Fig. 6. Natural frequency of Box structures ($L = 0.75$ m) with and without internal stiffeners

Table 4. Natural frequency (Hz) and corresponding mode shapes of box section

BC	Finite Element Model	Mode number 1	2	3	4	5	6
CF Clamped – Free		277	277	309	438	475	547
		261.7	261.7	285.6	382.9	471.4	612.7
		266.7	266.7	283.9	401.5	455.3	596.5
Clamped – Clamped		457	518	618	646	646	730
		454.0	593.7	593.7	730.3	852.9	852.9
		433.6	566.6	597.2	597.2	731.4	731.4

Next, the similar analysis has been carried out on the thin walled structures having double in length i.e., length of the column or panel is taken as 1.5 m to study the influence of length of the column. The remaining material and geometrical (thickness and width of the plate/stiffeners) parameters considered are same. Fig. 7 shows the frequencies calculated for box structure ($L = 1.5$ m) having/not having stiffeners.

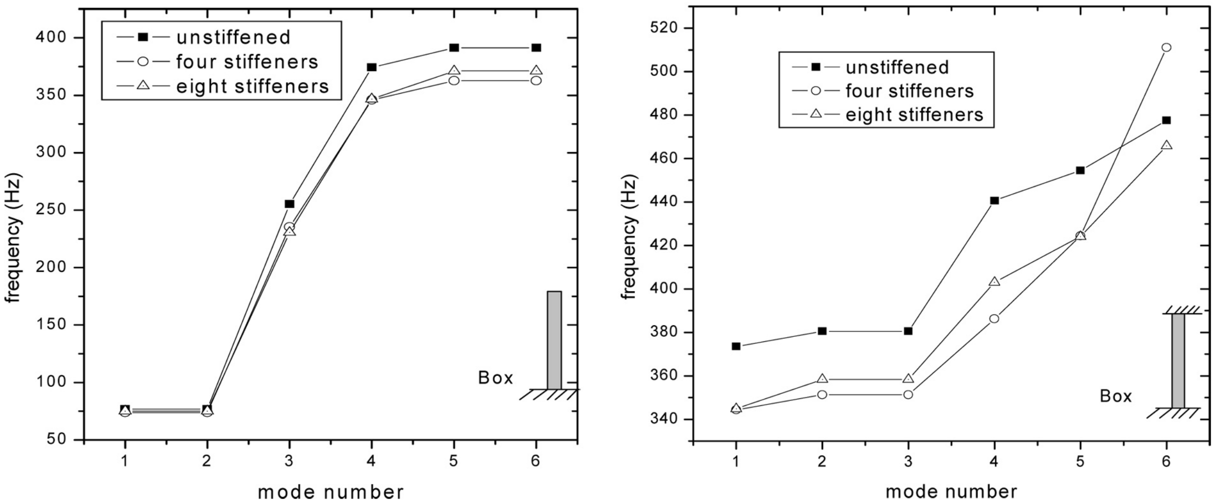


Fig. 7. Natural frequency of Box structures ($L = 1.5$ m) with and without internal stiffeners

From these Fig. 7, it shows that the influence of stiffeners on the frequency which means there is a frequency fall with increase in stiffeners in the box structures. Mostly for local modes the stiffness impact is greater than the mass one and, therefore frequency increases with stiffening, while for global modes the stiffness impact is less than the mass one and, therefore frequency decreases with stiffening.

Also as we expected that there is a frequency fall with increase in length of the column (for all the cross sections) from 0.75 m to 1.5 m. This is due to the influence of slenderness ratio. If the plate components are too slender, the column would fail by premature local vibration, if on the other hand the plate components are made compact, e.g. smaller width and/or greater thickness, then the column is more likely to fail by global vibration.

2.3. *Composite thin-walled structures*

The thermal buckling and vibration analysis on isotropic thin walled structures are carried out and presented in the previous section 2.2 has been extended to thin walled structures made up of composite materials. Material properties (Glass/epoxy) considered in this section are given in the Table 5.

Table 5. Composite material properties

Material	E_1 [GPa]	E_2 [GPa]	ν_{12} [GPa]	$G_{12} = G_{13}$ [GPa]	α_1 [10^{-6}/°C]	α_2 [10^{-5}/°C]	ρ [Kg/m^3]
Glass/epoxy	37.78	10.9	0.3	4.91	7.0	2.3	1 870

The study has been carried out on the composite thin walled section which is similar to the previous isotropic section in which the effect of length of the column, boundary condition, cross section and number of stiffeners are taken in to consideration. In addition the effect of fiber orientation and lay-up sequence on the frequency and buckling of such structures are examined.

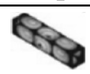
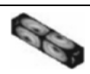
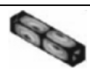
2.3.1. Thermal buckling

To start with thermal buckling analysis is carried out on the thin walled sections having lengths $(L) = 0.750$ and 1.5 m, width $(b) = 0.150$ m and thickness $(t) = 0.004$ m. The fiber orientation and the stacking sequence of the host structure and the stiffener are maintained same. The width of the stiffener is taken as 0.025 m (similar to isotropic thin walled structures). First six thermal buckling temperatures of three different composite thin walled structures having five layers with fiber orientation of [0/0/0/0/0], [45/45/45/45/45] and [90/90/90/90/90] are shown in the Tables 6, 7 and 8, respectively.

Influence of stiffener on thermal buckling

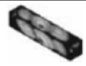
Thermal buckling studies have been carried out for, three different fiber angle orientations, stiffener configurations, and two different column lengths. The results from these studies have been tabulated in Tables 6–8. From these results, it is clear that there is a significant increase in the thermal buckling temperature when the stiffeners are introduced in the structure. This can be explained through buckling analysis of rectangular plate. Stresses developed while buckling analysis in the rectangular plate with and without stiffeners are shown in the Figs. 8, 9. From the Figs. 8, 9 one can observe that the stresses developed in the unstiffened plate structure is more compared to that of the structure with stiffener. So the K_g (geometric stiffness) matrixes for the structure with stiffeners are less compared to that of the structures without stiffeners and hence the thermal buckling temperature of the structure with stiffener is more.

Table 6. Thermal buckling temperatures of Box structures, [0/0/0/0/0]

$L = 0.75$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[0/0/0/0/0]	Box section	0	154.83	
		4	352.82	
		8	360.25	

$L = 1.5$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[0/0/0/0/0]	Box section	0	148.45	
		4	307.95	
		8	313.79	

Table 7. Thermal buckling temperature of Box structures, [45/45/45/45/45]

$L = 0.75$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[45/45/45/45/45]	Box section	0	211.32	
		4	361.38	
		8	374.39	

$L = 1.5$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[45/45/45/45/45]	Box section	0	205.8	
		4	327.67	
		8	338.26	

Table 8. Thermal buckling temperature of Box structures, [90/90/90/90/90]

$L = 0.75$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[90/90/90/90/90]	Box section	0	158.39	
		4	330.38	
		8	336.45	

$L = 1.5$ m				
Composite Lay up	Cross Section	Number of Stiffeners	Critical Buckling Temperature (°C)	Mode Shape
[90/90/90/90/90]	Box section	0	155.73	
		4	311.68	
		8	317.30	

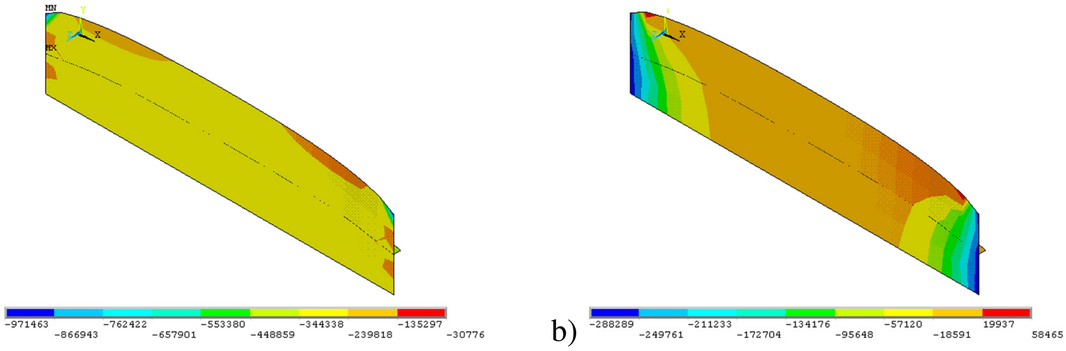


Fig. 8. Stress plot for the stiffened rectangular plate (a) X direction (σ_{xx}), (b) Y direction (σ_{yy})

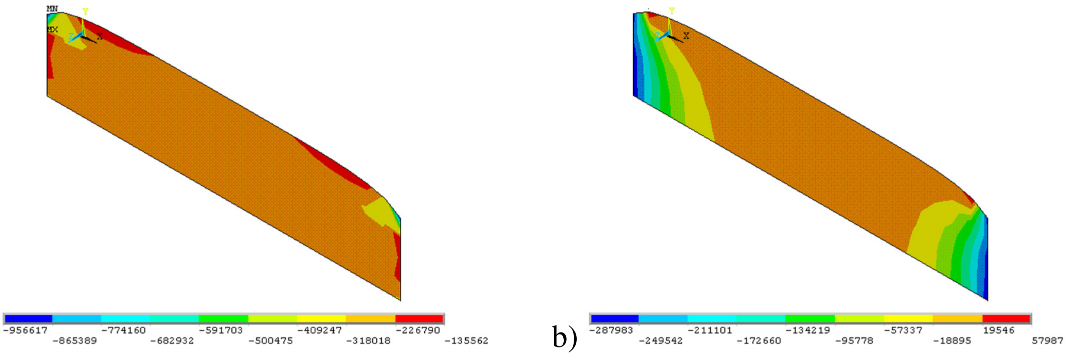


Fig. 9. Stress plot for the un-stiffened rectangular plate (a) X direction (σ_{xx}), (b) Y direction (σ_{yy})

From the Tables 6–8, it is clearly seen that the effect of stiffeners on the shorter length column (0.75 m) is more when compared to that of the longer length column (1.5 m). The reason may be due to the domination of rotational terms over the stresses in Kg matrix. The structural columns with $45°$ fiber orientation have more thermal buckling temperature than the structural columns having $0°$ and $90°$ fiber orientation because the structural columns with $45°$ fiber orientation will have bending stretching coupling.

2.3.2. Free vibration

Next, a free vibration analysis is carried out on the thin walled sections having the same geometrical parameters which are given in the section 2.3.1. Similar to thermal buckling, here also the fiber orientation and the stacking sequence of the host structure and the stiffener are maintained same. Width of the stiffener is taken as 0.025 m (similar to thermal buckling). First five frequencies of three different composite thin walled structures having five layers with fiber orientation of [0/0/0/0/0], [45/45/45/45/45] and [90/90/90/90/90] are shown in the Tables 9, 10 and 11, respectively.

Influence of number of stiffeners on free vibration

From the Tables 9, 10 and 11, irrespective of fiber angle, stacking sequence and cross section, it is observed that the influence of stiffener on the natural frequency of the structural column is more in clamped – clamped boundary condition when compared to that of the clamped – free boundary condition. The above statement is true only for shorter structural column (0.75 m) whereas for longer structural columns (1.5 m), the effects of stiffeners are not predominant for both the boundary conditions (see Tables 12–14). In case of box section structural columns

Table 9. Natural frequencies of different thin walled structures ($L = 0.75$ m), [0/0/0/0/0]

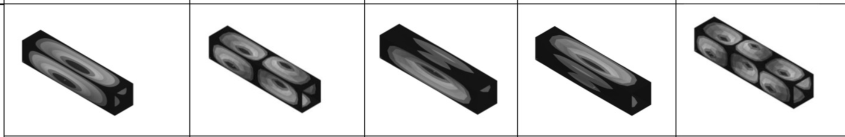
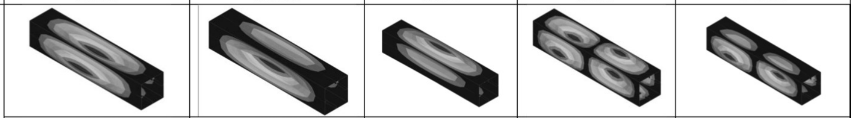
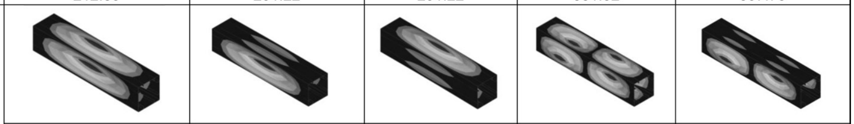
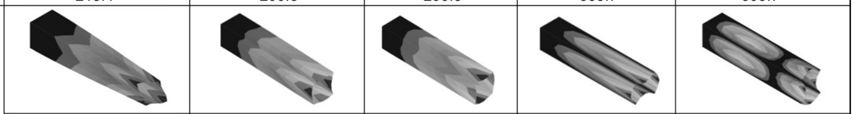
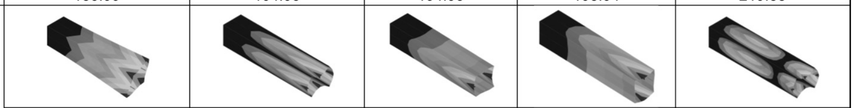
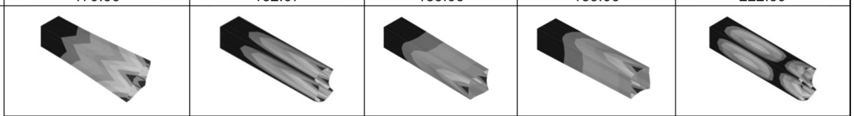
Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz) 1	2	3	4	5
Box section 0/0/0/0/0 L=0.75m	Clamped Clamped	0					
			209.9	251.22	303.5	303.5	325.05
		4					
			212.85	291.22	291.22	331.52	397.75
		8					
			213.4	290.8	290.8	333.7	395.7
	Clamped Free	0					
			186.99	194.95	194.95	198.94	219.88
		4					
			179.36	182.67	185.06	185.06	222.39
		8					
			173.2	182.5	182.8	182.8	224.3

Table 10. Natural frequencies of different thin walled structures ($L = 0.75$ m), [45/45/45/45/45]

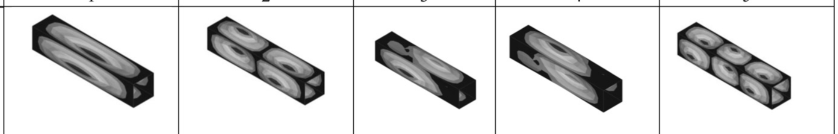
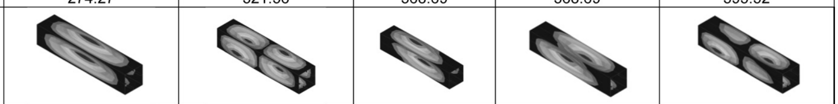
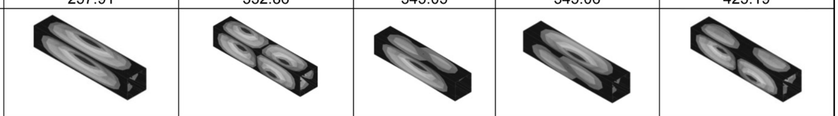
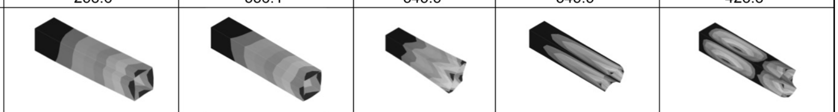
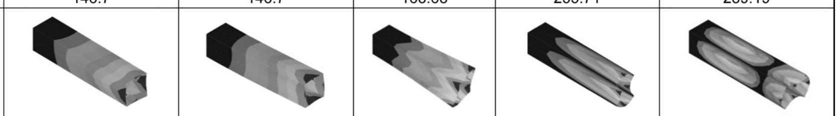
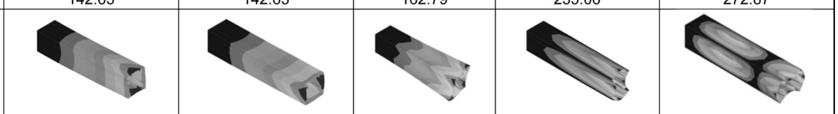
Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz) 1	2	3	4	5
Box section 45/45/45/45/45 L=0.75m	Clamped Clamped	0					
			274.27	321.56	368.69	368.69	395.52
		4					
			257.91	332.86	345.05	345.06	425.19
		8					
			258.6	336.1	345.5	345.5	428.8
	Clamped Free	0					
			146.7	146.7	168.68	255.71	289.19
		4					
			142.63	142.63	162.79	235.06	272.87
		8					
			141.6	141.9	156.1	235.3	274.9

Table 11. Natural frequencies of different thin walled structures ($L = 0.75$ m), [90/90/90/90/90]

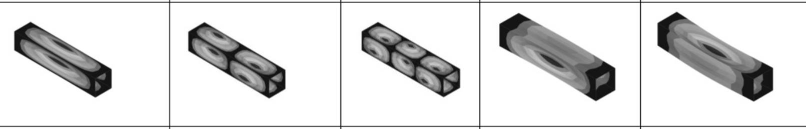
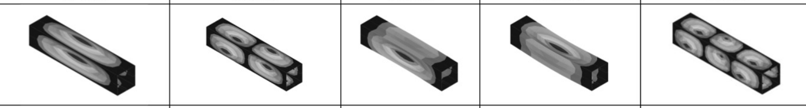
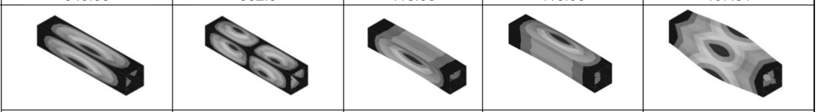
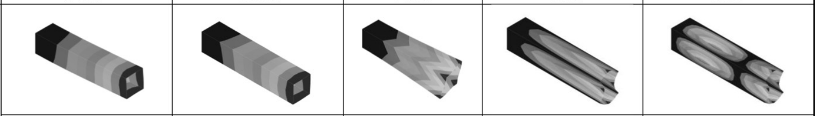
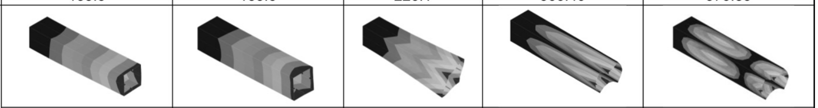
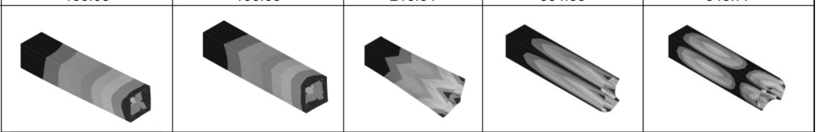
Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz) 1	2	3	4	5
Box section 90/90/90/90/90 L=0.75m	Clamped Clamped	0					
			371.61	389.48	421.08	449.36	449.36
		4					
			343.38	382.3	418.95	418.95	467.31
		8					
			343.1	383.6	413.3	413.3	458.1
	Clamped Free	0					
			133.5	133.5	226.1	366.49	376.85
		4					
			130.08	130.08	216.84	334.88	348.77
		8					
			129.2	129.2	205.5	334.1	349.3

Table 12. Natural frequencies of different thin walled structures ($L = 1.5$ m), [0/0/0/0/0]

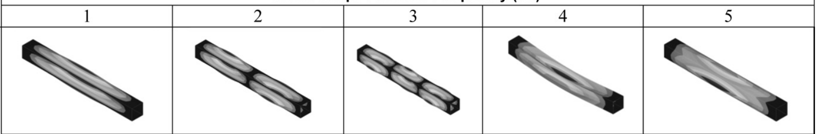
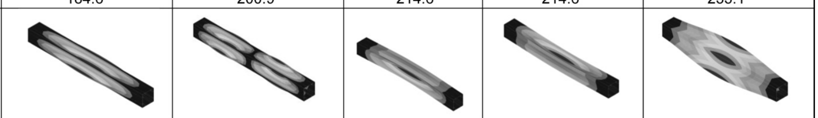
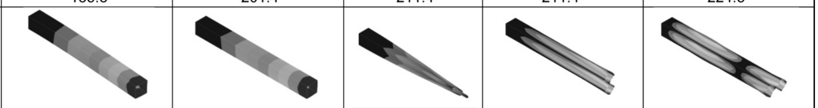
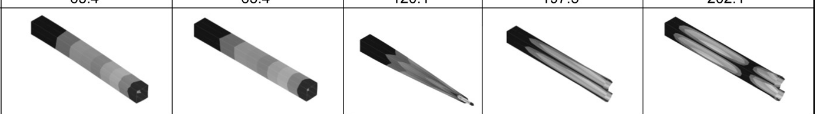
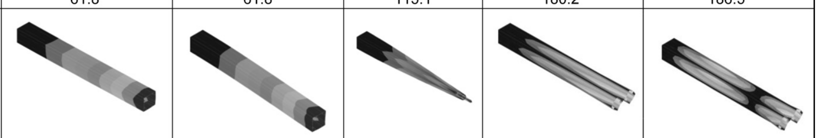
Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz) 1	2	3	4	5
Box section 0/0/0/0/0 L=1.5m	Clamped Clamped	0					
			199.7	207.8	222.2	229.4	229.4
		4					
			184.6	200.9	214.6	214.6	233.1
		8					
			183.8	201.4	211.4	211.4	224.3
	Clamped Free	0					
			63.4	63.4	120.1	197.3	202.1
		4					
			61.8	61.8	115.1	180.2	186.5
		8					
			61.4	61.4	108.9	179.8	186.7

Table 13. Natural frequencies of different thin walled structures ($L = 1.5$ m), [45/45/45/45/45]

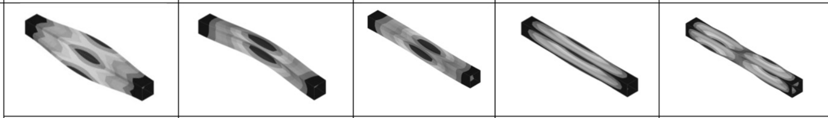
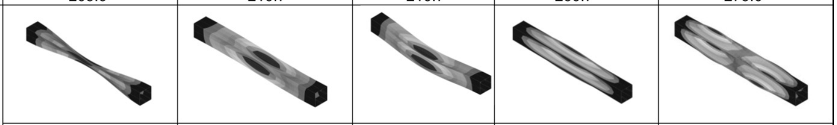
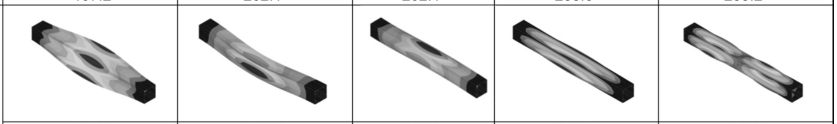
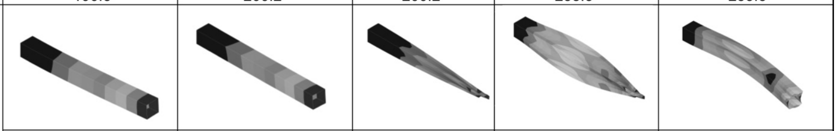
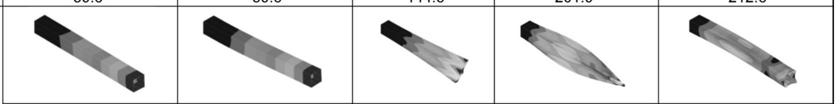
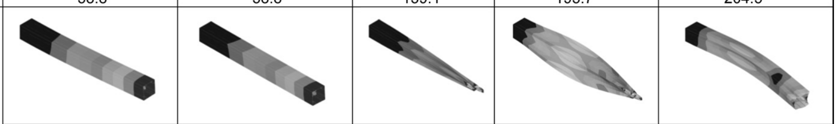
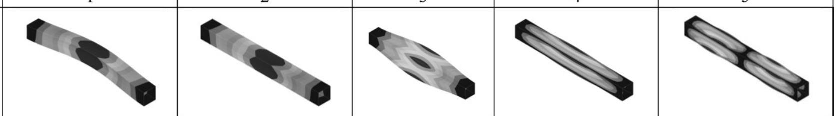
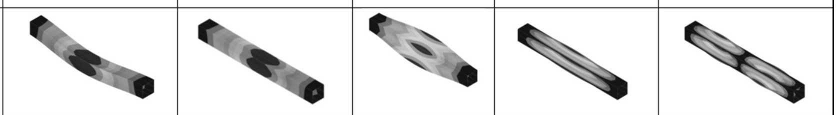
Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz)				
			1	2	3	4	5
Box section 45/45/45/45/ 45 L=1.5m	Clamped Clamped	0					
			205.9	210.7	210.7	260.7	275.0
		4					
			197.2	202.1	202.1	238.6	255.2
		8					
			190.3	200.2	200.2	238.3	255.5
	Clamped Free	0					
			39.6	39.6	144.5	201.9	212.6
		4					
			38.8	38.8	139.1	193.7	204.5
		8					
			38.7	38.7	131.4	187.5	203.2

Table 14. Natural frequencies of different thin walled structures ($L = 1.5$ m), [90/90/90/90/90]

Cross Section	Boundary Condition	Number of Stiffeners	Mode Shape / Natural Frequency (Hz)				
			1	2	3	4	5
Box section 90/90/90/90/ 90 L=1.5m	Clamped Clamped	0					
			190.4	190.4	246.7	367.1	371.1
		4					
			183.8	183.8	236.9	335.5	340.8
		8					
			181.3	181.3	225.2	334.6	340.5
	Clamped Free	0					
			35.9	35.9	195.2	195.2	211.3
		4					
			35.1	35.1	189.1	189.1	202.5
		8					
			35.1	35.1	187.3	187.3	190.7

the natural frequency will not get affected by increasing the number of stiffeners from 4 to 8 even for the short column subjected to clamped – clamped boundary condition. The structural columns with $45°$ fiber orientation have more stiffness and so the natural frequency than the structural columns having $0°$ and $90°$ fiber orientation because the structural columns with $45°$ fiber orientation will have bending stretching coupling.

3. Study on dynamic response of fluid-filled structures

Thin wall structures are used to store or to transport bulk materials of liquid or granular form. They cover a diverse range of structures from simple package to nuclear reactors and fuel tanks in aerospace structures. The amount of bulk material contained may vary throughout their service life and they are often subjected to dynamic loads including random vibrations, for example due to land and sea transportation loads or earthquakes. Their dynamic characteristics are very important to designers to evaluate their dynamic response and fatigue service life. It is now well recognized that the fluid-structure interaction influences the dynamic response of both the liquid and the structure.

3.1. FE model description

The commercial finite element package ANSYS is used to carry out all the analyses in this chapter. SHELL63/SHELL99 and FLUID30 elements are used to model the structure and fluid domain respectively (see Fig. 10). Similarly SHELL99 has both and membrane capabilities and in addition this can be used for layered applications of a structural shell model.

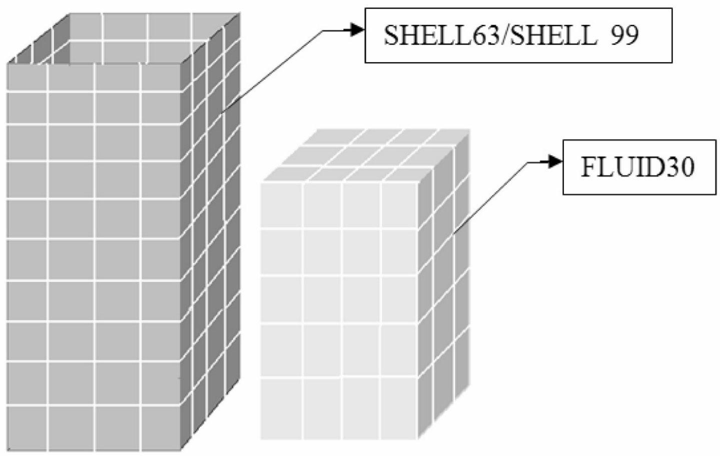


Fig. 10. FE model for structure and fluid

FLUID30 element is used to model the fluid medium and elements presents in the interaction (fluid-structure) in ANSYS. This element has eight corner nodes with four degrees of freedom per node: translations in the nodal x, y and z directions and pressure. The translations, however, are applicable only at nodes that are on the interface. Moreover the element has the capability to include damping of sound absorbing material at the interface. The element can be used with other 3-D structural elements to perform un-symmetric or damped modal, full harmonic response and full transient method analyses. Typical applications include sound wave propagation and submerged structure dynamics. The governing equation for acoustics, namely the 3-D wave equation, has been discretized taking into account the coupling of acoustic pressure and structural motion at the interface. Free vibration analysis involves determining the natural frequencies and mode shapes of fluid filled box structure. The container and the liquid within the container are discretised; the finite elemental matrices are generated and assembled to form the global matrices. The global matrices obtained are as denoted as follows. $[K]$ and $[M]$ are respectively the stiffness and mass matrices of the container, $[H]$ and $[G]$ are respectively the stiffness and mass matrices of the liquid, $[S]$ is the fluid-structure interaction matrix. The fluid mass and stiffness matrices are obtained from the governing wave equation of a compressible, in-viscid fluid is given as

$$\nabla^2 p - \frac{1}{c^2} \cdot \ddot{p} = 0, \tag{3}$$

where p is the acoustic pressure and c is the velocity of sound in the fluid medium. At the fluid-structure interface, to ensure contact between the fluid and the container wall, the boundary condition is given by

$$\frac{\partial p}{\partial n} = -\rho_l \frac{\partial v_n}{\partial t}, \tag{4}$$

where n indicates the direction normal to the surface of the container wall, ρ_l is the density of the liquid and v_n is the velocity of the container wall in a direction normal to the wall. This boundary condition gives the interaction matrix. Zero acoustic pressure has been imposed as a boundary condition wherever the liquid medium is exposed to atmosphere. The above equation leads to the coupling matrix $\mathbf{S}$, so that the coupled eigen value problem (Ashok [1]) can be stated as

$$\begin{pmatrix} \mathbf{K} & -\mathbf{S}^{\mathrm{T}}/\rho_f \\ 0 & \mathbf{H} \end{pmatrix} = \omega_c^2 \begin{pmatrix} \mathbf{M} & 0 \\ \mathbf{S} & \mathbf{G} \end{pmatrix}, \tag{5}$$

where ρ_f is the density of the fluid and ω_c is the coupled natural frequency. The final form of coupled dynamic equation for a structure in contact with fluid is given by

$$\begin{bmatrix} M & 0 \\ \rho_l S^{\mathrm{T}} & G \end{bmatrix} \begin{Bmatrix} \ddot{\delta} \\ \ddot{p} \end{Bmatrix} + \begin{bmatrix} K & -S \\ 0 & H \end{bmatrix} \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \{0\}, \tag{6}$$

where δ denotes the structural degrees of freedom of the container and p denotes the pressure degrees of freedom of the liquid. The resulting eigen problem can be solved to obtain the natural frequencies and mode shapes of the system which will lead to structural frequencies and as well as acoustic frequencies. Liquids are in general incompressible. However, the liquid mass matrix $[G]$ representing the compressibility effect is included in the formulation. The presence of the liquid compressibility matrix generally may not have much influence on the natural frequencies and mode shapes of the structure. When a structure is pre-loaded, the natural frequencies of the structure are modified as these loads produce stresses which change the structural stiffness. Pre-stressed modal analysis is carried out to find the natural frequency of the pre-loaded structure. The natural frequency at any given temperature can be calculated by evaluating the geometric stiffness matrix K_g at that temperature and by solving the coupled eigen value problem (Ashok [1]) as given below:

$$\begin{pmatrix} \mathbf{K} + \mathbf{K}_g & -\mathbf{S}^{\mathrm{T}}/\rho_f \\ 0 & \mathbf{H} \end{pmatrix} = \omega_c^2 \begin{pmatrix} \mathbf{M} & 0 \\ \mathbf{S} & \mathbf{G} \end{pmatrix}. \tag{7}$$

3.2. Fluid filled isotropic box container

3.2.1. Validation with previous work

A box structure (Fig. 11) made of steel rectangular plates of 1m thickness with overall dimensions of $L = 20$ m and $b = 30$ m analyzed by Ashok [1] is considered for validation of added mass frequencies. The container material properties are Young's Modulus, $E = 2.85 \times 10^{10}$ Pa, Poisson's ratio, $\nu = 0.2$ and density $\rho = 2\,500$ kg/m^3. Container is filled with water whose density is $\rho_F = 1\,000$ kg/m^3. Ashok [1] used added mass based finite element method to predict the coupled natural frequencies while ANSYS is used in the present work. Table 15 compares the natural frequencies of a square container obtained by using ANSYS with those results reported by Ashok [1]. From the Table 15 it is clear that there is a good agreement of frequencies obtained using ANSYS with the results reported by Ashok [1] for both empty tank and fully filled tank.

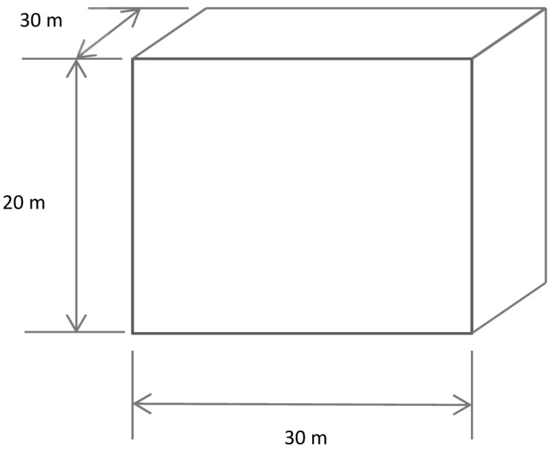


Fig. 11. Square box structure

Table 15. Comparison of added mass frequencies (rad/sec) of a square container

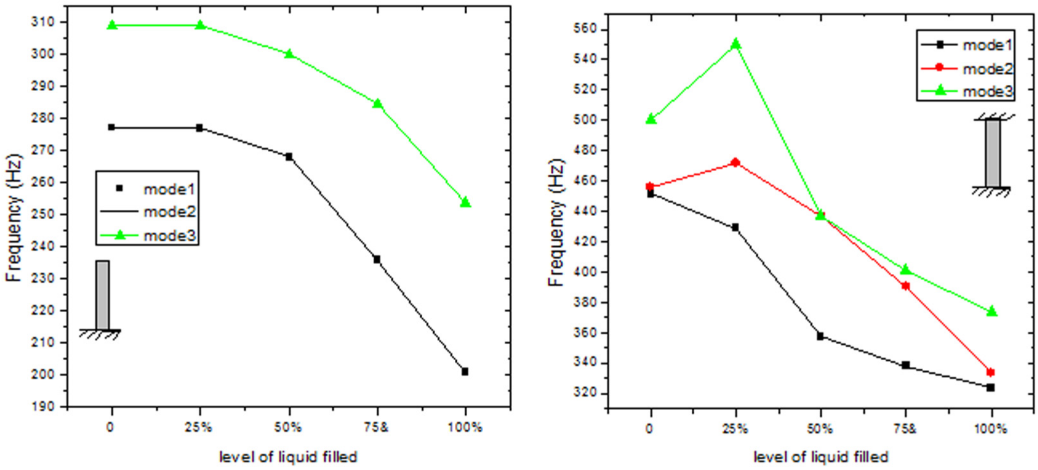
mode	empty tank		full tank	
	Present	Ashok (2001)	Present	Ashok (2001)
1	19.15	19.11	12.78	13.52
2	23.55	23.52	14.85	15.44
3	23.55	23.52	14.85	15.49
4	29.51	29.73	15.22	15.65

3.3. *Free vibration analysis of fluid filled isotropic box structure*

After testing the accuracy of the present model, free vibration analysis is carried out for two different boundary conditions on the box member (steel) having $L = 0.750$ m, width 0.15 m and thickness 0.004 m with/without inside fluid. Based on the convergence study, the box structure is meshed with 320 shell and 320 fluid elements, respectively. The frequencies of box type container with/without fluid obtained are tabulated in Table 16. Also the effect of fluid level in the container on the added mass frequencies are computed and plotted in Fig. 12.

Table 16. The effect of added mass on frequencies of a square steel container ($L = 0.750$ m and width 0.15 m)

mode	Natural Frequencies(Hz)			
	CF		CC	
	Without fluid	With fluid (Full tank)	Without fluid	With fluid (Full tank)
1	277	201	452	324
2	277	201	456	333
3	309	254	501	374



(a) (b)

Fig. 12. Effect of fluid level in the container on the added mass frequencies (a) C-F and (b) C-C

As expected the natural frequency decreases as the level of fluid increases for the both CF and CC boundary conditions. This is due to the total mass (addition of liquid mass with structural mass) influencing the system and hence there is a reduction in the frequency value as the mass increases (see Eq. (7)). Figs. 13, 14 show the coupled modes associated with first six modes obtained using modal analysis for various fluid levels for CF and CC boundary condition. When the fluid filled container is subjected to vibration, different types of modes such as bending, bulging, individual plate-bending and torsion mode can be observed for CF and CC box member as seen in Fig. 13, 14. The mode shapes of vibration of such structures were found to fall within these four or five types or a combination of two or more of these basic types. The acoustic cavity frequencies are obtained for various levels of fluid to know about the free vibration behavior of acoustic cavity and given in Table 17.

Table 17. Acoustic cavity frequencies for various fluid levels

mode	Acoustic frequencies (Hz)			
	Height of fluid level			
	25% filled	50% filled	75% filled	Fully filled
1	2 008	1 001	667	500
2	5 508	3 028	2 008	1 504
3	5 508	5 129	3 372	2 517
4	6 224	5 226	4 777	3 544
5	7 527	5 226	5 173	4 594
6	8 065	5 956	5 173	5 154

When the fluid level is equal to or more than half of the tank height similar kind of mode shape is observed for both CF and CC boundary condition. This can be observed from mode shapes shown in Figs. 13, 14 for various fluid levels. It can be clearly seen between 3rd mode of CF and 1st mode of CC and between 5th mode of CF and 2nd mode of CC for 50 % filled case. Similarly between first modes of CF and CC for 75 % filled and fully filled case. This is due to the domination of fluid coupling with structure. So the mode shape and natural frequency is not influenced by boundary condition. Even though some excellent agreement is observed for

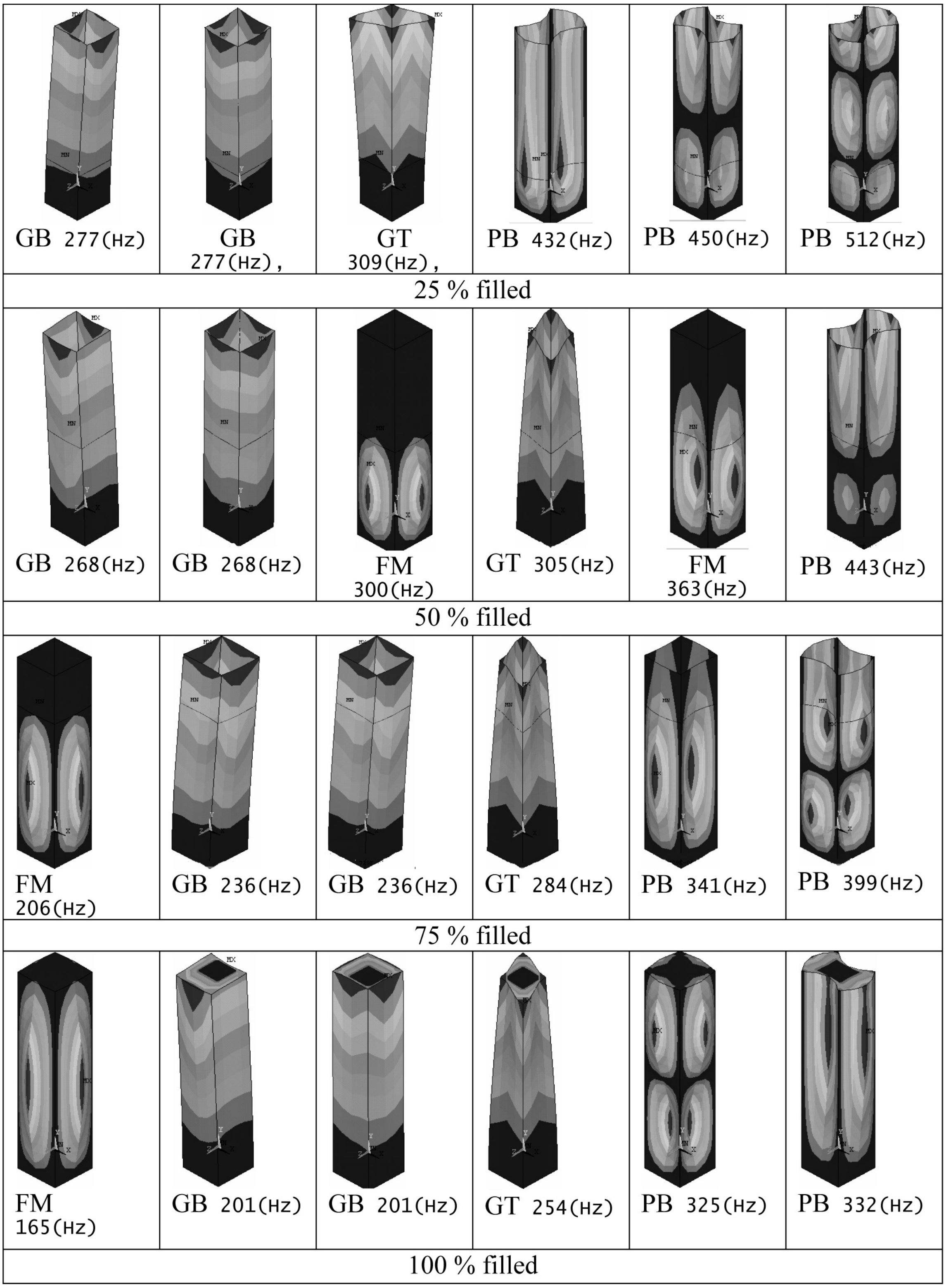


Fig. 13. First six mode shapes of CF box container filled with different level of liquid. (GB-Global bending mode; GT-Global twisting mode; FM- fluid dominating mode; PB-Plate bending mode)

natural frequency between some modes of CF and CC boundary condition, there is a significant change in mode shape due to the influence of boundary condition. For example 5th mode of CF and 2nd mode of CC for 75 % filled case. 5th mode of CF and 2nd mode of CC for fully filled case. It can be clearly seen from Figs. 13, 14, the fluid dominant modes are shifting towards

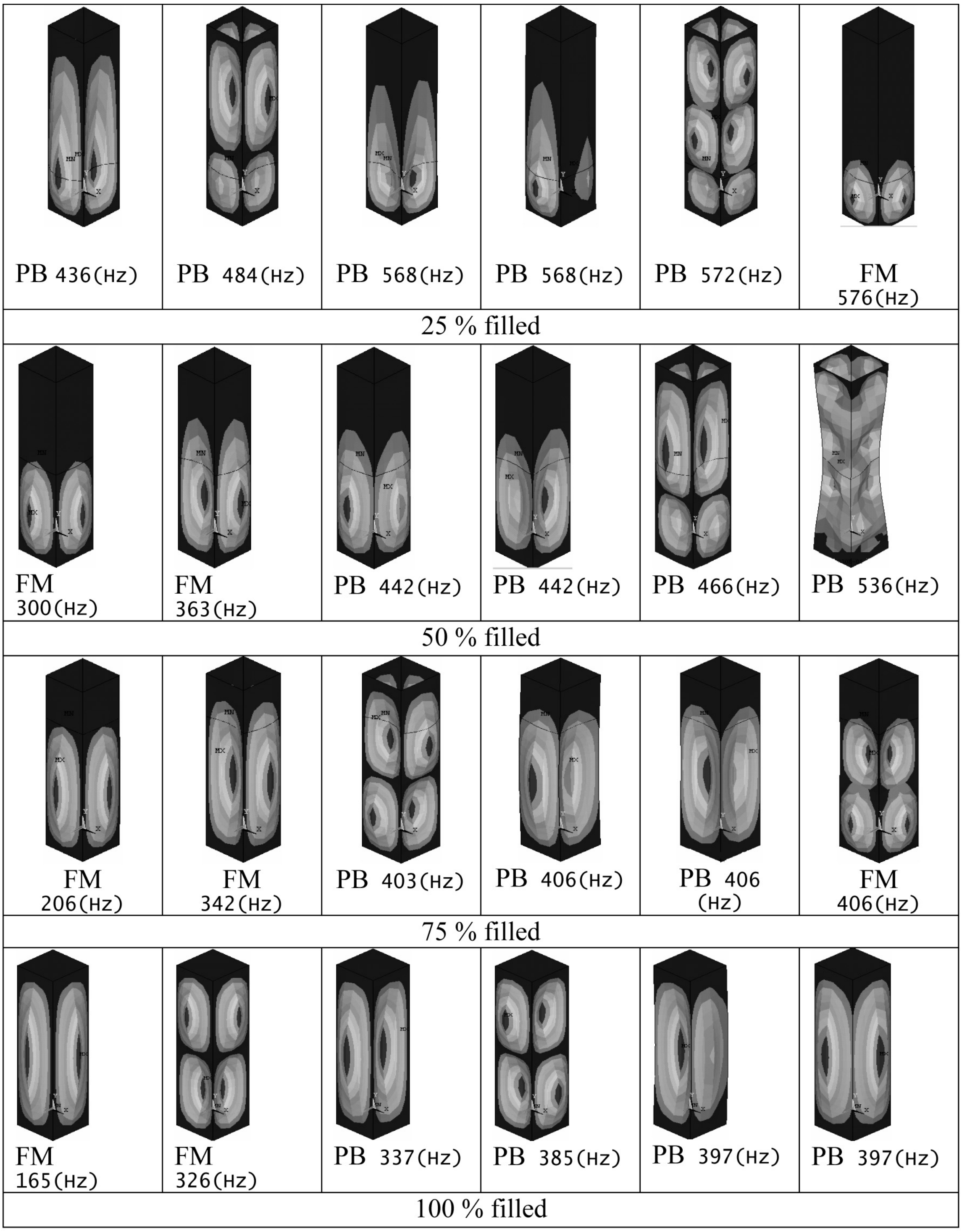


Fig. 14. First six mode shapes of CC box container filled with different level of liquid (GB-Global bending mode; GT-Global twisting mode; FM- fluid dominating mode; PB-Plate bending mode)

lowest natural frequencies as the fluid level increases. It is observed from the mode shapes of CF box that the global modes are not affected by the level of fluid.

A square fluid filled container with level of 0.75 times height of the tank, subjected to a uniform temperature rise has been analyzed to know the influence of temperature on the added mass frequencies of such container under thermal environment. For that a pre-stressed modal analysis is carried out on the 75 % fluid filled container for CC boundary condition by assuming a uniform temperature of 200 °C (this is less than the critical buckling temperature of pure structure which is above 200 °C) and results obtained are tabulated in Table 18. The natural frequencies given in Table 18 are frequencies associated with structural modes, acoustic modes and combination of these structural and acoustic modes. From Table 18, it is also clear that, irrespective of mode type as expected there is a significant reduction in natural frequencies for CC box member due to reduction in stiffness caused by the thermal pre-load.

Table 18. Frequency of both ends clamped box structure under temperature

mode	Natural Frequencies (Hz)	
	at room temp	at 200 °C
1	207	143
2	342	149
3	403	187
4	406	201
5	406	247

3.3.1. Harmonic analysis of partially fluid filled isotropic box structure

The box analyzed in the modal analysis is considered for the harmonic response analysis of fluid filled box also. The level of fluid is varied as 25 %, 50 % and 75 % of length of the box and corresponding variation in vibration and acoustic are analyzed. Fig. 15 show the finite element model of the box with 75 % fluid level for CF and CC boundary condition. A 1N mechanical force is applied at node 122, 82 and 47 for the box filled with 75 %, 50 % and 5 % fluid respectively for both CF and CC boundary condition. The vibration response at the point of excitation and acoustic response at the centre of the acoustic cavity for various levels are obtained to analyze the response characteristics of partially fluid filled box.

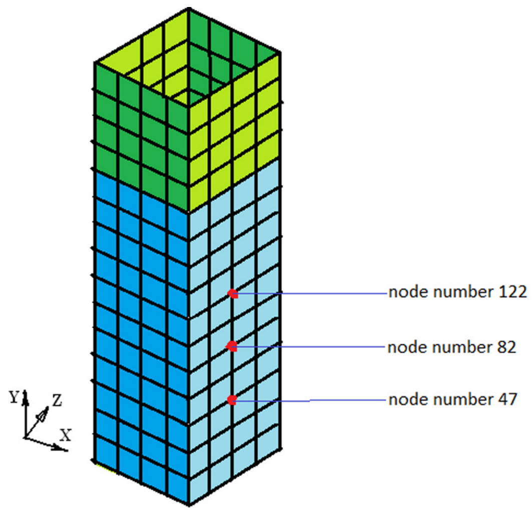


Fig. 15. Finite element model of rectangular box structure with 75 % fuel filled

A structural damping ratio of 0.01 is assumed throughout the analysis. Fig. 16 shows the vibration response for CF and CC boundary condition while Fig. 17 shows the acoustic response for CF and CC boundary condition.

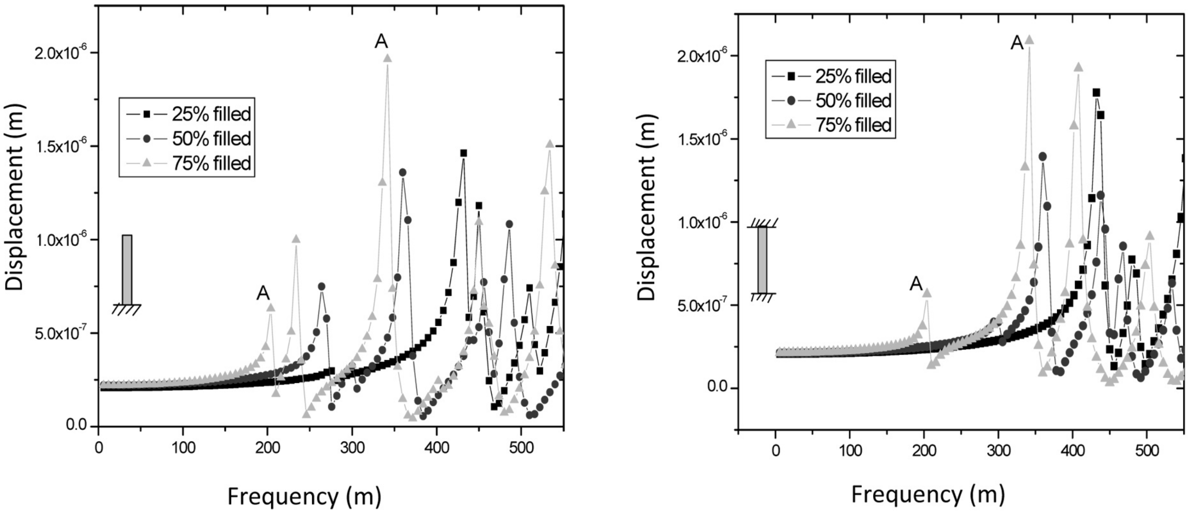


Fig. 16. Vibration responses at the excitation point for CF and CC box structure with 25 %, 50 % and 75 % liquid filled

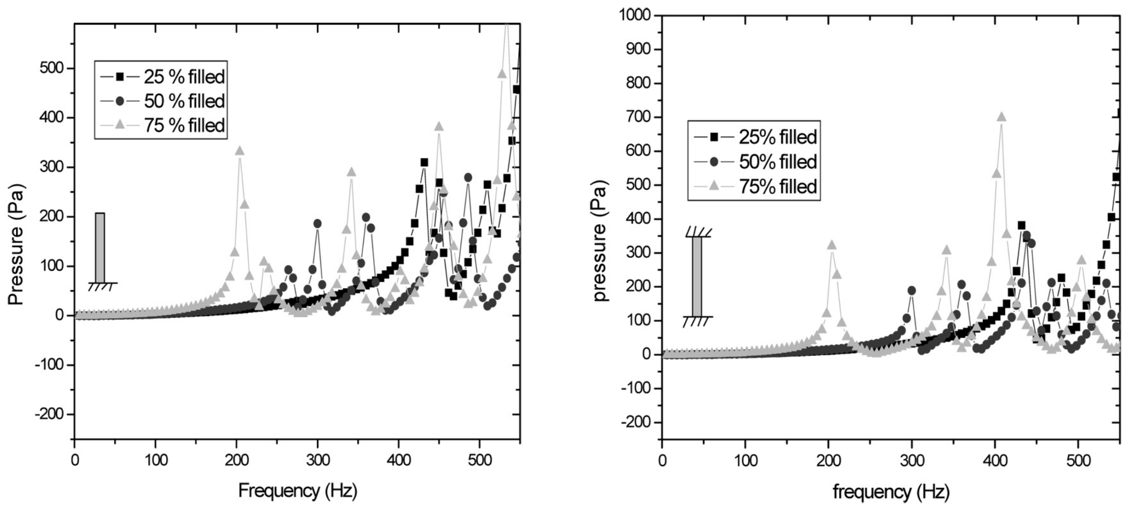


Fig. 17. Acoustic responses at the center of the acoustic cavity for CF and CC box structure with 25 %, 50 % and 75 % liquid filled

From Figs. 16–17, it is clear that the natural frequencies shifts towards lower frequencies as the fluid level increases due to added mass effect. It is also been observed from Fig. 16 that the displacement at frequencies near to fluid dominant mode increases with the level of fluid. Clear peaks can be seen at frequencies near 206 Hz and 340 Hz for 75 % fluid filled case. This indicates the strong fluid coupling with the structure. From Fig. 17, it is observed that the acoustic pressure response level is more in fluid dominating modes compared to structure dominating modes. This indicates the high fluid loading on the structure.

3.4. Fluid filled composite box container

A box container made up of glass/epoxy material and having same dimensions of isotropic box member analyzed in the previous section is considered for the study of fluid filled composite container. Two kind of analysis has been carried on the containers namely (i) modal analysis and (ii) frequency response analysis. The material properties of glass/epoxy composite material are given in the Table 19. The composite box container is assumed to be made up of five layers.

Table 19. Composite material properties

Material	E_{11} [GPa]	E_{22} [GPa]	ν_{12}	$G_{12} = G_{23}$ [GPa]	ρ [kg/m^3]
Glass/epoxy	37.78	10.90	0.3	4.91	1 870

3.4.1. Modal analysis of fluid filled composite box structure

Initially, modal analysis is carried out to find the coupled natural frequencies of fluid filled composite box container. The fluid filled composite box column is analyzed for three different fiber orientations by keeping level of the fluid as a parameter for CC and CF boundary conditions. The results obtained from the modal analysis for CC and CF boundary conditions for various fiber orientations and various fluid levels are shown in Figs. 18–21 and Figs. 22–24, respectively.

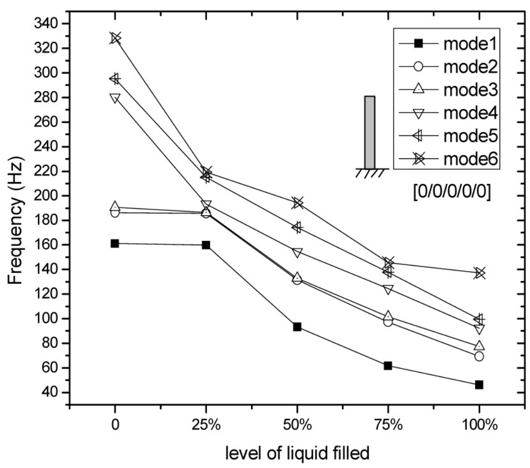


Fig. 18. Natural frequency variations with the level of liquid level for [0/0/0/0/0] lay up for CF box column

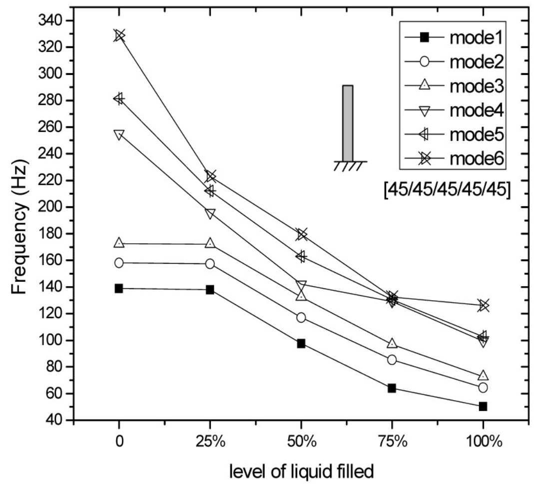


Fig. 19. Natural frequency variations with the level of liquid level for [45/45/45/45/45] lay up for CF box column

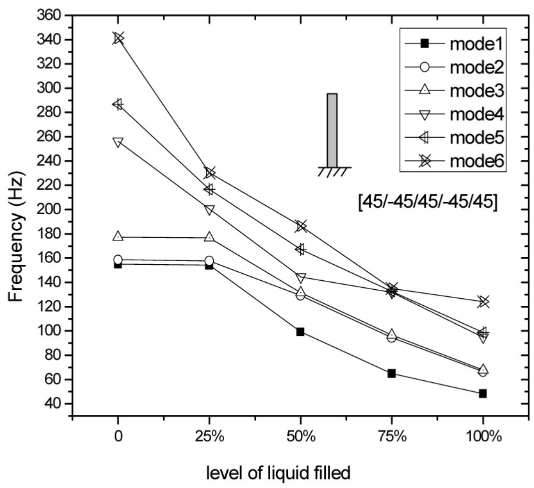


Fig. 20. Natural frequency variations with the level of liquid level for [45/-45/45/-45/45] lay up for CF box column

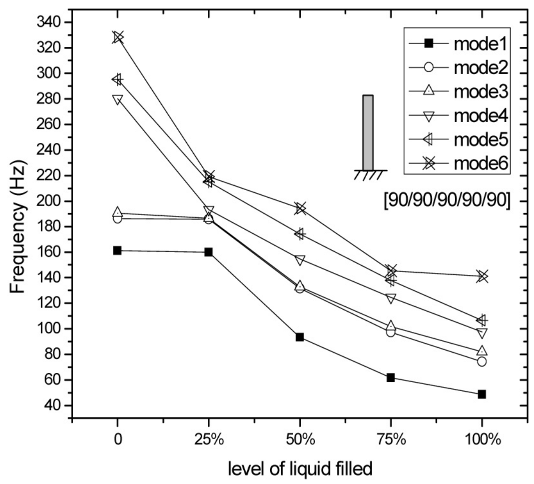


Fig. 21. Natural frequency variations with the level of liquid level for [90/90/90/90/90] lay up for CF box column

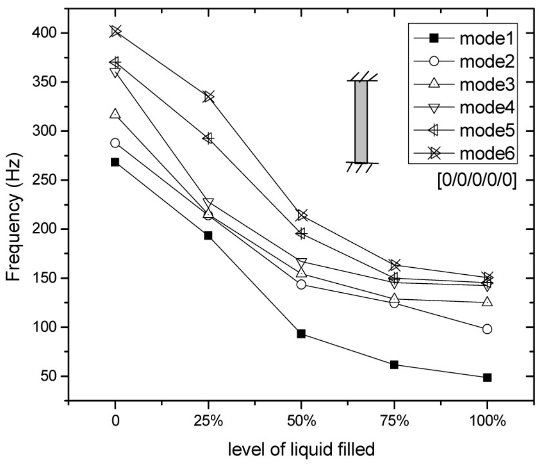


Fig. 22. (a) Natural frequency variations with the level of liquid level for [0/0/0/0/0] lay up for CC box column

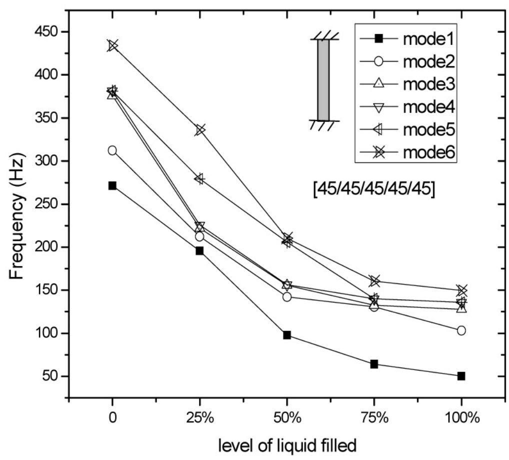


Fig. 23. (b) Natural frequency variations with the level of liquid level for [45/45/45/45/45] lay up for CC box column

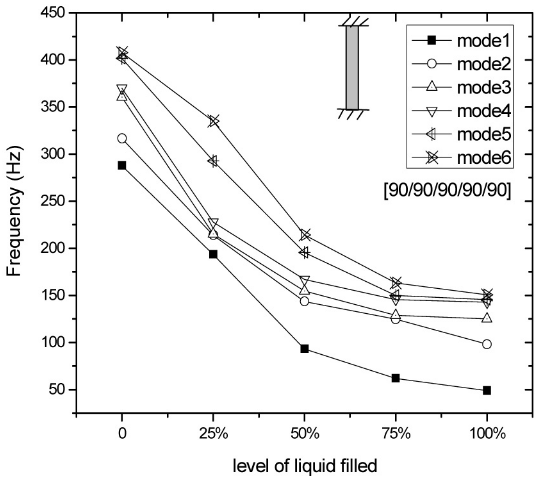


Fig. 24. (c) Natural frequency variations with the level of liquid level for [90/90/90/90/90] lay up for CC box column

From Fig. 18, it is observed that there is a variation of frequencies with increase in level of liquid fill for the higher modes (after 3rd mode) and not for the lower modes (below 3rd mode). As discussed for isotropic box, irrespective of liquid filling, first three modes are global modes for the case of CF case and remaining are local plates modes. The same trend has been observed for various fiber orientations analyzed which are shown in Figs. 19–21.

The influence of stacking sequence and fiber angle is clearly observed in the case of CC which is shown in the Figs. 22–24. In general, from Figs. 18–24, it is clear that the natural frequency reduces with increase in the level of the water irrespective of the fiber orientation and boundary conditions. This is due to increase in added mass with the level of the liquid.

3.4.2. Harmonic response analysis of partially fluid filled composite box structure

A square fluid filled composite box column having fluid level of 0.75 times the height of box, analyzed in the previous sections is considered to analyze the influence of fiber orientations on vibration and acoustic response of fluid-filled composite box column. Composite box column is analyzed for both CF and CC boundary conditions by keeping the same excitation locations as that of isotropic box column. The displacement and acoustic response obtained for different values of fiber orientations for CF and CC boundary conditions of a composite box column filled with water to a level of 0.75 times the height of the column are shown in Figs. 25–28 respectively.

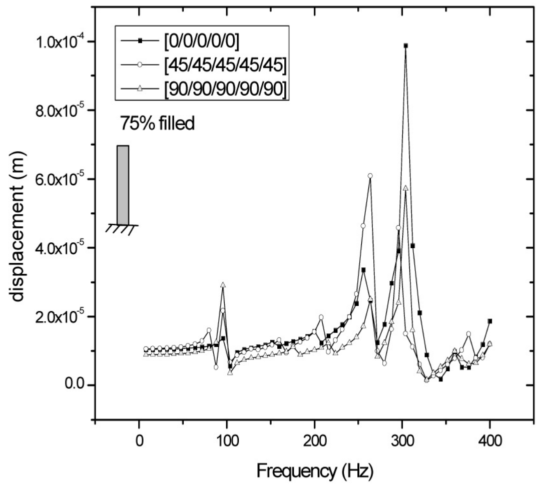


Fig. 25. Displacement at the excitation point for various values of fiber orientation for CF box column

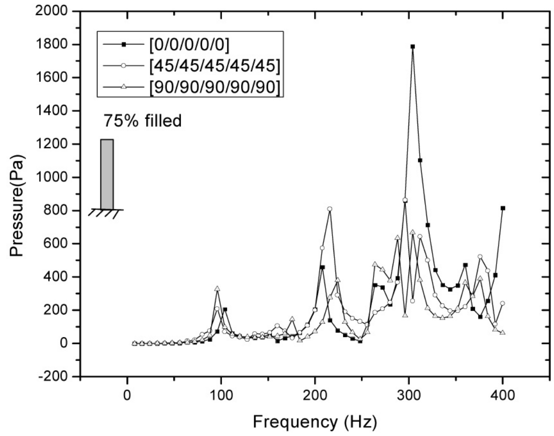


Fig. 26. Acoustic response various values of fiber orientation for CF box column

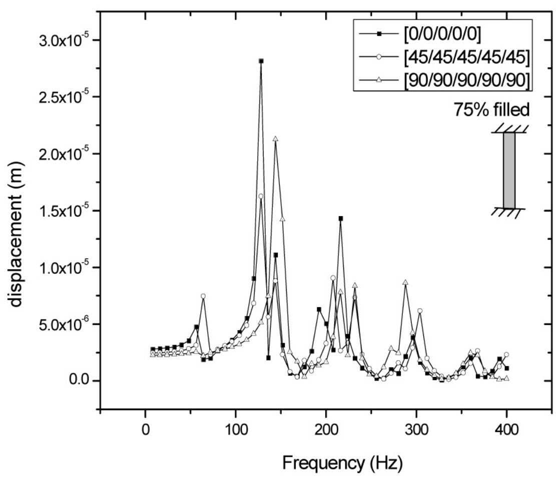


Fig. 27. Displacement at the excitation point for various values of fiber orientation for CC box column

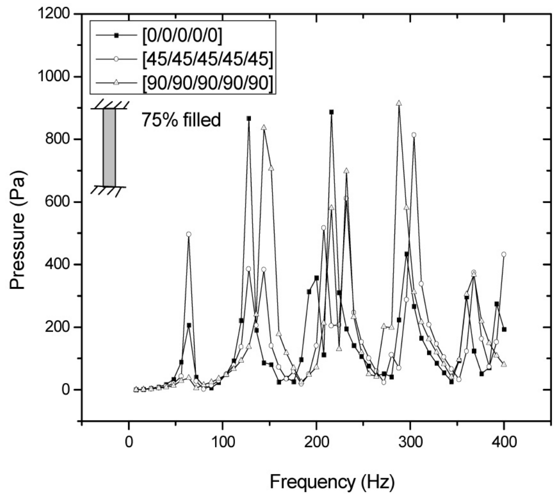


Fig. 28. Acoustic response various values of fiber orientation for CC box column

From Figs. 25–28, it is observed that the fiber orientation influences both the vibration and acoustic response.

4. Summary and conclusions

4.1. Buckling and vibration study

To know the influence of stiffeners on the static and dynamic behavior, the free vibration and thermal buckling analysis has been carried out on the un-stiffened and stiffened thin walled structures by FE solver ANSYS. The accuracy of the present procedure was checked by a comparison with previous numerical results. Extensive numeric results are obtained on the isotropic and composite thin walled structures for Clamped Free (CF) and Clamped Clamped (CC) boundary conditions. In order to study the nature of vibration modes such as local and global, two different lengths are taken in to consideration. From the study the following conclusions can be made on the different cross sections as follows:

- There is a significant increase in thermal buckling temperature when the stiffeners are introduced in the structural columns.
- There is no significant change in the natural frequency of the structural columns when the stiffeners are introduced in the case of clamped – free boundary condition but a considerable amount of change is observed in the natural frequency in case of clamped – clamped boundary condition.
- The influences of stiffeners on the natural frequency of long structural columns are very marginal. The structural columns with $45°$ fiber orientation have more thermal buckling temperature and natural frequency than the structural columns having $0°$ and $90°$ fiber orientation.

4.2. Study on dynamic response of fluid-filled structures

The free vibration and harmonic response analysis have been carried out for isotropic and composite liquid filled containers. The commercial finite element software ANSYS has been used to carry out the analysis. A detailed parametric study has been carried out to analyze the effect of level of liquid filling, boundary conditions, pre-stress and stacking sequences on dynamic behavior of fluid filled containers. The following conclusions are made based on the numerical studies carried out.

- The natural frequencies are reduces with increase in level of liquid fill for both CF and CC conditions as expected and the influence of level of liquid on coupled natural frequencies is more when the tank is fully filled with fluid.
- It is found that the square containers exhibit different types of mode shapes like torsion, bulging, and bending modes. The level of liquid filling in the container has a considerable influence on the mode shapes.

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